A critical reinterpretation of Shacklean decision theory

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ABSTRACT

Shackle was one of the representative critics of probability calculus. His alternative decision theory was mathematically reformalized by Katzner till 1990s. Following the Katzner's reformalized framework, this paper presents a new interpretation of Shacklean theory by focusing on the common stage structure of the decision-making. This paper shows that the characteristics of Shackle-Katzner framework can be explained as: (1) the non-distributive and non-additive ordinal measure of subjective uncertainty, (2) the incomplete list of imaginable future states, (3) the valuation of "importance" reflecting various types of loss-psychology, and (4) the final choice of the action is made on the set of importance intervals. The purpose of this paper is to investigate how the foundational stance of Shackle-Katzner framework about the decision-making elicits the different formal structure, and to provide a new interpretation of Shacklean decision theory.

Key words: Shackle, uncertainty, expected utility theory, non-additivity

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1. Introduction

In most economic decisions, choice is made under uncertainty. Uncertainty is, literally, uncertain phenomena; not only the ambiguity in obtaining the probability distributions but even the ignorance in delineating the possible range of future outcomes. So, it is difficult to deal with uncertainty with an observable numeric such as probability. This is inevitable circumstance which a decision-maker must face. Even a person who has the authority to access the highest level of information and to exert worldwide influence is not exempt from this. For example, Greenspan said:

…how…the economy might respond to a monetary policy initiative may need to be drawn from evidence about past behavior during a period only roughly comparable to the current situation… In pursuing a risk-management approach to policy, we must confront the fact that only a limited number of risk can be quantified with any confidence. [1]

Greenspan's statement brought up a fundamental issue of economic theory: how to measure or to represent the degrees of possibility of future events with some sort of numeric. Basically, probability is the justifiable concept only when the decision-maker has knowledge regarding the frequency or the possibility of each outcome in the trustable list of imaginable future states. But it is not debatable fact that, in actual economic decision such as the business investment for the production plant, the full list of future states is not available at that moment of decision-making. Information about the frequency or plausibility of a specific event in the future is nearly impossible to obtain. In this manner, the usage of probability was doubted by many economists such as Keynes (1921), Knight (1921), Mises (1949), Shackle (1949; 1954; 1969; 1972), Davidson (1983, 1991), and Lachman (1976). Such notion of uncertainty is called; radical uncertainty (Langlois, 1984), structural uncertainty (Langlois, 1994) or fundamental uncertainty (Davidson 1988).

Out of those skeptics of probability, Shackle is one of the most distinguished subjectivists, who influenced the methodology of some Post-Keynesians and Austrians, and one of representative critics to the modelization of uncertainty with probability. In Shackle's viewpoint, probability is unjustifiable concept due to the inescapable limit of the human decision-making, *ignorance*. If the probability is construed in the objective manner, the probability loses its meaning when the choice is a unique activity because the probability implies the convergence of outcomes when experiment is repeatable. However, the economic decision-making is almost always unique and irreversible. Besides, for example, representing the possibility of an event having no supporting evidence by zero probability is not a suitable way to reflect ignorance. The assignment of zero probability premises some "knowledge" or, at best, subjective speculation that an event is unrealizable. But this is different to disbeliefs due to the lack of supporting information.

For Shackle, future is basically unknowable, and ignorance is fundamental circumstance of the decision-making in time. First, the knowledge of decision-maker regarding the possible scenarios of the future is predestined incomplete. Even when the decision-maker can conjecture a list of possible future states, it is easily updatable through the passage of time. Any changes in the list must alter the probability distribution, which was assigned to the previous list of the possible futures. In this context, Shackle suggested to introduce his own measure of uncertainty, *potential surprise* which is constructed on the collection of hypotheses over the incomplete list about the future. 'Potential surprise' implies the degree of decision-maker's disbelief to each hypothesis about uncertain future events. The fragility of the currently imaginable list of the future states determines an important feature of potential surprise, *non-distributivity*. Distributivity indicates allocating the degree of certainty to each possible state from a fixed limit of total value, *e.g.*, the unitary probability in sum.

Furthermore, in Shacklean framework, it is presupposed that the decision-maker is isolated at each moment in a unique path of time. While the list of possible future states is unlimitable and not fully imaginable, the state to be realized must be eventually unique in time. Thus, when multiple hypotheses composite a hypothesis by connecting them with a conjunction, 'or', the potential surprise value of the union hypothesis cannot be obtained by just 'summing' potential surprise values of constituent hypotheses because they are not realizable in a single path of time. This perspective about time defines another feature of potential surprise, *non-additivity*.

However, as Gorgescu-Roegen (1958) pointed out, the formal structure defining the potential surprise in Shackle's original manuscript was relatively descriptive and technically less rigorous. Such limit of Shacklean framework prevented it to be accepted in the current of modern decision theory. However, there was a small but significant movement at the University of Massachusetts at Amherst during the last quarter of the 20th century to rehabilitate Shacklean framework. A series of economists emphasizing methodological reflection such as Bausor (1982, 1984), Crotty (1994), Vickers (1978, 1987, 1994)) and Katnzer (1986-7; 1987-88; 1989-90, 1990-91; 1995; 1998; 2001) adopted on Shackle's insights in seeking an alternative theoretic framework in sharp contrast to that of mainstream economics. Specifically, Katzner totally reformalized Shackle's decision theory as mathematical modellings.

Hence it should be emphasized that, in this paper, "Shacklean theory" does not literally mean "Shackle's theory" but represents an interpretation of Shackle's decision theory based on Katzner's formalization. So, from now, readers can regard the adjective, "Shacklean" as Shackle-Katzner framework. It is unrealistic to satisfactorily reflect Shackle's abstruse insight on the human decision-making and the related epistemology with a single current of formalization. Thus, the interpretation presented here may have some limitations in hermeneutical sense and creative deviation from strict reading of shackle's original monographs. Instead, we can expect to open a room for diversified interpretations and to expand the range of applications in order to alleviate the current isolated status of Shacklean theory.

The purpose of this work is two folds. First, based on Katzner's formalization of Shacklean decision theory, we will provide a new narrative onto Shacklean decision process to construct a common ground of interpretation in comparison to the expected utility theory (hereafter, EUT). Second, we will decompose the algorithm of the decision model into several sub-steps and correspond each of them between the EUT and Shacklean framework. In this work, the originality of Shacklean framework is clarified in terms of this common stage structure as follows.

