Corner Solutions in Duality Models: A Cross-Section Analysis of Dairy Production Decisions

Robert D. Weaver and Daniel A. Lass

Corner solutions are often observed in cross-section samples of farm-level production decisions. An estimation strategy is presented and applied to a uniquely comprehensive data set for Pennsylvania dairy farms. A complete set of choice functions is derived consistent with multiple outputs and multiple inputs, expected profit maximation, and the existence of corner solutions with respect to the labor hiring decision. Results illustrate that substantial estimation bias may occur if the existence of corner solutions is not recognized. Estimated elasticities of choices with respect to input and net output prices indicate substantial responsiveness of choice to price. Results indicate that changes in education and acreage operated result in substantial changes in output and input mixes and that the differences in results for farms, with and without hired labor, are substantial.

Key words: corner solutions, distributional effects, duality, milk supply.

Although all farmers may face common technological possibilities, variations in prices and fixed factor flows lead each farmer to different choices. In fact, some farmers may find corner solutions optimal and not use (or produce) particular inputs (or outputs). Corner solutions are often observed in cross-sectional samples of farm budget data reporting revenues, expenses, and various farm characteristics. In time series where data are aggregated across individuals, zero output or input levels are obscured by the process of aggregation. While the same result could occur through aggregation across products in cross-sectional data, corner solutions often remain.

The primary objective of this paper is to present an estimation strategy for cross-sectional data sets that describe economic behavior where corner solutions are observed. In the process of presenting an estimation strategy, the effects of ignoring these corner solutions (either through dropping those observations from the sample or by ignoring the occurrence of zeroes in estimation) will be apparent. Methods introduced by Heckman and by Lee, Maddala, and Trost will be extended to estimate a seemingly unrelated system of equations.

Design of effective dairy policy requires knowledge of short-run elasticities of output supply and input demand by dairy farmers. To illustrate the importance of recognizing corner solutions as well as the potential usefulness of farm record data sets, the estimation strategy is applied to a cross-section of data for Pennsylvania farmers and a complete set of short-run elasticities of production choices is presented which is consistent with the hypotheses of (*a*) short-run expected profit maximization, (*b*) multiple output and multiple input technology, and (*c*) existence of fixed input flows. In addition, the estimated results are used to analyze the effects of changes in two types of fixed factors on the relative utilization of variable inputs. The first factor is scale of crop production as measured by crop acreage, which is an important target of federal government intervention to control crop production. Following Weaver (1978), changes in acreage controls may induce cross-commodity distortions of input use and

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Robert D. Weaver is an associate professor, Department of Agricultural Economics and Rural Sociology, The Pennsylvania State University, and Daniel A. Lass is an assistant professor, Department of Agricultural and Resource Economics, University of Massachusetts.

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output supply. A second factor of production considered is operator characteristics. Griliches and, more recently, Lopez have corroborated the role of operator characteristics as measures of stocks of human capital services that affect agricultural production decisions.

Results reported here demonstrate that cross-sectional data presenting a complete account of revenues, expenses, prices, and fixed factor flows can be useful in modeling farm production decisions and their response to market- or policy-originating changes in prices or fixed factor flows.

Theoretical Model

The theoretical foundation of a model of production decisions, which is applied to cross-section data, must explicitly incorporate a behavioral hypothesis which recognizes the possibility of corner solutions for some inputs or outputs. In the data set analyzed here, only 64% of the farms employed hired labor. For those farms which did not use hired labor, no data for the wage rate are available, and continuous relationships do not exist between observed hired labor (definitionally zero) and other choices, and the market wage rate for hired labor. The multiple output, multiple input profit function presented in Weaver (1982, 1983) was adapted to consider this problem.

Assume firms base their production decisions on the solution of the following choice problem:

$$\max \Pi = PY' - RX' \quad \text{s.t. } F(Y, X, \Theta) = 0,$$

where P and Y are $1 \times m$ vectors of expected net output prices and levels; R and X are $1 \times n$ vectors of variable input prices and quantities flows; Θ is a $1 \times p$ vector of fixed input service flows; Π is short-run profits or, equivalently, Ricardian rents available as returns to Θ; and $F(\cdot)$ is a production technology satisfying the usual neoclassical properties.

Suppose interior solutions are optimal for all choices except X_n. The following Kuhn-Tucker conditions provide the basis for deriving different sets of choice functions and associated expected profit functions, depending upon the occurrence of corner solutions.

$$(1) \qquad P_i + \mu \partial F/\partial Y_i = 0 \quad Y_i^* > 0 \quad i = 1, \ldots, m,$$

$$(2) \qquad -R_h + \mu \partial F/\partial X_h = 0 \quad X_h^* > 0 \quad h = 1, \ldots, n-1,$$

$$(3) \qquad (-R_n + \mu \partial F/\partial X_n)X_n = 0 \quad X_n^* > 0, \qquad (-R_n + \mu^* \partial F/\partial X_n) \leq 0 \quad X_n^* = 0,$$

$$(4) \qquad F(Y, X, \Theta) = 0 \quad \mu^* > 0.$$

Depending on the value of X_n, (1)–(4) define two distinct sets of continuous choice functions written in implicit form. Recognizing each set is homogenous of degree zero in prices, we solve each set for explicit choice functions, and by substitution, the normalized expected profit function (NEPF). The derivative property links choices to respective elements of the gradient vector of the NEPF.

Summarizing these statements for $X_n^* > 0$:

$$(5) \qquad \pi^* = \Pi^*(\tilde{P}, \tilde{R}; \Theta, X_n^* > 0) = \pi^*(P, R; \Theta, X_n^* > 0)/P_1,$$

$$(6) \qquad Y_i^* = \partial \pi^*/\partial \tilde{P}_i = \partial \Pi^*/\partial P_i = Y_i^*(\tilde{P}, \tilde{R}; \Theta, X_n^* > 0) \quad i = 2, \ldots, m,$$

$$(7) \qquad -X_h^* = \partial \pi^*/\partial R_h = \partial \Pi^*/\partial R_h = -X_h^*(\tilde{P}, \tilde{R}; \Theta, X_n^* > 0) \quad h = 1, \ldots, n,$$

$$(8) \qquad Y_1^* = \pi^*(\cdot) - \tilde{P}\tilde{Y}^{*\prime} + \tilde{R}X^{*\prime},$$

where $\tilde{P} = P/P_1$, $\tilde{R} = R/P_1$, and $\tilde{Y}$, $\tilde{P}$ are $1 \times (m - 1)$. Concavity of $F(\cdot)$ implies convexity of $\pi^*(\cdot)$. A second set of choice functions is defined as the explicit form of (1)–(4) when $X_n^* = 0$. These functions would relate optimal choices and expected profits conditional on $X_n^* = 0$ denoted (π^c, Y^c, X^c) to (P, R^c, Θ) where X^c and R^c are $1 \times (n - 1)$.

