2018

A Note on Krugman's Liquidity Trap

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November 2018

Abstract: The 1998 stylized model of Krugman constituted a ground-breaking contribution explaining the long lasting Japanese stagnation as the consequence of a 'liquidity trap' situation featuring a negative natural interest rate. Our critique to such a proposal will focus on three aspects. First, we will question the logical structure of the model, providing an alternative interpretation of its closure. Second, we will argue that aggregate demand has no role in the explanation, as the cause for the persistent excess of savings over desired investment is the result of a supply side shock plus a financial rigidity on the nominal interest rate. Finally, we will discuss the restrictive assumptions needed to get a negative natural interest rate, the concept that lies at the foundation of the entire theoretical apparatus. Our conclusion is that the explanation offered within the 1998 contribution does not provide a satisfying rationale for the Japanese stagnation.

Keywords: Liquidity trap, Japanese stagnation, natural interest rate
JEL Codes: E31, E40, E52, E58

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1. Introduction

Krugman (1998) discusses extensively the Japanese prolonged stagnation of the late eighties – nineties, many times referred to as the Japanese 'lost decade', trying to trace back its ultimate causes. His aim was to provide a possible explanation for that situation by means of a new analytical concept. Later on such contribution has been often cited by the author himself as one of his best pieces, and has gained widespread acceptance in the academy as a path-breaking article. Its importance rests also in the fact that the analytical germ of what, after the Great Recession, will be called the 'demand side Secular Stagnation' strand of thought (Summers 2014, 2015) can be found in this contribution. In particular, in this stylized model we find a formal treatment of how a negative natural real interest rate can appear in an economy; when the latter is coupled with the zero lower bound on the nominal interest rate, an underemployment equilibrium emerges. The analysis carried out in this paper has a twofold goal: on the one hand, we want to ascertain whether the novelty constituted by an equilibrium position featuring a negative natural real interest rate is analytically justified, on the other hand, we analyse what is the role of aggregate demand in the explanation for the long term economic stagnation of the Japanese economy.

In our discussion we will highlight how the conclusions that Krugman drew from the model appear unwarranted from both a theoretical and an empirical viewpoint. In particular, we will see that the steps for the convergence to an equilibrium can be read differently from what the author proposed, and that the negative natural interest rate appearance is tied to a series of overtly restrictive assumption. Moreover, on the empirical side there is no confirm for the fundamental hypothesis of a decreasing potential output, which lies at the basis of his explanation. We further complement these arguments by maintaining that aggregate demand does not play any role in this theoretical apparatus, which basically relies on a supply side shock and a rigidity on the nominal interest rate to interpret the Japanese stagnation. Given that Summers has recently described the issue of Secular Stagnation for the US economy using a close analogy to the reasoning of Krugman, this work can also serve to recognize what are the roots of the demand side Secular Stagnation theory. In section 2 we are going to reconstruct the model of Krugman, which starts from an endowment economy and is then refined with the price rigidity case and an example to study investment, in section 3 we discuss the questionable assumptions needed to get the underemployment equilibrium and the irrelevant role that aggregate demand plays in such a model, while section 4 concludes.

2. The 1998 seminal contribution of Paul Krugman about Japan

Krugman, in the writing of the model, was mainly interested in retrieving the Keynesian 'liquidity trap' category from the old – fashioned IS – LM analysis.[1] This is the objective of the first half of the paper, while the second is devoted to applying the conclusions of the model to the Japanese case. In working out such task however, Krugman aimed also at updating that tool with three features: i) an intertemporal structure based on rational expectations, ii) an open economy treatment with foreign trade and capital mobility, iii) the role of financial intermediaries. In what follows, we are going to deal with the basic formulation of the model. Our interest points mostly to the general formulation, since in that framework the natural real interest rate is shown to be negative given some specific condition, and the zero lower bound on the nominal interest rate prevents the economy from attaining full employment. As Krugman formulates it, a 'liquidity trap' is