Constructing the List of the possible future states

To define an uncertainty measure like probability or potential surprise, first, the list of possible future states should be defined. Shacklean decision theory and EUT are distinguished by whether that list is complete or not.

The degree of uncertainty

Next, a sort of numeric representing the degree of uncertainty should be constructed. The degree of uncertainty is defined as a functional form, whose domain is the list of future states and its range can be real numbers, possibly in the interval, [0, 1].

Constraint on the uncertainty measure

If the value of uncertainty measure is allocated from the given limit, *e.g.*, $\sum p_i = 1$, it is said to be distributive. Whether the degree of uncertainty is distributive or not is an important feature distinguishing potential surprise and probability.

Degree of uncertainty for the union hypothesis

For the event/hypothesis derived by the unions, *i.e.*, $A_1 \cup A_2 \cup \ldots$, if its degree of uncertainty can be defined by summing the uncertainty values of constituent events/hypotheses, then that uncertainty measure is said to be additive. The additivity of probability is also a main target of Shackle's criticism about probability calculus.

Payoff

The payoff will rely on two factors: which action the decision-maker takes, and which state is supposed to be realized. This means that the payoff is defined as a function of two variables, actions and hypothetical events. In the EUT, the payoff function is either objective payoffs or subjective utility, in which the attitude of the decision-maker to risk is congealed such as Arrow (1965)-Pratt (1964) measure. Meanwhile, Shackle did not explicitly reflect the preference in defining the payoff function.

Valuation of importance

By choosing an action in the thought experiment by the decision maker, two measures, degree of uncertainty and payoff are hypothetically determined. Then the next step is to process these two speculated data into a value-scale for the final decision-making. By processing these two numeric, the decision-maker will evaluate and rank the importance of each combination of payoff and the degree of uncertainty. From now, we introduce a new generalizing term, *importance function*, i, which is defined on all the combinations of uncertainty values and payoffs. By using the concept of importance, we can encompass both the expected utility, *i.e.*, the multiplication of probability and payoff,[2] of the EUT and the *attractiveness function* of the Shacklean framework.

Potential payoff

The "potential payoff" is distinguished to the payoff. The payoff value defined above is just nominal values. In other words, it can be given to the decision maker only when it is supposed to be realized. But at the moment of decision-making, the influence of uncertainty is not reflected yet in the valuation. In the EUT, once an action and a future state are supposed to be decided, the potential payoff is computed by the multiplication of payoff and its corresponding probability. Here, the probability is used to discount the value of payoff, and the importance in the EUT simply means the degree of potential payoff. Meanwhile, in Shacklean framework, although the potential payoff can be regarded as a particular form of attractiveness function, the attractiveness function itself is defined as the abstract form.

Final decision

Now the importance has been evaluated at each combination of payoff and uncertainty value. Then each action generates a series of importance values through all imaginable future states. At this step, this data should be transformed into a single representative value for each action. Then the final decision, in the EUT, will be completed by choosing the action giving the highest $\sum_{\Omega \ni \omega} u_x(\omega) p(\omega)$ or $\int_{\Omega \ni \omega} u_x(\omega)\, p(d\omega)$. In Shacklean framework, the outcomes of the importance valuation in the previous step are distinctive to the EUT and it requires an additional step to elicit the ultimate choice function. Details will be discussed later.

Based on this stage structure, we will compare and analyze each step between the EUT and Shacklean framework. In section 2, the outline of Shacklean decision process is briefly presented based on the Katzner's reformalization. In section 3 and 4, the two distinguished features of potential surprise, *non-distributivity* and *non-additivity* will be examined. In section 5, the ordinality of potential surprise will be examined. In section 6, the importance functions of two frameworks will be compared in detail. In section 7, the final functional step of Shacklean decision-making, *decision index*, will be prescribed as the functional representation of interval order defined on importance values.

2. Choice under uncertainty in Shacklean framework

Encountering a problem requiring a choice for the uncertain future, the decision-maker recognizes a set $\boldsymbol{X}$ of all available actions, and a set $\boldsymbol{\Omega}$ of all imaginable future states of the world. Being different to the probability theory, $\boldsymbol{\Omega}$ is supposed to be an incomplete list. Strictly speaking in the aspect of mathematical formalization, there is no meaningful difference between the incomplete list and the complete sample space because both cases commonly contain all the states which is currently imaginable by the decision-maker. However, the incompleteness of the list Ω determines the conditions in defining an uncertainty measure.

In the reformalized Shacklean framework, we also endorse a concept, *σ-field*, which is also used in the probability theory. A nonempty collection $\boldsymbol{F}^*$ of subsets of **Ω** is called *σ-field* over **Ω**, if for any A in $\boldsymbol{F}^*$ and countable collection $\{A_i | A_i \in \boldsymbol{F}^*\}$, it satisfies A^c in $\boldsymbol{F}^*$, $\cup_i A_i$ in $\boldsymbol{F}^*$, and $\cap_i A_i$ in $\boldsymbol{F}^*$. In Shacklean terminology, each element of $\boldsymbol{F}^*$ is called a *hypothesis*. For any A in $\boldsymbol{F}^*$, a hypothesis B in $\boldsymbol{F}^*$ is called *rival* to A if $A \cap B = \emptyset$.

Based on the recognition of available actions in ***X*** and hypotheses in $\boldsymbol{F}^*$ over **Ω**, the decision-maker imagines (1) the future payoff when a state of the world is realized together with his/her chosen action from ***X***, which is represented by a payoff function (*e.g.*, a utility or a profit function) defined on **X** × **Ω** to the real space **R**, and (2) the degree of potential surprise he/she would feel now upon the future realization of an element of hypothesis in $\boldsymbol{F}^*$. This notion of potential surprise is the original concept conceived by Shackle (1954, 1969), and redefined in functional form by Katzner (1986-87, 1998) as follows.[3] A potential surprise function on $\boldsymbol{F}^*$ is a function s: $\boldsymbol{F}^* \rightarrow [0, 1]$ such that: (*i*) For all A in $\boldsymbol{F}^*$, $0 \leq s(A) \leq 1$; (*ii*) For any (possibly infinite) collection $\{A_i\}$ of nonempty subsets in $\boldsymbol{F}^*$, $s(\cup_i A_i)$ = infimum of $s(A_i)$; (*iii*) If $\{A_i\}$ is a complete list of rival hypothesis and a partition of **Ω**, then $s(A_i) = 0$ for at least one i. If a hypothesis A in $\boldsymbol{F}^*$ has zero potential surprise value, *i.e.*, $s(A) = 0$, then the hypothesis A is said to be *perfectly possible*. If $s(A) = 1$, then the hypothesis A is said to be *perfectly impossible*.