The comparative-statics of choice are conditional on whether $X_n^* > 0$ or $X_n^* = 0$ and are derived from differentiation with respect to prices of the appropriate set of choice functions, e.g., (6)–(8) where $X_n^* > 0$ (Weaver 1982). Continuity of the NEPF in prices implies that these comparative-statics for each set of choice functions satisfy the symmetry property. The comparative-statics with respect to exogenous changes in fixed factors can also be derived from the profit function, providing the basis for determining individual choice elasticities as well as the Hicksian biases in relative product mixes and input use patterns. These comparative-statics also depend on whether $X_n^* = 0$. For $X_n^* > 0$, following Weaver (1983), the allocative effect of a change in Θ_r on the relative use of X_k and X_h can be summarized by the rule:

A change in Θ_r is Hicks'

(9) X_h saving / neutral / using relative to X_k as $B_{hk} \gtreqless 0$,

where $B_{hk} \equiv -\partial \ln\left(\dfrac{X_h^*}{X_k^*}\right)/\partial \ln \Theta_r$, or using (6) and (7)

$$(10)\ B_{hk} = \left(\frac{\partial^2\pi^*}{\partial R_h \partial \Theta_r}\frac{1}{X_h^*} - \frac{\partial^2\pi^*}{\partial R_k \partial \Theta_r}\frac{1}{X_k^*}\right)\Theta_r.$$

Expressions (9) and (10) indicate corrections of typographical errors in expressions (13) and (14) in Weaver (1983).

The relationship of these results to farm budget analysis is of interest to note. Traditional budgets of interest are illustrated using (6) and (7):

$$(6')\quad E_i^* = P_i Y_i^* = E_i^*(\tilde{P}, \tilde{R}; \Theta, X_n^* > 0) \qquad i = 2, \ldots, m;$$

$$(7')\quad E_h^* = -R_h X_h^* = E_h^*(\tilde{P}, \tilde{R}, \Theta, X_n^* > 0) \qquad h = 1, \ldots, n.$$

The dual model (5)–(8) provides a basis for systematic modeling of the variation in budgets and, thereby, the distributional impact of exogenous changes. Further, all comparative-statics of net revenues or expenses can be written in terms of choice elasticities, e.g.,

$$(\partial E_i^*/\partial \theta_r)(\theta_r/E_i^*) = (\partial Y_i^*/\partial \theta_r)(\theta_r/Y_i^*).$$

Estimation of Duality Models When Corner Solutions Occur

An important implication of corner solutions is that a dual relationship between a single dual function and the technology no longer exists for all observations. When $X_n^* > 0$ a function $\pi^*(\cdot)$ is dual to the technology, whereas when $X_n^* = 0$, a function $\pi^c(\cdot)$ is dual. In terms of parameters, if a vector Γ characterizes the dual and Δ the production technology, then Γ^* would be dual to Δ for $X_n^* > 0$, while Γ^c would be for $X_n^* = 0$. The implication is that if corner solutions are ignored in a data set and a "profit function" estimated, the resulting parameter vector, say Γ°, will not be dual to the technology described by Δ. Only estimates of Γ^* and Γ^c can be used to describe Δ and the comparative-statics of choice through dual relationships.

We employ quadratic forms for the profit functions conditional on X_n^*; for example, for $X_n^* > 0$ we assume $\Gamma^* = [\alpha, \beta]$ where α is a vector of first-order coefficients and β is a matrix of second-order coefficients. A different dual system involving $\Gamma^c = [\alpha^c, \beta^c]$, $(\tilde{P}, \tilde{R}^c, \Theta)$, and (Y^c, X^c) can be written for the case where $X_n^* = 0$. In general, elasticities for the case where $X_n^* > 0$ are expected to differ from those for the case where $X_n^* = 0$.

The systems of choice functions derived from the quadratic NEP functions are written in more compact notation in order to consider estimation. For the case where $X_n^* > 0$,

$$(11)\qquad Y^* = Z\Gamma^* + U^*,$$

where $Y^* = \begin{bmatrix} Y^* \\ -X^* \end{bmatrix}$ a $MT_1 \times 1$ vector, $M = m + n$, T_1 is the number of observations where $Y_M^* > 0$; Z is an $MT_1 \times \sum_{i=1}^{M} K_i$ matrix where $Z_i = [1 \tilde{P}\ \tilde{R}\ \Theta]$, a $T_1 \times K_i$ matrix of the exogenous determinants of the ith choice function,

$$\Gamma^* = [\Gamma_1^{*\prime} \ldots \Gamma_M^{*\prime}]',$$
$$\Gamma_i^{*\prime} = [\alpha_i\ \beta'_{P_iP}\ \beta'_{P_iR}\ \beta'_{P_i\Theta}], \text{ and}$$
$$U^{*\prime} = (\epsilon'_{y_2} \ldots \epsilon'_{y_m}\epsilon'_{x_1} \ldots \epsilon'_{x_n}).$$

For $Y_M^* = 0$, the system of choice functions is

$$(12)\qquad Y^c = Z^c\Gamma^c + U^c,$$

where notation is analogous to that used in (11).

We expect $\Gamma_{ij}^* \neq \Gamma_{ij}^c$ for $i, j \neq M$. This suggests the data set should be sorted into those observations with $X_n^* > 0$ and those with $X_n^* = 0$. However, conventional estimation using these sorted data sets is complicated because the values taken on by (Y^*, U^*) in (11) are conditional on $Y_M^* > 0$ and those taken on by (Y^c, U^c) in (12) are conditional on $Y_M^* = 0$. To define the stochastic properties of these models, we assert Y^* and Y^c and, therefore, U^* and U^c are drawn from respective multivariate normal distributions. We further assume $E(U^*) = E(U^c) = 0$, $E(U^*U^{*\prime}) = \Sigma^* \otimes I_{T_1}$, and $E(U^cU^{c\prime}) = \Sigma^c \otimes I_{T_2}$. Condensing (11) and (12) we have

$$(13)\quad Y \equiv \begin{cases} Z\Gamma^* + U^* & \text{if } Y_M^* > 0. \\ \begin{bmatrix} Z^c\Gamma^c + U^c \\ 0 \end{bmatrix} & \text{if } Y_M^* = 0. \end{cases}$$

It follows that for the subsample where $Y_M^* > 0$,

(14) $E(Y^*) = E(Y|Z, Y_M^* > 0)$
$= Z\Gamma^* + E(U^*|Y_M^* > 0).$

Because $E(U^*|Y_M^* > 0) \neq 0$, estimation of $Z\Gamma^*$ would result in sample selection biased estimators of $E(Y^*)$ if all observations were drawn conditional on $Y_M^* > 0$. If $Y_M^* = 0$, then $Z^c\Gamma^c$ would similarly fail as an unbiased estimator of $E(Y^c)$.

