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Horizons Inside Classical Lumps*

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ABSTRACT

We investigate the possibility of having horizons inside various classical field configurations. Using the implicit function theorem, we show that models satisfying a certain set of criteria allow for (at least) small horizons within extended matter fields. Gauge and global monopoles and Skyrmions satisfy these criteria. Q-balls and Boson stars are examples which do not and can be shown not to allow for horizons. In examples that do allow for horizons, we show how standard 'no hair' arguments are avoided.

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1. Introduction

Black holes are intriguing objects and worth studying in all their possible varieties. In this paper we will study the possibility of having black holes inside various classical field configurations. Examples we consider include gauge and global monopoles, Skyrmions, Q-balls [1], and Boson stars [2].

Besides the basic search for black hole solutions, there are a number of physically motivated questions one can ask in this context. For instance, what happens when you drop such an object into a Schwarschild black hole? For a gauge or global monopole the result should be a black hole with the appropriate kind of hair, since these both involve non-trivial behavior of the fields at infinity. But in the case of a Skyrmion, one might think that the only possibility would be its vanishing without a trace. Our results show that there is another possibility, at least for horizons very small compared to the Skyrmion radius†. In the case of gauge monopoles, Lee et.al. [7] have argued that, besides the Reissner-Nordstrom type solutions [8], there also exist, for sufficiently small horizon radius, solutions in which the Higgs field and gauge field behave more like an extended monopole outside the horizon. Our results confirm their arguments, and show that global monopoles can also have horizons inside them.

Horizons inside extended field configurations may also be relevant in the late stages of black hole evaporation by Hawking radiation. Lee et.al. [9] have shown that extreme, magnetic Reissner-Nordstrom type black holes are unstable in a theory with extended monopole solutions. They conjecture that the extended solution discussed above is stable and that evaporation of the black hole proceeds through this configuration, leaving a non-singular magnetic monopole as the end state. Perhaps a Schwarschild black hole in a Skyrmion theory, for example, similarly becomes unstable (or metastable) when its radius is less than the characteristic

† numerical results on extended Skyrmion fields around a black hole are given in references [3,4,5,6].

Skyrmion radius. The evaporation process may then leave behind other stable remnants.

Finally, in the literature Q-stars (large Q-balls) [10,11] and Boson stars (see [12] and references therein), as well as strange matter [13] and other types of non-topological solitons, are discussed as candidates for compact astrophysical objects. We can ask what the possible final collapsed states of such matter are.

2. Existence of Solutions with Horizons

We will be looking for static, spherically symmetric solutions to Einstein's equation, which have nonsingular, nontrivial matter fields outside a horizon. The form of the metric will be taken to be

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega^2 \tag{2.1}$$

It is often convenient to define the function $m(r)$ by

$$\frac{1}{A(r)} = 1 - \frac{2Gm(r)}{r}. \tag{2.2}$$

A horizon occurs at coordinate r_H if

$$2Gm(r_H) = r_H. \tag{2.3}$$

When a horizon is present, one also expects that $m_o \equiv m(0) \neq 0$, so that the metric is not well behaved at the origin. This is like having a seed mass at the origin.

Let us agree to call a *star* a configuration of matter fields ϕ (not necessarily scalar) such that the stress-energy is static, spherically symmetric, and localized. Suppose a particlar matter field theory has star type solutions, without gravity. There is some force balance, without gravity, which keeps the field configuration from either collapsing to a point or expanding to infinity. One might expect that weakly gravitating solutions would then exist, and that even placing a small seed mass inside the star, wouldn't disturb the balance too much. This can be made more precise by considering the Oppenheimer-Volkoff (OV) equation of hydrostatic equilibrium, which states (in the case when the three principal pressures are not necessarily the same)

$$\frac{dp_{\hat{r}}}{dr} = -G\frac{(m(r) + 4\pi r^3 p_{\hat{r}})}{r(r-2Gm)}(\rho + p_{\hat{r}}) + \frac{2}{r}(p_{\hat{\phi}} - p_{\hat{r}}) \tag{2.4}$$

In the absence of gravity, only the second term on the right hand side is present and for weak gravity, this term may still dominate. However, from the first term in (2.4), we see that at a horizon the sum of the radial pressure and the energy density must vanish. In a normal, burning star, both these quantities are positive and a horizon is not possible. On the other hand for many field theories, it happens quite naturally that $(\rho + p_{\hat{r}})|_{r_H} = 0$.