Now two streams of information, payoffs and the potential surprise are given for each decision option x in ***X***, and they generate the two wing-shaped curves in center diagram of Figure 1, which indicates the potential surprise density functions $\mathcal{F}_a$ and $\mathcal{F}_b$ transposed to be defined over the conceivable possible payoffs resulting from actions *a* and *b* respectively. From now, we will simply call the graph of the potential surprise density function, the *potential surprise locus*.

Now the decision-maker is thought to focus on two pairs each consisting of a potential surprise and a payoff value which grabs the most attention of the decision-maker. It is said that these pairs are maximizing the attractiveness function. One pair of a potential surprise and a payoff value is associated with possible gains called the *focus-gain*, and the other pair with possible losses, and called the *focus-loss*. The attractiveness maximizing pairs for each action can be found at a tangency between an iso-attractiveness contour and the *potential surprise locus*. Iso-attractiveness contours for each action are elicited by *attractiveness function*, whose domain is the plane of payoffs and potential surprise values, *i.e.*, $\boldsymbol{R} \times [0, 1]$. Each iso-attractiveness contour is the set of all combinations of potential surprise and payoff in $\boldsymbol{R} \times [0, 1]$, which has a specific level of attractiveness in gains and losses, respectively.

[Figure 1]

In the left-hand diagram of Figure 1, a and b are two decision options in **X**. The diagram pointed to by curved arrow 1 indicates the determination of the two attractiveness maximizing pairs of potential surprise and payoff values from decision options a and b. The pairs of values for each action are then evaluated in terms of a *decision index* that expresses the decision-maker's final valuation of potential surprise and payoff. Here, it is worth noting that the attractiveness function excludes the less important combinations of payoff and potential surprise values generated by each action and restricts the focus of the decision-maker to only the most attractive pairs of payoff and potential surprise value. The decision index operates only on those selected pairs. Among those points selected by the attractiveness maximizing process, the decision-maker makes comparisons (the right-hand diagram after arrow 2) according to their values in the decision index. Then, the pair with the maximal value in decision index determines which *action* is to be finally chosen.

3. non-distributivity

Probability calculus premises the complete set of all possible states, called 'sample space', on which the domain of probability measure is constructed. Then the given value, 1, is distributed to each event by its degree of expected frequency of, or subjective belief for, the occurrence of each event. We will call this property *distributivity*, *i.e.,* the summation or integration of all probability values is to be 1. Under the fixed sample space, keeping the unity of the summation/integration implies that the outcomes of the experiment or the future state must be extracted from only the current list of possible states. If not, the current probability "distribution" should be redefined.

Many cases of modern decision theory, *e.g.*, in prospect theory (Kahneman and Tversky, 1979), the summation of decision weight $\pi(p_i)$, which is transformed from the probability, times the value function $V(x_i)$ at a payoff x_i, can be unequal to 1.[4] Like this, just the suspension of distributivity in constructing the uncertainty measure cannot be said extraordinary in modern decision theory. However, in Shacklean framework, the negation of distributivity is not just from the subjective distortion of probability and its corresponding utility value as the prospect theory. The kernel of Shacklean negation to the distributivity is elicited from the emphasis on the *ignorance* of human decision-making.

In Shacklean framework, ignorance comprehends not only ambiguity, *i.e.*, the incapability to evaluate and enumerate the accurate degree of possibility about the occurrence of the future event, but also the fundamental impossibility to constitute the full list of possible future states. Unlike simple problems of 'class probability'[5] like coin-tossing, obtaining the complete list of possible states is impossible in the actual business decision-making. Therefore, it is impossible to construct an uncertainty measure, whose values are distributed from the fixed unity. Potential surprise is introduced to explain such situation, which is a non-distributive measure of subjective uncertainty defined on an incomplete collection of future states. The potential surprise indicates just the emotional degree of current disbelief about the realization of a hypothesis.

Suppose that the potential surprise value of a hypothesis is zero. Perfect possibility in Shacklean framework does not mean that the decision-maker has the strongest confidence about the realization of that event, but the decision-maker cannot confirm any evidence, which blocks the progress of the situation toward that direction. In decision-maker's mind, future is open to that direction, but the exact plausibility is unknown. Shacklean framework intendedly supposes the room for the unspecified possibility beyond the current expectable list, which is called *residual hypothesis*. Specifically, in Katzner's formalization, the empty set $\emptyset$ in the space $\boldsymbol{F}^*$ of the hypotheses, which is constructed on $\boldsymbol{\Omega}$ of the imaginable future states, is interpreted as the residual hypothesis.

Note that, while the probability implies the degree of "certainty" or "belief", potential surprise is related to the degree of "uncertainty" or "disbelief". In other words, two measures have contrasting implication. Apparently, it is not pathological to acknowledge that infinitely many events have zero probability, and in Shacklean context, this is relevant to the perfectly impossibility having the unitary potential surprise. Hence, although there is no fixed constraint on the summation of the potential surprise, the spillover of the summed value of potential surprise than unity can be justified even in the familiar sense of probability. On the other hand, zero potential surprise means the 'lack' of disbelief. In the actual decision-making, it is not abnormal that a decision-maker has numerous, even infinitely many hypotheses which can be thought perfectly possible to happen. Like this, the potential surprise is not simply a reversed version of subjective probability. The non-distributivity is the natural result from the original meaning of potential surprise.

4. non-additivity

The next feature of Shacklean framework is the non-additivity of the potential surprise. If multiple hypotheses are connected as a single hypothesis, *e.g.*, *tomorrow it will snow or it will rain*, the degree of confidence about this new hypothesis should be higher than or, at least, equal to the one of each hypothesis but not the less because the new hypothesis is explaining the wider range of future possibility. Apparently, this is a natural property of human reasoning, so this should be reflected in constructing the axioms of an uncertainty measure. In probability, it is the additivity: for any countable collection $\{\mathrm{A}_i\}$ of mutually disjoint subsets of the sample space, if $p(\bigcup_i \mathrm{A}_i) = \sum_i p(\mathrm{A}_i)$ holds, then it is called *countably additive*. If $\{\mathrm{A}_i\}$ is finite, it is said to be *finitely additive*.