By an extension of Heckman's suggestion, the conditional nature of the distributions of Y^* and Y^c can be summarized with an unobservable index L^*. Using the first-order conditions (1)–(4) and previous definitions, the following rule can be written:

$$(15)\quad Y_M^* > 0 \quad \text{if} \quad -Z_M + \mu\frac{\partial F}{\partial Y_M} = L^* > 0,$$

$$Y_M^* = 0 \quad \text{if} \quad -Z_M + \mu\frac{\partial F}{\partial Y_M} = L^* \leq 0.$$

By (15), the decision to employ Y_M^* is determined by $(\tilde{P}, \tilde{R}, \Theta)$. The indicator L^* can be approximated by

$$(16)\qquad L^* = W_T\delta + \epsilon_L,$$

where

$$W_T \equiv [\tilde{P}\tilde{R}^c\Theta],\ T \times (M - 1 + p).$$

Although the index L^* is unobservable, an observable binary indicator L may be defined as $L = 1$ if $L^* > 0$, or $L = 0$ if $L^* \leq 0$.

Equations (11) and (16) fully describe choices made by the firm when $Y_M^* > 0$. To proceed, we assume the vector $[U^{*\prime}\epsilon_L^{\prime}]$ is multivariate normal,

$$E(U^{*\prime}\epsilon_L^{\prime}) = 0, \text{ and}$$

$$E\left(\begin{bmatrix} U^* \\ \epsilon_L \end{bmatrix}[U^{*\prime}\epsilon_L^{\prime}]\right) = \begin{bmatrix} \Sigma^* & \sigma^* \\ \sigma^* & 1 \end{bmatrix} \otimes I_{T_2},$$

where

$$\sigma^* = (\sigma_{U_1^*\epsilon_L} \ldots \sigma_{U_M^*\epsilon_L}).$$

A convenient estimation method follows by extension of Heckman and of Lee, Maddala, and Trost, who noted that

$$(17)\quad E(U^*|Y_M^* > 0) = E(U^*|\epsilon_L > -W_1\delta) = \Lambda^*\sigma^*,$$

where $\Lambda^* = I_M \otimes \lambda^*$, λ^* is $T_1 \times 1$ with $\lambda^* = \phi(-W_1\delta)/[1 - \Phi(-W_1\delta)]$; $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard and cumulative normal density functions; and W_1 contains the first T_1 observations of W_T. By substitution,

$$(18)\qquad Y^* = Z\Gamma^* + \Lambda^*\sigma^* + v^*,$$

where

$$v^* = U^* - \Lambda^*\sigma^* \text{ and}$$
$$E(v^*|Y_M^* > 0) = E(v^*) = 0.$$

Estimates of δ in (16) can be obtained by maximum likelihood (MLH) probit methods. Using $\tilde{\delta}$, we can determine $\tilde{\Lambda}^*$ and estimate (18) using iterative Zellner methods. By extension of Barten's results, this method will produce MLH estimates of (Γ^*, σ^*) conditional on $\tilde{\Lambda}^*$. Such estimators are easily shown to be consistent. However, variances of these estimators are conditional upon the use of $\tilde{\delta}$. Covariance matrices ignoring this will underestimate the correct asymptotic variances. This follows from the fact that given $\tilde{\delta}$ we obtain residuals $\tilde{U}^* = \tilde{v}^* + \tilde{\Lambda}^*\sigma^*$, not $U^* = v^* + \Lambda^*\sigma^*$. Estimators and their properties are derived in the appendix for the multiple equation case estimated here. A similar estimation method can be motivated for the system (12). Defining $L^c = 1 - L^*$, a selection rule for the system of optimal choices conditional on $X_n^* = 0$ can be written. The independence of the drawings of U^* and U^c allows each system to be estimated independently.

The Pennsylvania Dairy Sample

The data were from a sample of 117 Pennsylvania dairy farms which were randomly selected and individually enumerated in the 1974 U.S. Department of Agriculture's (USDA) Cost of Production Survey (COPS). Mandated by Congress, this COPS resulted in a uniquely comprehensive account of output and variable input prices, quantities produced or employed, fixed farm input flows, and operator characteristics. Although the 1974 COP survey was updated in 1980, this more recent survey was not comprehensive. Instead of obtaining a complete set of data characterizing all variable and fixed input use, output levels, prices, and farm characteristics for each farm, a variety of surveys was administered focusing on different aspects of the farm operation. Examples are machinery complements, equipment sets, irrigation components, and materials application rates. Annual specialized surveys update various of these past data sets. Current data is systematically combined with price data from still other surveys, and budgets are generated for regional, states, and national levels (see USDA).

A careful review of farm budget data collected by state experiment stations or extension services indicated that this COPS presented the most comprehensive farm-level data base available. Typical farm record systems, including those available for Pennsylvania, are not representative samples and report data only for particular enterprises or an incomplete set of farm outputs and inputs. Often, sales rather than production data are reported.

The value of a complete data set for estimating systems of choice functions follows from the requirement that they be consistent with a behavioral hypothesis. Elementary econometrics suggests that exclusion of relevant independent variables will bias estimates. Incomplete choice and dual functions could be defined based on data which do not completely account for all choices and fixed factors maintained in the behavioral hypothesis. By this definition, they exclude prices and fixed factors which the behavioral hypothesis defines as relevant determinants of choice. By exclusion of relevant prices from the profit function, biased and inconsistent estimates could be expected because prices are typically intercorrelated. Further, this bias would not allow imposition or test of the cross-equation constraints of symmetry.

We maintain the hypothesis that net milk and net grain crop outputs and commercial fertilizer and lime, herbicides, commercial feeds, hired labor, capital services (buildings and machinery), other livestock inputs, and other crop inputs are variable in the short run. The size of the dairy herd directly affects production possibilities and is hypothesized to be variable in the short run through sales or purchases of dairy cows. Acreage operated was hypothesized to be fixed in the short run because of the absence of short-run rental markets. Finally, production possibilities are hypothesized as conditional upon farm operator characteristics: age of operator, experience, and education.

Cross-sectional estimates of production choice models, such as (11) and (12), require adequate variation in prices and fixed factors across observations. Spatial variation in prices paid and received for products could be expected to follow from variation in transportation costs, quality, and market scale or efficiency and will be assumed to reflect variation in expected prices. Quality variation must be recognized through proper use of consistent aggregation procedures and sufficiently disaggregated data to construct constant quality indexes. Variation in prices which results from transportation costs, and market scale or efficiency represents price variation to which consistently aggregated choices would be expected to respond. Where output prices received are determined by central markets (whether through market auctions or government decree), a similar argument would apply. Products were first partitioned into product type and quality categories hypothesized to be weakly separable. Next, detailed price and quantity data were aggregated by product category using static forms of the Divisia index.