> "a situation in which conventional monetary policy has become impotent, because nominal interest rates are at or near zero: injecting monetary base into the economy has no effect, because base and bonds are viewed by the private sector as perfect substitutes". (Krugman 1998, p. 141)

Let us notice that by coupling this description with the negativity of the natural interest rate, we get the same picture described by the intuition of Summers (2014, 2015) about the US stagnation. Summers handles the US case proposing a reasoning in which the real natural interest rate turns negative when the demand for investment is peculiarly weak while the supply of savings is high. This causes the emergence of an underemployment situation since the zero lower bound on the nominal interest rate and the low inflations prospects prevent the Central Bank from being able to hit the equilibrium real interest rate. Therefore, it is in our opinion interesting to discuss the 1998 model as the first in which a negative real natural interest rate is presented as an impediment to full employment output realization, and to single out its relevance also in light of the recent discussion about Secular Stagnation.

Given the very stylized nature of the model we are going to review, it is in our opinion useful starting with a narrative description of the most important messages that Krugman wants to convey to the reader. Then, we are going to follow Krugman in putting those clues in a simple model. The American economist aimed at recreating in a modern intertemporal two periods model a situation of Keynesian flavour in which, given a rigidity on the nominal interest rate determined on

[1] Whose first proposer in a formalized model had been Hicks (1937).

the financial market, the real natural interest rate is not attainable by monetary policy.[2] The initial version of the model is built upon these hypothesis:

- agents in the model are described by a single representative agent with a given utility function;
- there are two time periods, today and tomorrow. Once tomorrow comes about, the economy remains in the state described by the second period situation. Only the transition between the two periods is described;
- there is no production, as only endowments of a single consumption good are given to the representative agent today and tomorrow;
- the Quantity Theory of Money holds, as the Central Bank can raise the price level by injecting money into the economy;
- there is price flexibility for what concerns the price of the single consumption good available, but there is also a rigidity on the nominal interest rate, which cannot go below zero;
- the price level of tomorrow is given.

Krugman, in the first place, wants to reproduce analytically a situation in which a liquidity trap emerges. As we will see, by supposing that the endowments of the consumption good are given for the two periods, and given the utility function of the representative agent, he can arrive at the real natural interest rate. When a specific assumption upon the amount of endowments available is made (namely, that the endowment tomorrow is adequately lower than the one of today), such a natural interest rate becomes negative. Given the zero lower bound on the nominal interest rate, the Central Bank can push the nominal interest rate at most down to zero, but no further; yet the attainment of the equilibrium real rate could be brought about by stimulating inflation. Indeed, even if the nominal interest rate is stuck at zero, raising inflation expectations can succeed in appropriately lowering the real interest rate down to the level of the natural rate, thus achieving equilibrium.

It is here that the hypothesis about the given price level of tomorrow acquires an important role. Such an hypothesis in fact represents a way to formalize the idea that agents are so convinced about the reliability and willingness of the monetary authority to preserve price stability that they will not believe that the price level tomorrow will be considerably higher than today. Krugman therefore fixes the price level tomorrow and utilizes such an hypothesis as a metaphor for the fact that, even though the Central Bank can today raise prices by boosting money supply, agents believe

[2] We are not going to deal with the differences that are recognizable between the new liquidity trap of Krugman and the old versions of Keynes and Hicks; for a treatment of this topic we refer to Taylor (2014), Boianovsky (2004), Kregel (2003).

that such a behaviour will not be kept in the future, and the CB will revert to its conservative approach. This hypothesis comes from the fact that, according to Krugman, entire decades have been spent by monetary authorities in convincing the public that their primary mission would have been keeping inflation low and stable; a single case of necessary monetary expansion will not be sufficient to change their ingrained expectations about the future monetary conduct. Were it not for such an hypothesis, indeed, an appropriate policy on the future money supply would have been sufficient to boost inflation expectations and thus get the negative real natural interest rate, whatever the rigidity on the nominal interest rate.