Our main result will be to show that given a matter theory which (1) has star type solutions without gravity and (2) satisfies $(\rho + p_{\hat{r}})|_{r_H} = 0$ "automatically" (in a sense defined below), then there exist star solutions when the matter theory is coupled weakly to gravity, and there also exist solutions with horizons inside.

More precisely, the non-gravitating matter theory is described by a Lagrangian L_m. A star solution is found by evaluating the action on field configurations consistent with a particular static, spherically symmetric ansatz. The Lagrangian restricted to this class of fields will be written $L_m(\phi)$. We will assume that $-L_m(\phi)$ is positive definite. When the matter theory is coupled to gravity, we will assume that the sum of the energy density and radial pressure is given by

$$(p_{\hat{r}} + \rho) = \frac{1}{A}K(\phi), \tag{2.5}$$

Where K is a functional of the matter fields only. Then there exist regular star type solutions to the Einstein equation, and there also exist star type solutions horizons, which have nontrivial, nonsingular matter fields outside the horizon, for G and r_H sufficiently small. The argument, as follows, is an application of the implicit function theorem.

First define new gravitational variables,

$$e^x = \sqrt{\frac{B}{A}} \qquad \text{and} \qquad e^y = \sqrt{AB}. \tag{2.6}$$

The action for fields outside a horizon is then taken to be $S = \tilde{S}_E + S_m$, with

$$\begin{aligned} \tilde{S}_E(x,y) &= -\frac{1}{8\pi G}\int_{r_H}^{\infty} dr y'((r-r_H)e^y - re^x) \\ S_m(\phi_n, x, y) &= \int_{r_H}^{\infty} dr r^2 e^y L_m \end{aligned} \tag{2.7}$$

$\tilde{S}_E$ differs from the usual Einstein action by a boundary term, which has been chosen so that varying $\tilde{S}_E$ imposes the correct boundary condition at the horizon (see reference [14]). Varying the action with respect to x and y gives the equations of motion

$$y' = -8\pi G r e^{y-x}\frac{\delta L_m}{\delta x} \tag{2.8}$$

$$\frac{d}{dr}(r(e^y - e^x)) = -r^2 e^y (e^{y-x}\frac{\delta L_m}{\delta x} + L_m + \frac{\delta L_m}{\delta y}) \tag{2.9}$$

and the boundary condition

$$re^x|_{r_H} = r\sqrt{\frac{B}{A}}\,|_{r_H} = 0. \tag{2.10}$$

Note that from the definition of the stress tensor $-2\frac{\delta L_m}{\delta x} = p_{\hat{r}} + \rho$. Equations (2.8) and (2.9) can be used to solve for the gravitational fields x and y in terms of the matter fields alone if and only if $p_{\hat{r}} + \rho = \frac{1}{A}K(\phi)$, where K is a function of the matter fields alone. This was one of our assumptions. This is equivalent to the matter lagrangian having the form

$$L_m(\phi) = -\frac{1}{A}K(\phi) - U(\phi, AB) \tag{2.11}$$

We can then define a positive definite functional of the fields, $E(\phi, G, r_H)$, by

$$E(\phi; G, r_H) = -S = \int\limits_{r_H}^{\infty} dr e^y (r(r - r_H)K + r^2 U) \tag{2.12}$$

In (2.12), $y(r)$ is given in terms of the matter fields by

$$y(r) = -8\pi G \int\limits_{r}^{\infty} dr' r' K(\phi) \tag{2.13}$$

Note that for a given configuration of the fields ϕ, $E(\phi, G, r_H)$ is a continuous, differentiable function of G and r_H.