Empirical Estimates of Production Choice Functions

Sample selection bias must be tested prior to a test of symmetry to ensure consistency of estimates of β. The hypothesis that sample selection bias does not exist was tested by the joint restriction $\sigma^* = 0$ and $\sigma^c = 0$. Log-likelihood ratio test results implied the restrictions can be rejected at the 99% level of confidence for both subsamples. Conditional upon the inference that selection bias exists in the present samples, $\tilde{\Lambda}^*$ and $\tilde{\Lambda}^c$ were retained in the models. Results were consistent with monotonicity and convexity at each observation in each subsample. Symmetry was tested conditional on the existence of sample selection bias by imposing the appropriate set of linear restrictions on $\hat{\beta}$ and $\hat{\beta}^c$.

The primary objective of the paper was to demonstrate an estimation strategy for samples where corner solutions are observed. The importance of accommodating corner solutions in estimation is illustrated by (*a*) the statistical significance of $\tilde{\sigma}^*$ and $\tilde{\sigma}^c$, indicating that biased estimates of Γ^* and Γ^c would have resulted if $\tilde{\Lambda}^*$ and $\tilde{\Lambda}^c$ were excluded from the models, and (*b*) differences in estimated parameters $\hat{\Gamma}^*$ and $\hat{\Gamma}^c$ as is apparent from tables 1 and 2. The importance of the method of estimation of asymptotic variances presented in the appendix is also illustrated in these tables by comparison of unadjusted with adjusted estimates of t-statistics. In general, the adjust-

Table 1. Estimated Parameters for the Net Supply and Input Demand Equations for Farms with Hired Labor, Symmetry Imposed

Equation	Crops	Dairy Cows	Lime and Fertilizers	Feeds	Other Livestock Inputs	Energy	Labor	Capital	Other Crop Inputs	Herbicides
Intercept	−169.19	145.65*	84.682*	20652*	3773.1*	7984.0*	2715.7	31675	1393.5	0.2246
	(1.58)[b]	(3.48)	(1.77)	(4.12)	(3.02)	(2.07)	(1.08)	(1.33)	(0.89)	(1.38)
	[1.62]	[6.05]	[2.02]	[4.39]	[4.10]	[2.13]	[1.21]	[1.33]	[0.91]	[1.50]
Prices[a]										
Crops	1.2614									
	(1.01)									
	[1.04]									
Dairy cows	0.2656	−0.4809								
	(0.78)	(0.99)								
	[1.19]	[2.37]								
Lime and fertilizers	−0.0790	0.7972	−2.6661*							
	(0.18)	(1.10)	(3.65)							
	[0.20]	[3.43]	[4.65]							
Feeds	62.394	−29.9928	15.103	−12502*						
	(1.25)	(0.90)	(0.47)	(2.75)						
	[1.34]	[2.07]	[0.54]	[3.29]						
Other livestock inputs	1.5785	−3.1743	21.751	−1126.7	−2000.0*					
	(0.14)	(0.14)	(1.27)	(.126)	(2.83)					
	[0.17]	[0.52]	[2.00]	[1.77]	[5.12]					
Energy	14.385	12.938	13.850	5929.7*	48.454	−28377*				
	(0.39)	(0.24)	(0.29)	(2.22)	(0.03)	(3.81)				
	[0.39]	[0.48]	[0.30]	[2.28]	[0.04]	[3.84]				
Labor	1.5889	0.9527	71.053*	−1256.2	225.65	694.12	−5421.8*			
	(0.07)	(0.03)	(2.76)	(0.72)	(0.28)	(0.30)	(2.90)			
	[0.07]	[0.08]	[3.25]	[0.84]	[0.43]	[0.31]	[3.44]			
Capital	4.5978	0.0061	−3.7445	724.17	21.253	−58.265	119.10	−8977.0*		
	(0.30)	(0.00)	(0.61)	(1.02)	(0.15)	(0.11)	(0.39)	(2.28)		
	[0.31]	[0.00]	[0.64]	[1.08]	[0.18]	[0.12]	[0.41]	[2.28]		
Other crop inputs	2.4701	0.4357	0.1999	−12.121	30.827	−85.109	−52.014	400.45*	−61.485	
	(0.89)	(0.45)	(0.20)	(0.09)	(1.16)	(1.06)	(0.98)	(1.66)	(1.29)	
	[0.94]	[0.79]	[0.21]	[0.10]	[1.50]	[1.06]	[1.06]	[1.71]	[1.39]	
Herbicides	0.0001	0.0020	−0.0008	0.0530	0.0322	−0.2354	0.1345	−0.0093	−0.0060*	−0.0001
	(0.04)	(0.80)	(0.37)	(0.53)	(0.51)	(0.97)	(1.46)	(0.46)	(1.79)	(1.16)
	[0.04]	[1.28]	[0.42]	[0.57]	[0.59]	[1.00]	[1.58]	[0.47]	[1.79]	[1.53]

Table 1. (Continued)

Equation	Crops	Dairy Cows	Lime and Fertilizers	Feeds	Other Livestock Inputs	Energy	Labor	Capital	Other Crop Inputs	Herbicides
Fixed factors										
Number of acres	0.0350	−0.0157	0.0864*	−2.1205	0.1057	4.8079*	2.8679*	21.611	4.7203*	0.0001
	(0.47)	(0.66)	(2.80)	(0.62)	(0.14)	(1.94)	(1.85)	(1.16)	(3.89)	(0.69)
	[0.48]	[1.00]	[3.02]	[0.65]	[0.18]	[1.99]	[1.98]	[1.16]	[3.98]	[0.73]
Age of operator	−2.9848	17.839	−14.062	807.70	639.44*	−106.21	1373.2*	−5010.4	358.84	−0.0776*
	(0.09)	(1.29)	(1.04)	(0.48)	(1.74)	(0.10)	(1.93)	(0.66)	(0.66)	(1.72)
	[0.10]	[2.72]	[1.17]	[0.60]	[2.53]	[0.10]	[2.25]	[0.67]	[0.76]	[1.83]
Experience	0.5310	−0.8985	−0.3551	−127.69*	−31.343*	18.768	−3.7257	41.509	−5.6896	−0.0002
	(0.37)	(1.24)	(0.65)	(1.66)	(1.81)	(0.47)	(0.12)	(0.13)	(0.23)	(0.09)
	[0.45]	[3.54]	[0.78]	[2.44]	[3.25]	[0.48]	[0.16]	[0.14]	[0.30]	[0.10]
Education	5.6532	−5.5600*	−4.3652*	−595.42*	−125.80*	−471.42*	−88.942	−583.09	−89.144	−0.0082
	(0.95)	(2.98)	(1.78)	(3.02)	(2.15)	(2.41)	(0.73)	(0.40)	(0.92)	(0.99)
	[0.99]	[4.50]	[1.92]	[2.30]	[2.67]	[2.47]	[0.78]	[0.40]	[0.96]	[1.04]
λ	8.5985	−6.7652	−2.7530	−567.20	−143.99	−29.058	−173.74	1016.87	−165.14	0.0002
	(1.15)	(1.54)	(0.88)	(1.38)	(1.38)	(0.13)	(1.06)	(0.64)	(1.27)	(0.02)
	[1.44]	[5.11]	[1.15]	[2.11]	[2.86]	[0.14]	[1.44]	[0.68]	[1.74]	[0.03]