At this point, Krugman argues, the degree of price flexibility becomes crucial: if prices today are fully flexible, then it will be possible to attain the negative real natural interest rate by deflation today. Let us expand on this last point: the Central Bank target is to set a negative real interest rate equal to the negative natural rate, but it cannot do it directly by setting adequately the nominal policy interest rate because of the zero lower bound on it. In addition to this, the price level tomorrow is given, and thus the strategy passing through agents' expectations is not available either. Being outside equilibrium, if prices are flexible today, for a given level of prices tomorrow a drop of them today causes inflation expectations to rise. Despite the zero lower bound on the nominal interest rate and the fixed expectation about tomorrow prices, price flexibility today can still ensure the attainment of the negative real natural interest rate.

- *2.2 - The stylized model of Krugman*

After the narrative introduction, let us see the stylized model.[3] In what follows we have two periods: period 1 (today) and period 2 (tomorrow, variables with asterisk). After period 2 the variables are supposed to remain constant, with the change happening between the two moments in time. Thus, after the transition from the first period to the second, the economy reaches a steady state. In the economy we have that the only consumption good is inelastically supplied, and the representative agent derives his utility from consuming it.[4] The utility function of the representative agent has the following form:

[3] Krugman (1998), p. 143, labelled the "Minimalist Model".

[4] The representative agent is subject to a 'cash in advance' constraint. In the text the acquisition of the only available good is described as a 'two-stage process' (cfr. Krugman 1998, pp. 143); in the first one individuals trade cash and one period bonds, while in the second they trade cash for consumption. The aggregate amount of cash, constituting the money supply is exogenously given by the Central Bank. Obviously the amount acquired in the first phase limits the magnitude of attainable consumption. There is also space for government intervention in the form of open market operations and imposition of lump sum taxes, but the author does not explicit model these aspects.

$$U = \frac{1}{1-\rho}\sum c_t^{1-\rho}\,\beta^t \tag{1}$$

where c_t is consumption at time *t*, ρ the relative risk aversion, β the discount factor; there is, in this first version of the model, no investment. In a steady state position, when endowments and money supply remain constant, we would have the price level and interest rate so defined:

$$P^* = \frac{M^*}{y^*} \tag{2}$$

$$r^* = \frac{1-\beta}{\beta} \tag{3}$$

where P^* is the price level tomorrow, M^* the money supply tomorrow, y^* the endowment tomorrow, r^* the equilibrium real interest rate. The price level is set once we have the endowment of the unique consumption good and the money supply delivered exogenously by the Central Bank.[5] The equilibrium real interest rate in equation 3 is pinned down by the discount factor of the representative agent, which means that the term 1+*i** equals the ratio of the two periods' marginal utilities.[6] From the initial utility function we are going to arrive at a relationship between the price level today and the nominal interest rate. Such a relationship is derived by obtaining the ratio of the two periods' marginal utilities, and exploiting the Euler equation for intertemporal utility maximization:

$$U = \frac{1}{1-\rho}\sum c_t^{1-\rho}\,\beta^t \rightarrow U_1' = c^{-\rho}, U_2' = \beta(c^*)^{-\rho}$$

$$\frac{U_1'}{U_2'} = \left(\frac{c}{c^*}\right)^{-\rho} = \beta(1+i)\frac{P}{P^*};\; c = y \rightarrow\; 1+i = \frac{P^*}{\beta P}\left(\frac{y^*}{y}\right)^{\rho} \;: CC\ curve \tag{4}$$

Equation 4 is obtained by firstly employing the Euler equation, thus equating the ratio between the marginal utilities to the product of the discount factor of the representative agent times the gross real interest rate (the gross nominal rate multiplied by the ratio of the price level today and tomorrow), and then by substituting in the ratio between marginal utilities the two amounts of endowments, since as we have said they are wholly consumed (there is indeed no investment).