The additivity of probability also determines the formula of the expected utility. Historically, calculating the expected utility by the weighted average is a customary heritage since the emergence of the concept of mathematical expectation around mid 17c. The first major step of the development in the notion of mathematical expectation is Huygens' book written in 1657; *Calculating in Games of Chance*.[6] Since the mathematical expectation contrived, it has been used to calculate the fair price of lottery or gambling such as the value of fair coin toss whose average payoff is $\frac{a+b}{2}$ where a, b is two payoffs of head and tail respectively. However, we can easily notice that such functional form of expected payoff/utility is not solely an accepted custom, but it is also a natural result inherited from the additive probability. For a given mutually exclusive collection $\{\mathrm{A}_i \mid i = 1,2, \dots n\}$ from the sample space $\mathbf{\Omega} = \bigcup_i \mathrm{A}_i$, the additive probability gives $p(\mathbf{\Omega}) = p(\mathrm{A}_1) + p(\mathrm{A}_2) + \cdots + p(\mathrm{A}_n)$. For a chosen action *x*, reflecting the scenario stream of all possible payoffs $\{u_x(\mathrm{A}_1), u_x(\mathrm{A}_2), \dots, u_x(\mathrm{A}_n)\}$ onto $p(\mathrm{A}_1) + p(\mathrm{A}_2) + \cdots p(\mathrm{A}_n) = p(\mathbf{\Omega})$ naturally generates $p(\mathrm{A}_1)u_x(\mathrm{A}_1) + p(\mathrm{A}_2)u(\mathrm{A}_2) + \cdots + p(\mathrm{A}_n)u_x(\mathrm{A}_n)$.

In Von Neumann and Morgenstern's monumental work (1944), the customary functional form of expected utility is beyond the range of proof but only the existence of utility function for a series of given bets is proven. Even when the probability is defined as subjective belief such as Savage (1954), the additive functional form of the expected utility itself is embedded in the sequential process of mathematical proofs to construct the subjective probability distribution and the utility function over a series of postulates about the choice behavior.

Meanwhile, in Shacklean framework, the additivity is regarded as a major caveat of probability. Corresponding to the additivity axiom of probability, the formal definition of the degree of potential surprise for the unionized hypotheses can be deduced from Shackle's original axiom stated in the descriptive manner as follows.

The degree of potential surprise associated with any hypothesis will be the least degree amongst all those appropriate to different mutually exclusive sets of hypotheses ... whose truth appears to the individual to imply the truth of this hypothesis (axiom 4).[7]

Shacklean criticism to additivity is not just toward the specific functional form but from his fundamental stance about the intrinsic limit of human decision in time. In Shacklean framework, the uniqueness of decision moment in time must be considered in analyzing the decision-making under uncertainty. Once the investment decision is finalized, the history will evolve toward the irreversible future. Under the impossibility of repetitive trials, simply adding the probabilities of disjoint events is never justifiable because it premises the consolidation of multiple events over different time paths into the single 'average' value, which is never realizable as a single event in the reality.

They (hypotheses) are rivals in the sense that the claim of each is a denial of the claims of all the others.[8] *In a non-divisible experiment, where the hypotheses are rivals which deny each other's truth, it is a contradiction to treat them as all true together in some degree.*[9]

Hence Shacklean framework did not accept the equality, $s(A \cup \mathrm{B}) = s(\mathrm{A}) + s(\mathrm{B})$. Instead, the above statement can be represented by $s(A \cup B) =$ infimum of$\{s(A), s(B)\}$ as Katzner's formalization, which means that rival hypotheses must be reduced to the one of the most believable hypotheses belonging to a single time path in the unrepeatable history.[10] [11]

The recent development in decision theory, for example, Choquet expected utility theory (Gilboa and Schmeidler, 1989) and maximin expected utility theory (Gilboa and Schmeidler, 1994) is constructed in terms of the non-additive probability, however still importance function of each action includes summation or integration. Not only that, many different currents of non-expected utility theories also include the addition in the importance valuation such as prospect theory (Kahneman and Tversky, 1979), rank-dependent expected utility (Quiggin, 1982) and many others. So, we can assert that the context of 'non-additivity' in Shacklean potential surprise stands on the most radical understanding about self-destructiveness and uniqueness of the decision-making in time.

5. scale of potential surprise

For the deeper understanding about the characteristics of potential surprise, let us think about the scale problem, specifically whether it is ordinal or cardinal.

As an illustrative example, suppose that two pieces of pencils are given: the pencil A is 6 inch long and the pencil B is 3 inch long. Then first we can notice from is the different length of pencil A and B. This is *nominal scale*. If we say A is longer than B, then the length of pencil has the property of *ordinal scale*. In *cardinal scale*, we can say that A is 3 inches long than B. If we can compute the ratio of length like A is two times as long as B, it is *ratio scale*.[12]

While order, differences and ratios of probability are computable and meaningful, the potential surprise implies just a sort of amorphous emotion. To examine the scale problem of the potential surprise in depth, we need to distinguish two usages of uncertainty measure in general. First, the uncertainty measure can be used in conjunction with the payoff stream to evaluate and to rank the importance of each pair of payoffs and uncertainty value. For instance, as we discussed in the previous section, probability in the EUT is summoned as a weight to be imposed to its corresponding payoff value. Second, the uncertainty measure can be independently used without considering payoff values. In this case, there is no need to consider the payoff stream. Just exemplary computations of probability in the first chapter of introductory probability textbook belongs to this case.

As the first case, since potential surprise should be used in conjunction with the payoff to derive the attractiveness, any kind of order-preserving functional representations of potential surprise are acceptable, provided that the final rank of attractiveness derived from both uncertainty and payoff is not violated. In the payoff side, Shacklean theory does not suppose the possibility accompanying subjective distortion of utility such as St. Petersburg paradox, Allais paradox, *etc*. The payoff in Shacklean theory is usually the uniquely represented value such as the discounted sum of yields from the investment, so the payoffs can be thought fixed, at least, as cardinal scale. Then the final order of attractiveness values is solely determined by the order of potential surprise. Since all order-preserving representations of the potential surprise gives the same ordering of the attractiveness, potential surprise can be regarded as the ordinal scale. In Shackle's original manuscript, there is a room to accept the ordinality of potential surprise.