[a] Prices are relative to milk price $R^2_{SYS} = 1 - \dfrac{V^{*\prime}(\hat{\Sigma}^{-1})\,V^*}{y^{*\prime}(\hat{\Sigma}^{-1})\,y^*} = 0.4053$, where y^* represents a vector of deviations from the means and $\hat{\Sigma} = (V^*V^{*\prime})$.

[b] Adjusted t-statistics are in parentheses. Unadjusted t-statistics are in brackets. Single asterisk indicates significant at 5% level.

Table 2. Estimated Parameters for the Net Supply and Input Demand Equations for Farms with No Hired Labor, Symmetry Imposed

Equation	Crops	Dairy Cows	Lime and Fertilizers	Feeds	Other Livestock Inputs	Energy	Capital	Other Crop Inputs	Herbicides
Intercept	−1.8137	55.696*	18.156	374.31	1530.2*	1019.9	20540	828.73	0.1177
	(0.03)[b]	(1.66)	(0.94)	(0.20)	(2.03)	(0.34)	(0.83)	(0.80)	(1.48)
	[0.03]	[3.59]	[1.04]	[0.21]	[2.79]	[0.42]	[0.96]	[0.80]	[1.60]
Prices[a]									
Crops	3.0754*								
	(2.27)								
	[2.31]								
Dairy cows	0.2901	−0.8252							
	(0.56)	(0.73)							
	[0.90]	[3.08]							
Lime and fertilizers	0.4946	0.5102	−1.1458*						
	(1.16)	(1.07)	(2.21)						
	[1.30]	[2.10]	[2.99]						
Feeds	4.4530	−1.0004	−10.224	−2610.1*					
	(0.14)	(0.04)	(0.70)	(1.72)					
	[1.14]	[0.09]	[0.84]	[1.81]					
Other livestock inputs	4.4995	−9.7424	2.4465	167.70	−2203.1*				
	(0.31)	(0.36)	(0.14)	(0.24)	(1.88)				
	[0.37]	[1.23]	[0.26]	[0.43]	[5.06]				
Energy	67.148	−62.119	20.489	4037.1*	541.36	−36002*			
	(1.23)	(0.98)	(0.42)	(1.99)	(0.24)	(3.56)			
	[1.36]	[2.00]	[0.54]	[2.53]	[0.41]	[4.91]			
Capital	154.18*	−21.915	−15.566	−206.76	−589.89	−6237.8*	−95378*		
	(3.03)	(0.75)	(1.10)	(0.13)	(0.82)	(2.75)	(4.03)		
	[3.10]	[1.87]	[1.15]	[0.13]	[1.43]	[3.14]	[4.62]		
Other crop inputs	−0.3909	−0.0042	−0.1544	−8.9366	−3.1542	−62.776*	18.128	−16.627	
	(0.51)	(0.01)	(0.52)	(0.37)	(0.30)	(1.69)	(0.06)	(1.13)	
	[0.52]	[0.02]	[0.66]	[0.38]	[0.42]	[1.96]	[0.06]	[1.13]	
Herbicides	−0.0017	0.0022	−0.0018	0.0828	0.0245	−0.3370	−0.0405	−0.0009	0.00001
	(0.98)	(1.19)	(0.90)	(1.39)	(0.30)	(1.27)	(0.67)	(0.76)	(0.21)
	[1.03]	[1.73]	[1.09]	[1.56]	[0.43]	[1.42]	[0.74]	[0.84]	[0.23]

Table 2. (Continued)

Equation	Crops	Dairy Cows	Lime and Fertilizers	Feeds	Other Livestock Inputs	Energy	Capital	Other Crop Inputs	Herbicides
Fixed factors									
Number of acres	0.2628* (2.41) [2.46]	0.0182 (0.29) [0.68]	0.1158* (3.19) [3.79]	−2.2376 (0.63) [0.67]	1.4579 (0.83) [1.52]	17.297* (3.24) [3.95]	115.32* (2.31) [2.64]	1.6209 (0.88) [0.89]	0.0004* (2.81) [3.14]
Age of operator	−7.6428 (0.28) [0.28]	7.688 (1.49) [2.58]	6.3670 (0.76) [0.81]	−237.03 (0.26) [0.27]	130.39 (0.43) [0.53]	404.55 (0.34) [0.36]	4980.5 (0.42) [0.46]	70.467 (0.13) [0.13]	−0.0218 (0.63) [0.66]
Experience	−0.0059 (0.01) [0.01]	−0.6015* (1.68) [2.57]	−0.1857 (0.65) [0.68]	−8.1128 (0.27) [0.27]	−7.2924 (0.78) [0.88]	9.7904 (0.24) [0.25]	46.389 (0.11) [0.12]	−4.5725 (0.24) [0.24]	0.0006 (0.47) [0.49]
Education	−3.7528 (0.75) [0.78]	−3.1309 (0.93) [2.61]	−0.5499 (0.36) [0.40]	290.40* (1.84) [1.95]	−68.523 (1.05) [2.67]	104.27 (0.44) [0.53]	−1568.6 (0.67) [0.81]	−21.668 (0.23) [0.23]	−0.0054 (0.81) [0.90]
λ	−1.1629 (0.25) [0.26]	3.4177 (0.94) [2.72]	0.2434 (0.13) [0.17]	46.774 (0.30) [0.32]	72.216 (0.87) [1.50]	−138.52 (0.52) [0.69]	1382.3 (0.63) [0.77]	3.9982 (0.05) [0.05]	−0.0033 (0.41) [0.47]

[a] Prices are relative to milk price $R^2_{SYS} = 1 - \dfrac{V^{*\prime}(\hat{\Sigma}^{-1})V^*}{y^{*\prime}(\hat{\Sigma}^{-1})y^*} = 0.6356$, where y^* represents a vector of deviations from the means and $\hat{\Sigma} = (V^*V^{*\prime})$.

[b] Adjusted t-statistics are in parentheses. Unadjusted t-statistics are in brackets. Single asterisk indicates significant at 5% level.

ment leads to substantial reduction in the t-statistics.