We assume that for $G = r_H = 0$, the functional $E(\phi, 0, 0)$ has a minimum $\bar{\phi}_o$. This is our non-gravitating star. For G and r_H nonzero, we seek solutions $\bar{\phi}$ to

$$F((\bar{\phi}; G, r_H)) \equiv \frac{\delta E}{\delta \phi} = 0, \qquad (2.14),$$

which by construction will satisfy the equation of motion with the correct boundary conditions. By assumption $F(\bar{\phi}_o; 0, 0) = 0$. The implicit function theorem for Banach spaces⋆ [15] can then be used to show that for G and r_H sufficiently close to zero, there exist functions $\bar{\phi}(G, r_h)$ satisfying (2.14), such that $\bar{\phi}(0, 0) = \bar{\phi}_o$. This can be seen by expanding (2.14),

$$0 = \frac{\delta F}{\delta \phi} \cdot (\bar{\phi} - \bar{\phi}_0) + \frac{\partial F}{\partial G} \cdot G + \frac{\partial F}{\partial r_H} \cdot r_h + \ldots, \tag{2.15}$$

with all the derivatives evaluated at $\phi = \bar{\phi}_0$, $G = 0$, and $r_H = 0$. There will be a solution for $\bar{\phi}$ as long as the operator $\frac{\delta F}{\delta \phi}$ in (2.15) is an isomorphism between

⋆ In the appendix we sketch a finite dimensional version of the theorem, which illustrates the relevant points.

two Banach spaces H_1 and H_2, and the two functions $\frac{\partial F}{\partial G}$ and $\frac{\partial F}{\partial r_h}$ belong to the space H_2. The choice of particular function spaces depends on the system under consideration. However, roughly speaking, we can see that this will be true in general given that the flat space solution $\bar{\phi}_0$ is a minimum of the energy functional (2.12) , which is equivalent to

$$\left.\frac{\delta F}{\delta \phi}\right|_{(\bar{\phi}_0,0,0)} \cdot \delta\phi > 0. \tag{2.16}$$

Hence $\frac{\delta F}{\delta \phi}$ has no zero modes and is invertable. In the next section we indicate how to choose appropriate function spaces for global monopoles.

The OV equation implied that $(p_{\hat{r}} + \rho) \propto \frac{1}{A}$ at a horizon. Above, we found that this same condition was needed to integrate out the metric coefficients A and B from the action. This allowed us to use the existence of non-gravitating solutions to imply via the implicit function theorem the existence of gravitating solutions and solutions with horizons. If we take a theory, such as Q-balls, in which, as we will see below, A and B cannot be eliminated from the action, then to use the implicit function theorem, one would have to compute the variation including all the dependent functions, ϕ, A and B. But knowledge of the flat space solutions gives us no information analogous to (2.16) about variations in the A or B directions, so the argument can't proceed.

3. Global Monopoles

In this section we demonstrate the use of the implicit function theorem and selection of appropriate function spaces for global monopoles. The matter field theory for the basic global monopole is given by an $SO(3)$ invariant Lagrangian for a triplet of scalar fields ϕ^a,

$$\mathcal{L} = \tfrac{1}{2}\nabla^\mu \phi^a \nabla_\mu \phi_a - \tfrac{1}{2}\lambda \left(\phi^a\phi_a - v^2\right)^2, \tag{3.1}$$

where ∇_μ is the covariant derivative operator. The scalar field configuration for the monopole has the spherically symmetric form

$$\phi^a = v\phi(r)\hat{r}^a. \tag{3.2}$$

For solutions without horizons $\phi(r)$ interpolates between 0 at the origin and 1 at infinity. Evaluated on such field configurations (with the covariant derivative operator appropriate for the spherically symmetric metric (2.1)) the lagrangian has the form $L_m = \frac{1}{A}K + U$, where the kinetic and potential terms are given by

$$K = \frac{1}{2}v^2\phi'^2, \qquad U = \frac{v^2\phi^2}{r^2} + \frac{1}{2}\lambda v^4(\phi^2 - 1)^2, \tag{3.3}$$

Here $\phi' = d\phi/dr$. The equations of motion for the metric coefficients are

$$m'(r) = 4\pi r^2(\frac{1}{A}K + U), \qquad \frac{(AB)'}{(AB)} = 16\pi G r K. \tag{3.4}$$

The flat space global monopole solution has the following asymptotic behavior

$$\bar{\phi}_0(r) \sim \begin{cases} ar, & r \to 0; \\ 1 - \frac{1}{2\lambda v^2 r^2}, & r \to \infty, \end{cases} \tag{3.5}$$

where a and b are constants (the slope a at the origin must be determined numerically). From (3.5) and (3.3), one can see that the energy density for the global monopole falls off only as $1/r^2$, so that the total energy of a global monopole diverges,

$$\lim_{r\to\infty} m(r) = 4\pi v^2 r. \tag{3.6}$$

Hence the spacetime of a global monopole is not asymptotic to flat spacetime, but rather to flat spacetime minus a missing solid angle [16],

$$\lim_{r\to\infty} \frac{1}{A} = 1 - 8\pi G v^2. \tag{3.7}$$

In order to avoid a horizon at large radius (which is not of the sort we are interested in), we will keep $8\pi G v^2 < 1$.