Potential surprise can, of course, only serve a theoretical purpose if it is regarded as a function of some other variable or variables, and our assumption that there is nothing in its own nature to prevent it from ranging over all the real numbers within some interval does nothing to ensure that, in any concrete instance, potential surprise can in any sense be taken to assume all these numerical values. If, for example, the decision-maker has in mind, for some available action, just two distinct hypothetical outcomes, then, at most, just two distinct degrees of potential surprise will be involved.[13]

Since the usage of potential surprise is just a method of comparison, and even its role appears at an intermediate stage of the whole decision-making process, potential surprise can be defined as ordinal scale.

On the contrary, in the second case for the independent usage, not only the rank of potential surprise value, but also the intuitive meaning of each value should be preserved.

When weighting a hypothesis, the decision-maker will surely often say to himself: 'I find no ground for inclining either towards or away from this hypothesis: I can perfectly well imagine that the hypothesis will prove true, and I can perfectly well imagine that it will prove false.' In such a case the decision-maker is in effect saying: 'I can find ***equally plausible*** *both the hypothesis and its contradictory.' Now how can he translate this sentiment into terms of numerical probability? Not by assigning a probability of ½ to the hypothesis and a probability of ½ to its contradictory, for that contradictory may be capable of being split into a number of mutually exclusive hypotheses.*[14]

For let us suppose that all the answers which have been suggested to some questions are to be accorded equal status. Then this status will be lower, remoteness from 'certainly right' will be greater for each of them, the more of them there. This is the inescapable logical consequence of using a distributional uncertainty variable. ... In order that a distributional uncertainty variable may be used, it is necessary that the list of suggested answers should be specific and complete. [15]

When an incomplete set of future states is composed of more than two 'equally plausible hypotheses (½ of potential surprise) as the quoted paragraph above, the summation of those values must exceed 1. If the summation must preserve the unity, the original distribution of values should be redefined to a new uniform value less than ½ and then the intuitive implication of 'equal plausibility' disappears. In this example, we can find a trade-off between preserving the distributivity of the uncertainty measure or the nuance of specific values. The probability takes the former. If the intuitive meaning of specific values is to be preserved, such values cannot be arbitrarily transformable into another functional representation of ordering. Thus, the scale of potential surprise cannot accept arbitrary order-preserving transformations.

Consider the distance between the unitary potential surprise value of perfectly impossible hypothesis and 1/2 of potential surprise for hypotheses having the intuitive meaning: *'equal plausibility in both the hypothesis and its contradictory'*. Under preserving the specific meaning of two potential surprise values, the distance between two values must be preserved. Then the potential surprise seems more likely to be cardinal scale. In fact, Shackle also simply regarded that potential surprise is cardinal scale.

However, since the potential surprise is totally insulated from the frequentative context in either objective tendency or subjective speculation about it, but has only 'emotional' implication, it is still hard to conclude that every numerical value of potential surprise and the mutual difference have significant meaning. Let us think the second usage of potential surprise from a different perspective. If we put the payoff stream as a constant, then the independent use of uncertainty measure is reducible to a special case of the first category. For example, in computing the expected utility with the constant payoff 1 for all possible states, it is just identical to the sum or integration of all probability values over the given sample space. Then the problem of selecting a specific action disappears because the probability does not have decision-maker's action as its argument. Likewise, when attractiveness value can also be evaluated without considering payoff variations, if the different representations of potential surprise preserve the original ordering, then the attractiveness also generates the same final ordering. Thus, potential surprise can be ordinal scale. Even for the pedagogical purpose to understand the meaning of potential surprise, we can adopt any order-preserving representation of potential surprise ordering but keeping some exceptional and intuitively important values such as ½.

If the analytical range of the decision-making is extended to the case of ordinal payoffs such as the subjective utility, then it is possible to encounter the order reversion of attractiveness when specific functional representations of potential surprise and payoff (Carter, 1953) are chosen. For example, when the attractiveness value is defined as the Euclidean distance from the origin to each pair of potential surprise and the payoff value as $a(s,p) = \sqrt{s^2 + p^2}$ and the objective payoff is subjectively distorted as $u = 2p$, then the rank of the attractiveness order, *e.g.*, $(s,p)_A = (\frac{9}{10}, \frac{1}{10})$ and $(s,p)_B = (\frac{3}{10}, \frac{8}{10})$, can be reversed. In this case, the original ordering of attractiveness can be preserved by redefining the attractive function with respect to the given utility function (Gorman, 1957). Hence, for an arbitrary representation of ordinal potential surprise, the representations of ordinal utility function and attractiveness must be determined together.

6. Importance function

The expected utility/payoff of an action x is decomposable into two steps. First, computing the importance for each event A_i and an action x by $i(x, \mathrm{A}_i) = p(\mathrm{A}_i)u(x, \mathrm{A}_i)$. Second, for each action x, summing/integrating $i(x, \mathrm{A}_i)$ through possible events A_i. Then this assigns the ranking to the available actions. Since the result of this evaluation is scalar values, the last step is automatically picking the action maximizing $\sum_i i(x, \mathrm{A}_i)$.

In section 4, we have discussed about the implication of only 'summing' part in the importance valuation of the EUT as the natural result from the additive probability. Now let us examine the validity of each 'multiplicative' term, $p(\mathrm{A}_i)u(\mathrm{A}_i)$. Multiplying the payoff to its corresponding probability is intuitively justifiable only if there is a complete list of future states like coin toss. When the probability of the head in fair coin-toss is ½, then discounting the payoff value by multiplying ½ can be thought somewhat reasonable because ½ means the portion of the event over the entire possibility, represented by 1. But if only an incomplete list of possible future states is available, simply multiplying each payoff to the uncertainty cannot be the uniquely justifiable way. Actually, the expected payoff is a kind of Cobb-Douglass function, $p^{\alpha}u^{\beta}$, $\alpha = \beta = 1$ defined on the space of probability and payoff and it is increasing return to scale ($\alpha + \beta > 1$,). If the probability is subjective and the list of the future states are incomplete, why only $2(p, u) \to 4pu$ should be accepted? Definitely we can find logic to justify $2(p, u) \to 2pu$ or even other types of functional form.

Hence, in Shacklean theory, the importance function, which is called *attractiveness function*,[16] processing both potential surprise and payoff does not have any specific functional form. Attractiveness indicates the degree of stimulus felt by the decision-maker for each possible combination of payoff and potential surprise. We will see this in the next section. After assigning the importance value to all the combinations of potential surprise and gain/loss by the attractiveness function, the decision-maker is supposed to focus on, for each action, only a pair of the highest stimulation, 'focus-gain' and 'focus-loss.'