Adjusted t-statistics support the conclusion that own-price effects were in general highly significant and had signs consistent with profit maximization. Further, numerous cross-price effects and the effects of fixed factors were also highly significant. One exception is found in the sign of the own-price coefficient for herbicides for the no-hired labor case as reported in table 2. For this case, the coefficient has the wrong sign but is both statistically insignificant and close to zero. We proceed by maintaining the hypothesis that this coefficient is, in fact, zero. Because numerous cross-price coefficients are not significantly different from zero, collinearity was assessed and not found to characterize the data set. This supports the inference that insignificant coefficients indicate product pairs for which comparative-static responses are zero.

Based on these results, the second objective of the paper is achieved by reporting a complete set of estimated elasticities of choice with respect to expected prices (tables 3 and 4) and biases induced by changes in fixed factors (tables 5 and 6). These represent the first complete set of dairy farm production choice elasticities based on microlevel data. The short-run elasticity of milk is estimated to be .5131 and .8998 for the cases of hired and no-hired labor, respectively. In both cases, net crops, lime and fertilizer, and commercial feed demands show substantial own-price elasticity. The absolute values of all other own-price elasticities of input demand are less than one. A strong inelasticity of milk with respect to all prices except its own is apparent, suggesting cull prices and feed prices may be weak instruments with which to control milk supply.

Crops represent a net output used directly for feed or sold. Results reflect the predominant use of crops as feed. Accordingly, the estimated elasticities are negative. Both net crop and concentrated feed demands have substantial elasticity with respect to their own prices and are substitutes for each other. Substantial positive elasticity is found in the feed demands with respect to changes in the price of milk. The demand functions for dairy cows indicate low levels of short-run elasticity. The demand functions for commercial inputs indicate fertilizers and energy have significant and substantial own-price responsiveness, while the demand for herbicides has statistically insignificant and relatively inelastic own-price response.

The estimated set of choice functions and elasticities based on cross-sectional data provide a solid foundation for policy analysis. Predicted net output levels imply predicted net output revenues and input expenditures or, in traditional terms, farm budgets. While traditional methods report mean revenue and expenditure levels for various stratifications of a sample of budgets, the methods used in this paper suggest that a far richer set of budget analyses can be generated from this type of data set.

Tables 1 and 2 report results concerning the effects of operator characteristics on output supply and input demand functions. Elasticities are reported in tables 3 and 4. For the hired labor subsample, number of acres operated had a significant and positive effect on fertilizer, energy, and hired labor demand. The same effects were found for the no-hired labor subsample as well as a positive and significant effect on herbicide demand. Operator characteristics are found to play a significant role in affecting decisions. In the hired labor subsample, hired labor demand increases and herbicide demand decreases with age of operator. Concentrated feed demand is found to decrease with experience. Results for education suggest an efficiency effect of education. All input demands declined as operator education increases, an effect which is statistically significant for dairy cows, fertilizer, concentrated feed, other livestock inputs, and energy demands. For the no-hired-labor sample, a strikingly less significant role is found for operator characteristics. This result corroborates the importance of recognizing sample heterogeneity introduced by corner solutions.

Following Weaver (1983), the effect of changes in fixed input levels on product mixes can be considered in Hicksian terms. Measures of biases reported in tables 5 and 6 indicate how factor ratios would respond to change in (a) acreage operated and (b) operator education, respectively. Biases in net output mix induced by changes in these fixed inputs were also estimated and are available from the authors. The rule (14) provides a basis for interpreting these results. For example, table 5 indicates that increasing acreage operated by a dairy farmer increased lime and fertilizer use relative to feed as well as relative

Table 3. Elasticities of Choice for Farms with Hired Labor

Quantities	Milk	Crops	Dairy Cows	Lime and Fertilizers	Conc. Feeds	Other Livestock Inputs	Energy	Labor	Capital	Other Crop Inputs	Herbicides
Prices											
Milk	0.5131	2.3090	0.1636	−0.1182	1.1782	0.6477	−0.0633	0.2666	0.0514	0.0102	−0.1087
Crops	−0.0879	−0.7699	−0.0700	0.0231	−0.1660	−0.0178	−0.0374	−0.0082	−0.0018	−0.0210	−0.0056
Dairy cows	−0.0171	−0.1929	−0.1509	0.2776	−0.0950	−0.0425	0.0400	0.0058	0.0000	0.0044	0.2519
Lime and fertilizers	0.0109	0.0562	0.2450	−0.9097	0.0468	0.2856	0.0419	0.4256	−0.0017	0.0020	−0.0931
Feeds	−0.2915	−1.0791	−0.2241	0.1253	−0.9427	−0.3597	0.4366	−0.1829	0.0079	−0.0029	0.1551
Other livestock inputs	−0.0375	−0.0270	−0.0235	0.1786	−0.0841	−0.6319	0.0035	0.0325	0.0002	0.0074	0.0933
Energy	0.0031	−0.0476	0.0185	0.0220	0.0856	0.0030	−0.3999	0.0193	−0.0001	−0.0039	−0.1318
Labor	−0.0248	−0.0199	0.0052	0.4278	−0.0688	0.0523	0.0371	−0.5731	0.0009	−0.0091	0.2856
Capital	−0.0631	−0.0567	0.0003	−0.0222	0.0390	0.0048	−0.0031	0.0124	−0.0695	0.0690	−0.0195
Other crop inputs	−0.0031	−0.1671	0.0127	0.0065	−0.0036	0.0385	−0.0245	−0.0296	0.0170	−0.0580	−0.0681
Herbicides	0.0011	−0.0014	0.0236	−0.0099	0.0061	0.0158	−0.0267	0.0301	−0.0002	−0.0022	−0.3561
Fixed factors											
Number of acres	*[a]	−0.5237	−0.1017	0.6206	−0.1384	0.0292	0.3065	0.3616	0.2029	0.9862	0.1836
Experience	*	−0.6367	−0.4656	−0.2042	−0.6676*	−0.6938	0.0958	−0.0376	0.0312	−0.0952	−0.0324
Education	*	−3.5345	−1.5022	−1.3092	−1.6234	−1.4520	−1.2549	−0.4683	−0.2286	−0.7778	−0.8665

[a] Asterisk indicates not identifiable.