[5] In this simplified model the price level is thus arrived at by means of the Quantity Theory of Money, where the velocity of circulation of money is equal to 1, and so the equation takes the form described by Krugman.

[6] Let us notice that in an equilibrium position in which the interest rate is determined only by the discount factor of the representative agent it would not be possible to derive a negative natural real interest rate; indeed, as the beta factor is always supposed to be positive but lower than unity in order to represent the lower weight attached by individuals to future consumption with respect to present consumption, the equilibrium interest rate cannot but be positive in that formulation.

Equation 4 shows, Krugman comments, a simple relation between the nominal interest rate and the price level arising from the intertemporal choice of how to allocate consumption in order to maximize utility for a given level of endowments. It shows such an inverse relation for given future price level, present and future endowments, discount factor. The Central Bank can thus set the desired nominal interest rate by appropriately moving the current price level, which is determined by means of the exogenously given money supply, in analogy to equation 2:

$$P = \frac{M}{y} : MM\ curve \tag{5}$$

The real natural interest rate of the model is expressed without any reference to prices. Reformulating equation 4 in terms of the real interest rate we get:

$$1 + i = \frac{P^*}{\beta P}\left(\frac{y^*}{y}\right)^{\rho} \rightarrow (1 + i)\frac{P}{P^*} = \frac{1}{\beta}\left(\frac{y^*}{y}\right)^{\rho} \rightarrow \ 1 + r = \frac{1}{\beta}\left(\frac{y^*}{y}\right)^{\rho} \tag{6}$$

As we can see, with the subjective discount rate, the relative risk aversion and the two endowments as givens, the real natural interest rate is fully determined. Krugman suggests an interpretation for the latter formula:

> "[…]there is an *equilibrium interest rate*, which the economy will deliver whatever the behaviour of nominal prices. Meanwhile, since the future price level *P** is assumed held fixed, *any rise* in the current level creates *expected deflation*; hence higher *P* means lower *i*." (Krugman 1998, p. 145, italics added)

The suggestion of Krugman can be taken as a description of the natural real interest rate as an equilibrium variable which is set only by real phenomena. In the case under discussion, the individual preferences and the amount of endowments, which can serve as a metaphor for production. Thus, the behaviour of prices is immaterial to the determination of the natural real interest rate, while it is relevant when determining the nominal interest rate, as seen before (though, only the current price level variations can have an impact, since the future price level is assumed fixed by expectations). Therefore, with a given real interest rate, if the inflation rate is expected to drop when the price level today rises, for a given price level tomorrow the nominal rate drops as well to maintain the predetermined level of the real rate (and vice versa). Krugman depicts also graphically equations 4 and 5:

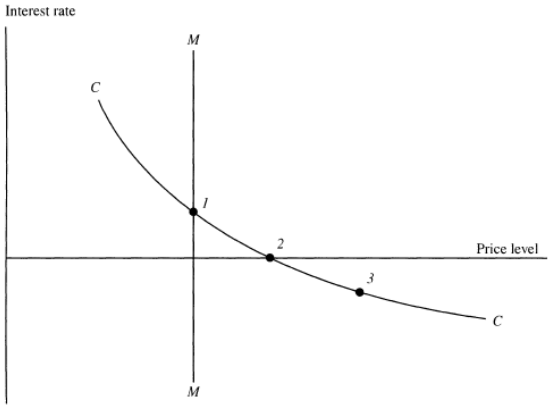

Figure 1 – The minimalist model, Krugman (1998), p. 145.