However, as a general framework of the decision-making, it seems that "stimulation" or "attractiveness" covers too narrow range of human emotion and cognition. In the positive context, why should only a specific mental activity, "exclusion" and "concentration" be modelized? Besides, excluding and concentrating does not actually happen in the model because the focus-gain and the focus-loss are already the results after considering all combinations of gain/loss and potential surprise by the attractiveness function. Furthermore, if the attractiveness function can exclude the pair of potential surprise and payoff having some "highest" importance, Shacklean theory cannot be justified in the normative sense. Therefore, to deal with these problems, the attractiveness-based interpretation of the importance function should be discarded. Instead, we will construct a different narrative by focusing on the meaning of the term 'importance' to capture a clearer vision about the context of Shacklean attractiveness in the broader frame.

[Figure 2]

For this task, first we should examine the shape of iso-attractiveness contours. For the case of positive payoffs, it is easily agreeable that the bigger payoff with the higher belief (lower potential surprise) is more important to the decision-maker. For the case of negative payoffs, it is not clear which direction is more 'important' to the decision-maker between the lower loss with the higher 'belief' and the lower loss with the higher 'disbelief. In the EUT, the predetermined $i(x, \mathrm{A}_i) = p(\mathrm{A}_i)u(x, \mathrm{A}_i)$ arranges the iso-importance map through the right-upward direction in the gain-side, and the right-downward direction in the loss-side in the plane of (p, u). Since the attractiveness does not have any pre-determined form like $p \cdot u$, the shape of iso-attractiveness contours in the loss-side is indeterminate.

If the importance of the loss-side in Shacklean approach also adopts a similar preference to EUT, *i.e.*, $i(x, \mathrm{A}_i) = p(\mathrm{A}_i)u(x, \mathrm{A}_i)$, then the lower loss with the lower belief (the higher potential surprise) may have the higher value. That is, the increasing direction of importance in the loss-side should be right-upward in the plane of loss and potential surprise as Figure 2.

[Figure 3]

On the other hand, as a different view, the lower loss with the lower belief (higher potential surprise) can be interpreted as the instability in such desirable, small degree of loss. Thus, the potential evolution of the situation to the bigger loss should be considered in advance. Hence it is also sensible to prefer the lower loss with 'higher' belief (lower potential surprise) because it allows the decision-maker to prepare the possible loss by hedging it in advance. Thus, in this case, the increasing direction of the importance is right-downward in the loss-side as Figure 3.

[Figure 4]

Note that the importance in two cases so far implies the degree of 'something valuable', not just a specific domain of human recognition, $i.e.$, emotional stimulus. Meanwhile, the expected utility pu also implies 'something valuable', which means the potential payoff.

Now let's consider the case that the higher loss with the higher belief (lower potential surprise) is more important. As a decision-maker, such tragic possibility must not be overlooked before making the decision. In this case, the increasing direction of importance in the loss-side should be left-downward (Figure 4). This interpretation of the importance, $i.e.$, the degree of how much it must not be overlooked is more adjacent to Shackle's original context, the degree of stimulus. Of course, it is also possible to imagine the case that the higher loss with the lower belief (higher potential surprise) gives the higher importance, which can be interpretable as paranoid's importance obsessing with the extremely rare disaster.

Finally, we can establish a generalized interpretation about the meaning of importance and categorize them with comparing to EUT as Table 5. Originally, the attractiveness in Shacklean framework gives just *two chosen points* having the strongest stimulus to the decision-maker. In the new interpretation proposed here, Shacklean importance is now an open concept to various kinds of 'loss' psychology as four cases. As we have observed through Figure 2, 3, 4 and paranoid's importance, the "range" of importance for a chosen action is thought as an interval to which the attention of decision-maker reaches. In figure 2, 3 and 4, the payoffs and the importance values are noted on the horizontal axis. In figure 2 and 3, the payoff values at the focus-gain and the focus-loss are denoted by G and L, and two importance values at the focus-gain and the focus-loss are the upper and the lower end points of the interval, $[\min i, \max i]$. Meanwhile, the case described in Figure 4 and the paranoid's importance, the range of importance is $[-max\, i(\text{loss}), max\, i(\text{gain})]$ for each action.

From the discussion so far, we can elicit the next topic to be examined. In the EUT, each action has a representative scalar value, $\sum_i p(\omega_i)u(x,\omega_i)$ or $\int_{\Omega\ni\omega} x(\omega)\, p(d\omega)$. So naturally the next step is just picking the action of the maximum value. On the contrary, outcomes of the attractiveness function are a series of intervals, and intervals are not simply comparable as the scalar values of the expected payoffs/utilities. Thus, to make the comparison for the final decision possible, an additional functional step is required to allocate the ranking to them.

[Table 5: Categorization of the importance]

7. Interval order

In the original construction, the standard point dividing the payoff space into two subspaces of gains and losses is constantly included in the graphical illustration of Shackle's monographs. However, if the presence of gain-loss dichotomy should be preserved explicitly, then it can impose an additional constraint in accepting Shacklean framework. On the contrary, although, in the EUT, gains and losses are already congealed within the algebraic and the order structure of the real number space **R**, the EUT does not need the explicit premise of the zero point dividing positive and negative payoffs because the signs of payoff values are absorbed and disappears in the calculation process of expected payoffs/utilities.

If we reinterpret the focus-gain and the focus-loss as the end points of importance interval as we discussed in the last section, then it is not necessary to explicitly separate the payoff space into positive and negative ones. Two end points of importance intervals are not necessarily separated by zero.

Then the next question is how to handle these interval data and assign the ranking as the final value for the decision-making.[17] In Katzner's formalization, it is conducted by the function, *decision index* defined on the four-dimensional space of the gain, the loss and their corresponding potential surprise values at the focus-gain and focus-loss, respectively. But here we will interpret the decision index as the functional representation of the interval order on the space of importance values. Mathematically, the interval order is defined as follows: for a set X, and $a, b, c, d \in$ X, if ($a \succ b$ and $c \succ d$) implies ($a \succ d$ or $c \succ b$), then the binary relation $\succ$ on X is called an *interval order*. In the current context, the set X is the space of importance values. Now let $[b,a]_x$ and $[d,c]_y$ be importance intervals given by two actions x and y. The representation theorems of interval order are well-known, and its detailed mathematical conditions is beyond the range of this work (Fishburn, 1985; Suppes et al., 1989).