Table 4. Elasticities of Choice for Farms with No Hired Labor

Quantities	Milk	Crops	Dairy Cows	Lime and Fertilizers	Conc. Feeds	Other Livestock Inputs	Energy	Capital	Other Crop Inputs	Herbicides
Prices										
Milk	0.8998	−0.8038	0.2431	0.4497	0.5701	0.9389	0.3730	0.6488	0.0503	0.5367
Crops	0.0224	−3.4959	0.0941	0.2151	−0.0275	0.0701	0.1874	0.0731	0.0051	0.2841
Dairy cows	−0.0252	0.3495	−0.2838	0.2351	−0.0066	−0.1608	−0.1838	−0.0110	−0.0001	0.3833
Lime and fertilizer	−0.0438	0.7502	0.2209	−0.6648	−0.0844	0.0508	0.0763	−0.0098	−0.0027	−0.4063
Feeds	−0.1001	−0.1734	−0.0111	−0.1523	−0.5531	0.0895	0.3861	−0.0034	−0.0040	−0.4756
Other livestock inputs	−0.0503	0.1346	−0.0832	0.0280	0.0273	−0.9030	0.0398	−0.0074	−0.0011	0.1080
Energy	−0.0224	0.4031	−0.1064	0.0471	0.1319	0.0445	−0.5307	−0.0156	−0.0043	−0.2984
Capital	−0.6383	2.5764	−0.1045	−0.0995	−0.0188	−0.1350	−0.2560	−0.6643	0.0034	−0.0998
Other crop inputs	−0.0283	0.1027	−0.0003	−0.0155	−0.0128	−0.0114	−0.0405	0.0020	−0.0500	−0.0350
Herbicides	−0.0051	0.0974	0.0354	−0.0399	−0.0259	0.0193	−0.0476	−0.0010	−0.0006	0.0000
Fixed factors										
Number of acres	*[a]	−5.2481	0.1038	0.8850	−0.2432	0.3989	0.8483	0.9600	0.3707	1.2358
Experience	*	0.0155	−0.4480	−0.1853	−0.1152	−0.2606	0.0627	0.0504	−0.1366	−0.2238
Education	*	4.0465	−0.9637	−0.2268	1.7039	−1.0123	0.2761	−0.7050	−0.2675	−0.8632

[a] Asterisk indicates not identifiable.

Table 5. Biases (B_{hk}) in Relative Input Utilization Resultant from Changes in the Fixed Factor—Number of Acres

h \ k	Dairy Cows	Lime and Fertilizers	Feeds	Other Livestock Inputs	Energy	Labor	Capital	Other Crop Inputs	Herbicides
Crops	0.4220[a] 5.3519	1.1443 6.1330	0.3852 5.0049	0.5529 5.6469	0.8301 6.0963	0.8852	0.7226 6.2080	1.5098 5.6187	0.7074 6.4837
Dairy cows		0.7223 0.7811	−0.0367 −0.3470	0.1309 0.2950	0.4082 0.7444	0.4633	0.3046 0.8561	1.0879 0.2668	0.2853 1.1318
Lime and fertilizer			−0.7591 −1.1282	−0.5914 −0.4861	−0.3142 −0.0367	−0.2591	−0.4178 −0.0750	0.3655 −0.5143	−0.4369 0.3507
Feeds				0.1676 0.6421	0.4449 1.0914	0.5000	0.3413 1.2031	1.1246 0.6138	0.3221 1.4788
Other livestock					0.2773 0.4494	0.3324	0.1737 0.5611	0.9570 −0.0282	0.1545 0.8368
Energy	(Symmetric in absolute magnitude but opposite in sign)					0.0551	−0.1036 0.1117	0.6797 −0.4776	−0.1228 0.3874
Labor							−0.1587	0.6246	−0.1779
Capital								0.7833 −0.5893	−0.0192 0.2757
Other crop inputs									−0.8025 0.8650
Herbicides									

Note: Evaluated at the means of the data.

[a] The top figures represent values for the "hired labor" case while the lower set of figures are the values for the "no hired labor" case.

Table 6. Biases (B_{hk}) in Relative Input Utilization Resultant from Changes in the Fixed Factor—Education

h \ k	Dairy Cows	Lime and Fertilizers	Feeds	Other Livestock Inputs	Energy	Labor	Capital	Other Crop Inputs	Herbicides
Crops	2.0323[a]	2.2254	1.9112	2.0825	2.2797	3.0662	3.3059	2.7567	2.6680
	−5.0103	−4.2734	−2.3427	−5.0588	−3.7705		−4.7516	−4.3141	−4.9098
Dairy cows		0.1930	−0.1212	0.0502	0.2473	1.0339	1.2736	0.7244	0.6357
		0.7369	2.6676	−0.0486	1.2398		0.2587	0.6962	0.1005
Lime and			−0.3142	−0.1428	0.0543	0.8409	1.0806	0.5314	0.4426
fertilizers			1.9307	−0.7854	0.5029		−0.4781	−0.0407	−0.6363
Feeds				0.1713	0.3685	1.1551	1.3947	0.8456	0.7568
				−2.7161	−1.4278		−2.4089	−1.9714	−2.5671
Other livestock					0.1971	0.9837	1.2234	0.6742	0.5855
inputs					1.2883		0.3073	0.7447	0.1491
Energy	(Symmetric in absolute magnitude					0.7866	1.0263	0.4771	0.3884
	but opposite in sign)						−0.9811	−0.5436	−1.1393
Labor							0.2397	−0.3095	−0.3982
Capital								−0.5492	−0.6379
								0.4375	−0.1582
Other crop									−0.0887
									−0.5956
Herbicides									

Note: Evaluated at the means of the data.
[a] The top figures represent values for the "hired labor" case while the lower set of figures are the values for the "no hired labor" case.

to energy use. More generally, table 5 indicates the expected percentage changes in relative input utilization in the sample if acreage were reduced by government intervention. Nonzero estimated biases indicate relative input use would change. For example, a reduction in acreage would reduce energy, labor, capital, and herbicide use and increase feed, energy, labor, capital, and herbicide use relative to fertilizer use.

Results reported in table 5 indicate that a decrease in acreage operated would result in greater crop supply to market (rather than home consumption as feed), fertilizer use would be reduced relative to all other inputs (except capital and herbicides for the no-hired labor sample), and use of concentrated feeds would increase relative to other inputs. For energy, capital, and herbicides, no generalization in shift of relative use can be drawn.

Table 6 indicates that allocative biases induced by changes in the level of educational attainment are not zero. Although few generalizations can be drawn, substantial shifts in factor use are apparent, suggesting that education is not neutral in its effect on input use. An increase in education appears to result in greater use of lime and fertilizer, energy, labor, capital, and herbicides and a decrease in concentrated feed use relative to lime and fertilizer for farms that hired labor; for farms that did not hire labor, an increase in feeds relative to lime and fertilizer is indicated. Increases in energy, labor, capital, and herbicide use relative to feed use are found for the hired labor case, while decreases in these ratios were found for the no hired labor sample. Similar results have been generated by the authors for age of operator and experience.

Conclusions

The above results suggest that dairy production decisions are responsive to prices in the short run, and that this responsiveness is conditioned by farm characteristics and fixed input flows. In general, results were consistent with the hypothesis that producers choose inputs and outputs in an attempt to maximize expected profits. Most own-price effects and a variety of cross-price effects were significant, the latter indicating complementarity or substitutability between product pairs. The complete set of elasticities of production decisions presented indicate that the extent of response is not quantitatively close to zero.