The quantity $\frac{\delta F}{\delta \phi}$ in (2.15) for the global monopole is given by

$$\frac{\delta F}{\delta \phi}\delta\phi = -\frac{d}{dr}\left(r^2\frac{d}{dr}\delta\phi\right) + \left(2 + r^2\left[6\bar{\phi}^2 - 2\right]\right)\delta\phi \tag{3.8}$$

Here we have rescale lengths by a factor $\sqrt{\lambda v^2}$. The variations $\frac{\partial F}{\partial G}$ and $\frac{\partial F}{\partial r_h}$ evaluated on the background solution can be seen to have the forms

$$\frac{\partial F}{\partial G} \sim \begin{cases} r, & r \to 0; \\ \frac{1}{r^2}, & r \to \infty, \end{cases} \qquad \frac{\partial F}{\partial r_h} \sim \begin{cases} const, & r \to 0; \\ \frac{1}{r^3}, & r \to \infty. \end{cases} \tag{3.9}$$

If we take the variation $\delta\phi$ to have the assymptotic behavior

$$\delta\phi \sim \begin{cases} const, & r \to 0; \\ \frac{1}{r^4}, & r \to \infty, \end{cases} \tag{3.10}$$

(with the standard L^2 norm in three dimensions), then we can accomodate the variations induced by (3.9). This can be seen by examining the asymptotic behavior of $\frac{\delta F}{\delta \phi}$ in (3.8). We then have to show that the operator $L = \frac{\delta F}{\delta \phi}$ is an isomorphism between these spaces. Since the operator is elliptic, this will be the case if neither it nor its adjoint have zero modes. Suppose that L has a zero mode, then we can write

$$0 = \int_0^\infty dr \left\{ -f\frac{d}{dr}\left(r^2\frac{d}{dr}f\right) + r^2\frac{\delta^2 U}{\delta\phi^2}f^2 \right\}. \tag{3.11}$$

Integration by parts yields

$$0 = -r^2\, f\frac{d}{dr}f\bigg|_0^\infty + \int_0^\infty dr r^2 \left\{ (\frac{d}{dr}f)^2 + \frac{\delta^2 U}{\delta\phi^2}f^2 \right\}. \tag{3.12}$$

The boundary term vanishes for functions f having the behavior (3.10). Equation (3.12) then leads to a contradiction if

$$\frac{\delta^2 U}{\delta\phi^2}\bigg|_{(\bar{\phi}_0,0,0)} \geq 0 \tag{3.13}$$

holds everywhere. We have checked numerically that (3.13) is satisfied for the flat space monopole. Therefore the operator L, which is self-adjoint has no zero-modes.

4. Examples

Three examples of field configurations which allow horizons inside are Skyrmions, gauge monopoles, and global monopoles. These three examples span a range of types: gauge monopoles have both a long range magnetic field and topological winding, global monopoles have only the topological constraint, and the Skyrmion field winds but is not topological. These all have $L_m(\phi)$ of the form (2.11), and so satisfy the condition $(\rho + p_{\hat{r}})|_{r_H} = 0$ at a horizon. The implicit function theorem argument shows that solutions with hair exist for G and r_H in some range about zero, but gives no information about how large this range is. One can deduce more information about the range from arguments based on the traditional positive 'no-hair' integrals, which we do below in Section 5.