Based on the concept of interval order, the importance can be represented by a pair of real valued functions $v \geq 0$ and w such that v is defined on the lower end points of importance intervals and w is defined on the end points of importance intervals, satisfying $a \prec b \Rightarrow v(a) + w(a) < w(b)$.[18] Here we can interpret that the function w preserves the order of importance values, and the function v determines the influence of the minimum importance value. Then the decision index can be set by $d_x(a,b) = \big(w(b) - w(a)\big) - v(a) > 0$. In other words, the decision index is reduced to two components: (1) $w(b) - w(a)$: how much the maximum importance dominates the minimum importance in an interval from each action, and (2) $v(a)$: the influence of minimum importance. Here, the function v reflects the attitude of the decision-maker toward the disappointing result, *i.e.*, the minimum importance value. By virtue of representing the decision index like this, it is possible to uniformize the impact of the minimum importance regardless of whether it is a loss or just a low but still positive payoff.

8. Closing remarks

It should be emphasized that the purpose of this study is not to 'internalize' Shacklean decision theory into the framework of mainstream decision theory, but to understand the original features of Shacklean approach under the common stage structure with the EUT. By exploring the algorithmic feature in the common stage structure, we can locate Shacklean theory within the map of modern decision theory to close the long period of oblivion. While the EUT seemingly has only a simple algorithm, *i.e.*, taking the action of the highest expected value, it has appeared that Shacklean theory has too many functional apparatuses including payoff, potential surprise, attractiveness and decision index. Also, ones who are not familiar with Shacklean decision theory may get impression that its narrative of decision-making relies on too restricted range of mental activity like surprise, stimulation, and exclusion. Hence the purpose of the new interpretation introduced here is to simplify the narrative of Shacklean decision-process and to clarify its formal structure with the common language of the decision theory.

Basically, the singularities of Shacklean framework are ultimately originated from the different domain and the focus of explanation. Shacklean theory is established for the problems of non-repetitive, irreversible, real decision-making. and it determines the distinctive formal structure of Shacklean framework as explained in table 6.

[Table 6]