Results reported demonstrate that farm-level data could play an important role in understanding the effects of price or other exogenous changes on production plans of all types of farms. At present, farm record data sets typically do not include prices paid or received; however, the present results suggest the usefulness of such data for the analysis of the level, responsiveness, and distribution across farms of production choices. Estimates could be used to generate econometrically based farm budgets and to analyze the effects of price and fixed factor changes on both the means and distributions of those budgets. With estimates of a full profit function, shadow prices of fixed factor flows could also be estimated. Such an approach could be used to estimate shadow prices of land or water resources.

Most important, this paper has demonstrated that the presence of corner solutions in such data sets requires adoption of a methodology which accommodates and explains their occurrence. The estimation strategy presented focuses on achievement of unbiased estimates of both the parameters of the conditional means of the dependent choice variables, as well as the error stucture. Results of the application presented illustrate that substantial bias in estimated parameters and the error structure can occur if the existence of corners is ignored.

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References

Barten, A. P. "Maximum Likelihood Estimation of a Complete System of Demand Equations." *Eur. Econ. Rev.* 1(1969):7–73.

Breusch, T. S. "Conflict Among Criteria for Testing Hypotheses: Extension and Comment." *Econometrica* 47(1979):1287–94.

Grilliches, Zvi. "Estimates of the Aggregate Agricultural Production Function from Cross-Sectional Data." *J. Farm Econ.* 45(1963).

Heckman, James J. "Sample Selection Bias as a Specification Error." *Econometrica* 47(1979):152–61.

Lee, Lung-Fei, G., S. Maddala, and R. P. Trost. "Asymptotic Covariance Matrices of Two-Stage Profit and Two-Stage Tobit Methods for Simultaneous Equations Models with Selectivity." *Econometrica* 48 (1980):491–503.

Lopez, R. E. "Estimating Substitution and Expansion Effects Using a Profit Function Framework." *Amer. J. Agr. Econ.* 66(1984):358–67.

U.S. Congress, Senate. *Costs of Producing Milk in the United States—1974.* Committee on Agriculture and Forestry, Committee Print, Washington DC, 1976.

U.S. Department of Agriculture. *Major Statistical Series of the USDA: Costs of Production.* Agricultural Handbook No. 671. Washington DC, Sep. 1978.

Weaver, Robert D. "Measurement of Allocative Biases of Production Control Policies." *S. J. Agr. Econ.* 10(1978):87–91.

———. "Multiple Input, Multiple Output Production Choices and Technology in the U.S. Wheat Region." *Amer. J. Agr. Econ.* 64(1983):45–56.

———. "Specification and Estimation of Consistent Sets of Choice Functions." *New Directions in Econometrics Modeling and Forecasting in U.S. Agriculture,* ed. Gordon Rausser. New York: North-Holland Publishing Co., 1982.

Appendix

A Consistent Estimation Method

Consider estimation for a mixed system of truncated and continuous choice functions for the subsample regime where $Y_M^* > 0$. Consistent parameter estimates are available from iterative Zellner's methods to (18) (Barten). Using (11), (17), and (18), note:

$$U^* = v^* + \Lambda^*\sigma^*, \tag{A.1}$$

where

$$\Lambda^* = I_m \otimes \lambda^* \text{ and } \lambda^* = \lambda(W_1; \delta).$$

Since probit estimates $\tilde{\delta}$ are used as estimates of δ in calculating an estimate $\tilde{\Lambda}^*$, we define

$$\tilde{v}^* \equiv \tilde{U}^* - \tilde{\Lambda}^*\sigma^*, \tag{A.2}$$

where $\tilde{v}^*$ is the residual encountered in (18) when $\tilde{\delta}$ is used. (A.1) and (A.2) imply

$$\tilde{v}^* = v^* - (\tilde{\Lambda}^* - \Lambda^*)\sigma^*. \tag{A.3}$$

The covariances for these residuals are defined:

$$E(\tilde{v}^*\tilde{v}^*) = \{\mathrm{var}(v^*) + (\sigma^*)^2 A W_1[\mathrm{var}(\tilde{\delta})]W_1'A - \sigma^* A W_1 \,\mathrm{cov}(\tilde{\delta}, v^{*\prime}) - \sigma^* \,\mathrm{cov}(\tilde{\delta}', v^*)W_1'A\}, \tag{A.4}$$

where $A = \mathrm{diag.}[-W_1\tilde{\delta}(\tilde{\lambda}^*) - (\tilde{\lambda}^*)^2]$. Using the results (Lee, Maddala, and Trost, p. 500):

$$\mathrm{Cov}\,(\tilde{\delta}, v^*) = 0, \tag{A.5}$$

$$\mathrm{Var}(\tilde{\delta}) = (W_T'SW_T)^{-1}, \text{ and} \tag{A.6}$$

$$\mathrm{Var}(v^*) = \Sigma_{v^*v^*} + (\sigma^*)^2 A, \tag{A.7}$$

we can write (A.4) as

$$E(\tilde{v}^*\tilde{v}^{*\prime}) = \{\Sigma_{v^*v^*} + (\sigma^*)^2 A + (\sigma^*)^2 A W_1(W_T'SW_T)^{-1}W_1'A\}, \tag{A.8}$$

where W_T is the matrix of regressors for the entire sample used in probit estimation, and $S = \mathrm{diag.}[\phi_t(\cdot)/\Phi_t(\cdot)(1 - \Phi_t(\cdot)]$ $(t = 1, \ldots, T)$. Using (A.8) the proper covariance for the Zellner estimators conditional on $\tilde{\delta}$ is

$$\mathrm{Cov}\,\frac{\hat{\Gamma}^*}{\hat{\sigma}^*} = \{[Z\tilde{\Lambda}^*]'(\Sigma_{v^*v^*}^{-1})[Z\tilde{\Lambda}^*]\}^{-1} + (\hat{\sigma}^*)^2\{([Z\tilde{\Lambda}^*]'(\Sigma_{v^*v^*}^{-1})[Z\tilde{\Lambda}^*])^{-1} \times [Z\tilde{\Lambda}^*]'[A + AW_1(W_T'SW_T)^{-1}W_1'A][Z\tilde{\Lambda}^*] \times [Z\tilde{\Lambda}^*]'(\Sigma_{v^*v^*}^{-1})[Z\tilde{\Lambda}^*])^{-1}\}. \tag{A.9}$$

The first term in the braces is the covariance matrix which results from iterated Zellner estimation of a seemingly unrelated system. The remainder represents the amount by which the asymptotic covariances are understated. The estimators appropriate when linear restrictions such as symmetry are imposed follow by straightforward extension of results in the text and above.