Field configurations which cannot support horizons include Q-Balls [1] and boson stars [2]. Q-Balls are star type configurations that exist without gravity [1], but, as we will see, fail to satisy the condition $(\rho + p_{\hat{r}})|_{r_H} = 0$ at a horizon. The simplest Q-balls occur in the theory of a single complex scalar field [1]. The Q-ball field has the form $\phi = f(r)e^{-i\omega t}$ where $f(r)$ vanishes at infinity. The lagrangian evaluated on such configurations is

$$L_Q = \frac{1}{2A}(f')^2 + \frac{1}{2}(m^2 - \frac{\omega^2}{B} f^2) + U(f^2), \tag{4.1}$$

where the mass-term in the potential has been separated out. The frequency ω must satisfy $\omega^2 > m^2$ for stability. From the definition of the stress tensor we then have

$$p_{\hat{r}} + \rho = -\frac{2}{A}\frac{\delta L_m}{\delta 1/A} + \frac{2}{B}\frac{\delta L_m}{\delta 1/B} = -\frac{1}{A}(f')^2 - \frac{1}{B}\omega^2 f^2. \tag{4.2}$$

We see that to satisfy $(\rho + p_{\hat{r}})|_{r_H} = 0$, f must vanish at a horizon⋆. But this means that the field is in its vacuum both at the horizon and at infinity, which is not a Q-Ball type solution.

⋆ We assume that the volume element $\sqrt{AB}$ is well behaved at a horizon, which implies that $B \sim r - r_H$ near the horizon.

Boson stars (see [12] for a review) are localised scalar field configurations which exist *only* with gravity. The matter lagrangian again has the form (4.2) (with different potential terms and with $\omega^2 > m^2$). Hence Boson stars satisfy $(\rho + p_{\hat{r}})|_{r_H} = 0$ only for $f(r_H) = 0$, implying again that the field be in its vacuum at the horizon, as well as at infinity.

A third example which probably does not allow hair is the Abelian-Higgs model [17]. If the scalar field has the form $f(r)$ and the gauge field is given by $A_t(r)$, then the matter lagrangian is

$$L_{AH} = \frac{1}{2A}(f')^2 - \frac{1}{AB}(A_t')^2 - \frac{1}{2B}e^2(A_t)^2 f^2 + \frac{\lambda}{2}(f^2 - v^2)^2 \tag{4.3}$$

This again is not of the form (2.11), and satisfying $(\rho + p_{\hat{r}})|_{r_H} = 0$ requires that $A_t^2 f^2 = 0$ at $r = r_H$. While this in itself is not enough to rule out solutions, it clearly makes it "harder" to satisfy the equations given this additional condition on the fields. Indeed, the 'no-hair' integrals discussed in section 5 further imply that if $A_t(r_H) = 0$, then the fields are in their vacuum states everywhere outside the horizon. Adler and Pearson [17] explicitly analyzed the Einstein equation for this system further, and have shown that this is indeed the case.

Finally, it is interesting to think about the case of a Coulombic electric field due to a point charge. This is outside the framework of the present discussion, because the non-gravitating configurations are singular, $A_t = q/r$. However, the Reissner-Nordstrom charged black holes *are* solutions with nonzero, nonvacuum, regular matter fields outside the horizon†. In this case, it is easy to check that the E&M Lagrangian reduces to

$$L_{EM} = \frac{1}{AB}(A_t')^2 \tag{4.4}$$

which has the form (2.11) and that, in fact, the combination $p_{\hat{r}} + \rho$ vanishes everywhere.

† Visser [18] has independently studied the condition $(\rho + p_{\hat{r}})|_{r_H} = 0$ in the context of various recent black hole solutions in field theories, such as dilatons and axions, coupled to gravity. He has also looked at the thermodynamics of such solutions.

In looking at these various examples, one notices that different kinds of mass terms play quite different roles. A "true" mass, or any potential U which is independent of the metric, makes no contribution to the sum $p_{\hat{r}}+\rho$, as in Inflation. A dynamical mass which comes from the coupling to the time component of a gauge potential, contributes a term to $p_{\hat{r}}+\rho \propto \frac{1}{B} f^2 A_t^2$, which tends to rule out hair. A dynamical mass which comes from coupling to the spatial components of a gauge field contributes zero, and contributes a winding term $\propto \frac{1}{r^2}$ to $p_{\hat{\phi}}-p_{\hat{r}}$, which is important in the OV equation (2.4).