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Tables and figures

Figure 1

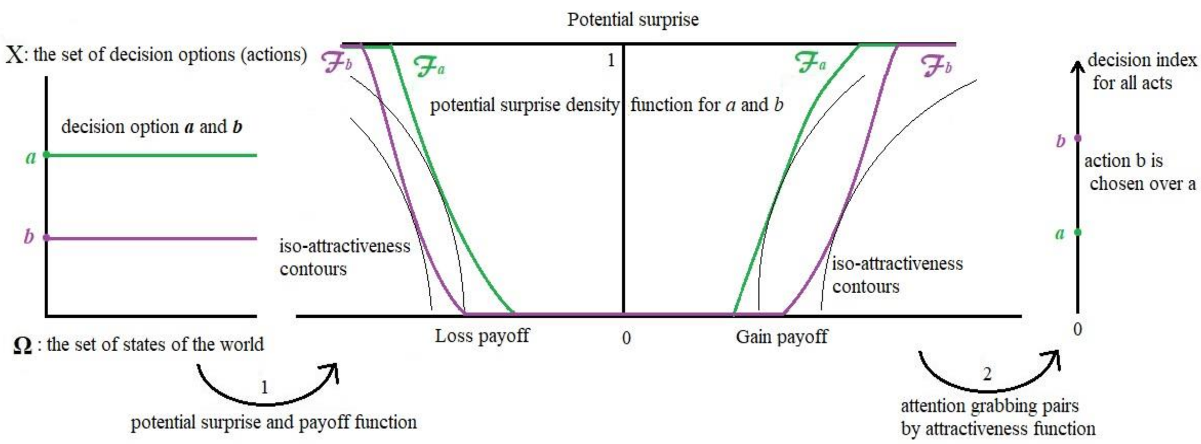

Figure 2

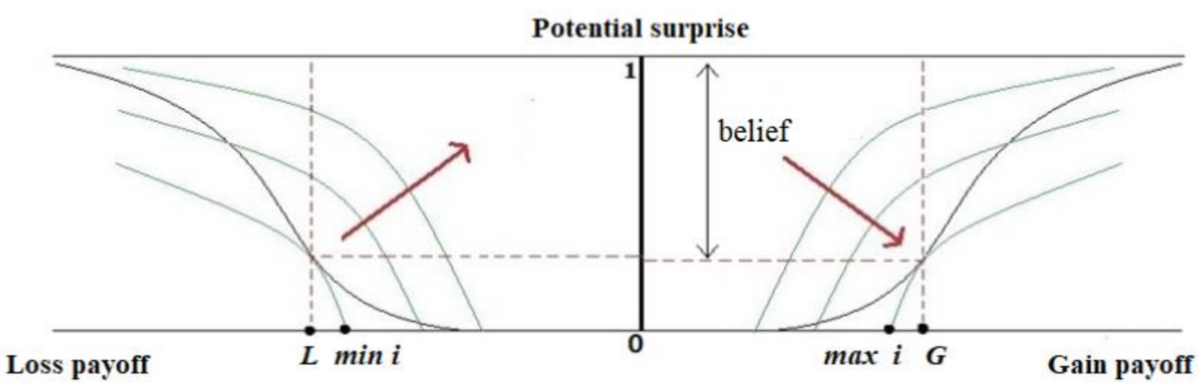

Figure 3

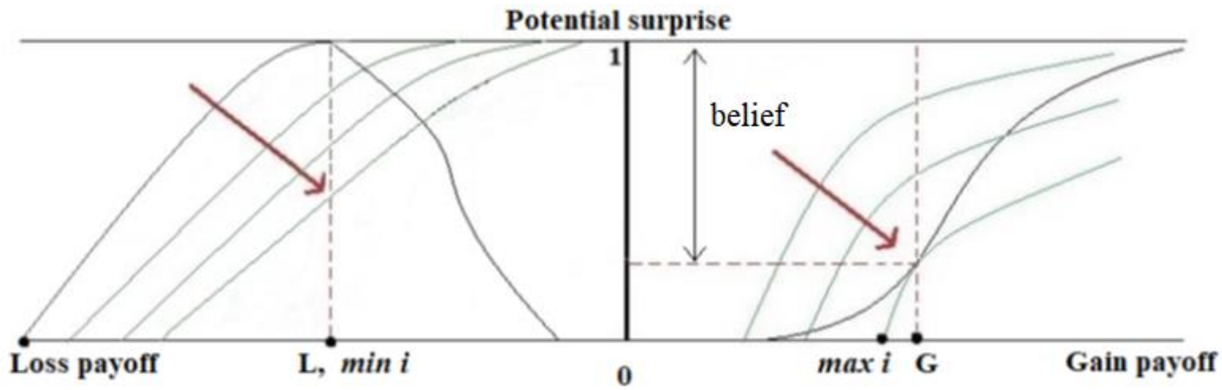

Figure 4

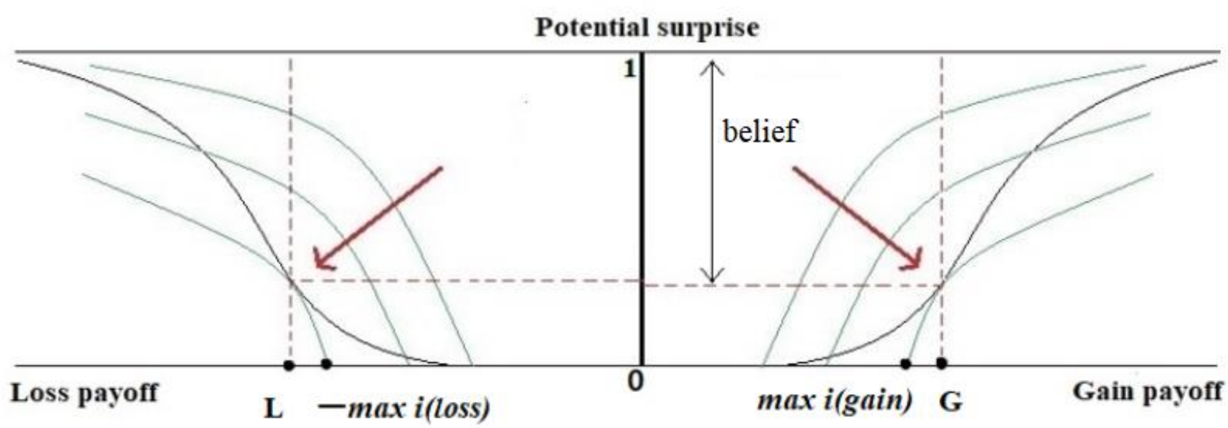

Table 5

		Formula	Increasing direction in the loss	Meaning of importance	The importance of each action
EUT		probability × payoff	Lower loss with lower belief	Something valuable: potential payoff	Scalar value: summation/integration of importance (potential payoff)
Shacklean	1	Abstract form			Interval: $[min\, i, max\, i]$ The range of importance from maximum potential loss to maximum potential gain
	2		Lower loss with higher belief		
	3		Higher loss with higher belief	Degree of attention needed → attention grabbing power (Shackle's original notion of attractiveness)	Interval: $[-max\, i(loss), max\, i(gain)]$ The range of attention needed
	4		Higher loss with lower belief (paranoid's importance)		

Table 6

		EUT	Shacklean
0. fundamental stance		Events of different time paths can be merged into the expected value.	The uniqueness of the decision moment : irreversible, non-repetitive : the single path of time (rival hypothesis)
		The complete list of future states	Predetermined incompleteness of the imaginable future states
1.Two information	Uncertainty measure	<u>Probability (objective/subjective)</u> : distributive, additive, ratio scale	<u>potential surprise</u> : non-distributive, non-additive (infimum), ordinal
	Payoff	<u>Utility function</u> - possibly ordinal - Subjective, the attitude toward risk is reflected in the utility function.	<u>Payoff function</u> - objective and cardinal
2. Importance	Importance (a point)	<u>Expected value</u> - A fixed functional form ($u_x \cdot p$) - Potential (discounted) payoff	<u>Attractiveness</u> - Unspecified functional form - Degree of importance which should be never overlooked
	Higher importance in the loss	Lower loss with lower certainty	Indeterminate
	Importance (an action)	- For each action *x*, $E[u(x)] = \sum_i U_x \cdot p$ or $\int u_x \, dp$.	- For each action *x*, $I(x) = [min\, i, max\, i]$ or $I(x) = [-max\, i(loss), max\, i(gain)]$

3. Completion of the decision-making	The final data from 2.	Well-ordered real values, $E[u(x)]$	Intervals of importance values, $I(x)$ Decision index, d
	The final choice	Choose x maximizing $E[u(x)]$.	Choose x maximizing $d(I(x))$.

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1 A. Greenspan (2004).

2 Strictly speaking, in the continuous form of probability, it is not possible to catch a probability value at a single value of random variable, so the multiplication of payoffs and probabilities is embedded in the integration form, $\int_{\Omega \ni \omega} x(\omega)\, p(d\omega)$ where x is payoff as a random variable, p is a probability measure, and Ω is the sample space.

3 In Shackle (1969, p.79-85), a series of axiom was provided. For the rigorous functional form, see p. 46-59 of Katzner (1998).

4 Kahneman and Tversky called this property subcertainty. In Kahneman and Tversky, 1992, the cumulative distribution function is adopted to derive the function of decision weight to consider subcertainty.

5 Mises (1949)

6 Huygens (1657). For detailed explanation on the development in the notion of mathematical expectation, see Hacking (1975).

7 Shackle (1969).

8 Shackle (1992), p.369.

9 Ibid, p.407.

10 Shackle's postulates of potential surprise do not follow the form of acute mathematical axioms like Kolmogorovian probability and have mutual interconnection to be considered within it. Thus, the axiomatic structure like Katzner (2006) may not be the only way to formalize it. For further discussions about the interpretation of Shackle's postulates, see Levi (1966), Ford (1994).

11 If rival hypotheses A "and" B occur *at the same time*, *i.e.*, in a single time path, then it must be represented by $A \cap B$ (not $A \cup B$). In $A \cap B$, since both payoffs A and B are given to the decision maker, the payoff of $A \cap B$ should be $u_{A \cap B} = u_A + u_B$. Since these three rival hypotheses, $A \cap B$, A/B and B/A, indicate totally distinguished events having different payoffs, they cannot occur in a single time path.

12 See Katzner (2006). Also, relevant discussions can be found in textbooks of measurement theory.

13 Shackle (1969, p.136)

[14] Shackle (1955, p.29)

[15] Shackle (1969, p.110)

[16] In Shackle's original construction, it is named *ascendancy function*.

[17] A similar concept to this in Shackle's original monographs is the gambler-preference map defined on the plane of the "standardized" focus-gain and focus-loss in Shackle's naming.

[18] See Suppes, et al. (1989), Bridges and Mehta (1995), Fishburn (1985), Aleskerov et al. (2007).