5. 'No-Hair' Integrals

It is interesting to see how the black hole solutions discussed above avoid being ruled out by standard 'no-hair' arguments. In the case of extended gauge monopole solutions, this was discussed in ref. [7]. We will see that Skyrmions and global monopoles escape in basically the same way. Necessary conditions for the existence of black hole solutions in a given field theory can be derived by constructing energy integrals from the equations of motion (see e.g. [17,19]). If the action in the region outside the horizon is given by

$$S=-\int_{r_H}^{\infty} dr J(r), \tag{5.1}$$

an extremum occurs when

$$\frac{d}{dr}\frac{\delta J}{\delta \phi'}=\frac{\delta J}{\delta \phi}, \tag{5.2}$$

with the boundary conditions $\delta J/\delta\phi' = 0$ at $r=r_H$ and the fields going to their vacuum values at infinity. Therefore

$$\int_{r_H}^{\infty} dr \left[\phi' \frac{\delta J}{\delta \phi'} + (\phi-\phi_\infty)\frac{\delta J}{\delta \phi}\right] = (\phi-\phi_\infty)\left.\frac{\delta J}{\delta \phi'}\right|_{r_H} = 0 \tag{5.3}$$

Consider the case at hand (2.12), where $S=-E$ and J is the positive definite integrand. Since we are assuming that regular solutions exist when $G=r_H=0$, the above is true with $r_h = 0$ and $e^y \equiv 1$ in J. Since typically the gradient term in the integrand is of the form $C^2(\phi)(\phi')^2$, this requires that as r ranges from zero to ∞, there are positive and negative contributions to the potential (the second) term in the integrand. Now, if the lower limit is taken to be r_H, there is still a possibility for positive and negative contributions to sum to zero above, if r_H is small enough. This point was discussed in [7] in reference to gauge monopoles, noting that the fields had to be Reissner-Nordstrom outside the horizon if r_H were sufficiently large. For Skyrmions, the structure of the no-hair integrals depends on what the response is of the Skyrmion field to gravity. But assuming that the effect of gravity is to further concentrate the energy density, again there will be a critical value of r_H, such that if the horizon is larger, the field must be in its vacuum outside the horizon. On the other hand, global monopoles have no such restriction on the value of r_H.

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APPENDIX A

Here we recall the arguement for the implicit function theorem for a system of N equations in N unknowns, and the limit as N becomes a continuous variable. Let g be the independent variable, and π , $i = 1, ..., N$ be N dependent variables. (These are numbers, not functions.) We seek solutions $\pi = \bar{\phi}_i(g)$ to the system

$$F_j(\pi, g) = 0 \ , j = 1, ..., N \ , \tag{A.1}$$

given that $\bar{\phi}_{io}$ is a solution when $g = 0$, $F(\bar{\phi}_{io}, 0) = 0$. Let $\pi - \bar{\phi}_{io} = \delta\pi$ and denote the matrix of first derivatives with respect to the independent variables by $O_{ji} = -\frac{\partial F_j}{\partial \pi}$, evaluated at $\bar{\phi}_{io}, g = 0$. Then Taylor expanding the equation $F = 0$, to linear order one needs to solve

$$O_{ji}\delta\pi = -\frac{\partial F_j}{\partial g} \cdot g \tag{A.2}$$

There is a solution $\delta\pi$ for any "source" on the right hand side of (A.2) if the matrix O_{ij} has no zero eigenvectors, i.e.,

$$O_{ij}v^i v^j \neq 0 \text{ , for all } v^i \tag{A.3}$$

For an implicit functional theorem, we would like the limit where the discrete index i becomes a continuous variable x, with $F_j \to F(x)$, $\pi \to \phi(x)$. Let $\{P_i(x)\}$ be a set of basis functions, and let $\phi(x) = \Sigma_i A_i P_i(x)$ and $\delta\phi(x) = \Sigma_i \delta A_i P_i(x)$. Then in this limit,

$$\Sigma_i \frac{\partial F_j}{\partial \pi}\delta\pi \to \int dy \frac{\delta F(x)}{\delta\phi(y)}\delta\phi(y) = \Sigma_i \frac{\delta F}{\delta A_i}\delta A_i. \tag{A.4}$$

Hence for a solution one needs that this last quantity, evaluated at the known solution, has no zero modes. In the main part of the paper, this condition was met since the second variation of the energy functional was nonzero, at the non-gravitating solutions.

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