In the graph it is recognisable the above mentioned inverse relation between the price level and the nominal interest rate. In normal times, with a positive natural real interest rate the policy maker intervention can move the nominal interest rate along the CC line with the only constraint being set by the zero lower bound, which is placed on point 2 in the graph. The movement along the CC is guided exploiting the MM equation: by exogenously controlling the money supply the Central Bank can fix the price level today, and by this means it can also set the nominal interest rate through the inverse relation between these latter two variables described by the CC curve. Yet, Point 3 would be out of reach due to the fact that with a negative nominal interest rate, Krugman states, "[…] money would then dominate bonds as an asset" (1998, p. 146). Therefore, at the zero lower bound bonds and money become perfect substitutes, and agents do not have any incentive to acquire bonds with the additional money provided by the Central Bank.[7, 8]

The question then is: why the Central Bank should target a negative nominal interest rate, such as the one placed at point 3 in the graph? In order to answer this question, Krugman argues that indeed the model can well feature a natural real interest rate that is negative. The policy maker would thus try to reach equilibrium by moving the nominal interest rate into the negative territory, and in so doing it would eventually hit the zero lower bound. As mentioned, at that point the MM curve would lose its power to push the price level above its maximum attainable, since any

[7] As explicitly posited by Krugman, in any case the zero interest rate situation characterizes only one period bonds, whereas longer term ones are still yielding a positive rate. It may be said, though, that the interest rate relevant for investment decisions ought to be the long-term one, which is compared by entrepreneurs with the expected profitability of their project, when one tries to apply the insights provided in the article to a real world situation. The close attention to the short-term rate can thus be understood only when a strict relation between the short and long term rates is warranted, i.e. a well-behaved and fixed interest rates' structure.

[8] Given the zero lower bound on the nominal interest rate, Krugman argues, there would be a minimum rate of inflation and a maximum rate of deflation attainable. This is due to the fact that once the zero lower bound becomes active, there is no possibility anymore for the monetary authority to control the price level, which in correspondence with the zero lower bound has attained its maximum level. For a given future price level, this set the minimum rate of inflation, or maximum rate of deflation that the economy can deliver.

additional amount of money exogenously supplied would just be substituted for bonds. But then, how would it be possible for the system to be characterized by a *negative equilibrium real interest rate*? Krugman observes that it would be the case for a negative rate "[…]*if the marginal utility of consumption in period two is greater than that in period one, which will be the case if the economy's future output is expected to be sufficiently less than its current output*." Analytically this is translated into the following condition[9]

$$\left(\frac{y^*}{y}\right)^{\rho} < \beta \tag{7}$$

This condition directly follows from equation 6:

$$1 + r = \frac{1}{\beta}\left(\frac{y^*}{y}\right)^{\rho} \rightarrow r = \frac{1}{\beta}\left(\frac{y^*}{y}\right)^{\rho} - 1 < 0 \rightarrow \left(\frac{y^*}{y}\right)^{\rho} < \beta$$

We have that, in order to get a negative natural real interest rate, we need that for given risk aversion and discount factor parameters, the level of endowment tomorrow has to be sufficiently lower than the endowment today.[10] Since the natural real interest rate is set by employing the Euler condition for intertemporal utility maximization, if tomorrow there is a lower quantity of consumption good, the marginal utility of consuming it will be higher tomorrow than today. The fact that the representative agent would thus prefer at the margin to consume tomorrow rather than today will render negative the equilibrium interest rate.

At this point the policymaker recognizes the necessity to bring about in the economy a real interest rate equal to the equilibrium one, and tries to push the price level up by injecting money into the economy. Once the nominal interest rate hits the zero lower bound money and bonds becomes perfect substitutes, and thus the additional money injected is no more effective in raising the price level. At this point, Krugman argues, given the inability of monetary authority to hit the negative natural real interest rate, the flexibility of prices still ensures the attainment of equilibrium. Indeed, the author argues, an adequate drop in the price level today, for a given price level tomorrow, creates expectations of inflation and thus even with a zero nominal interest rate it is possible to get the equilibrium real interest rate. In the words of the author:

> "[…]the economy deflates now in order to provide inflation later. That is, if the current money supply is so large compared with the future supply that the nominal rate is zero, but the real rate needs to be negative, P falls below P*; the public then expects the price level to rise, which provides the necessary negative real interest rate. And to repeat, this

[9] Krugman (1998), p. 147, italics added.

[10] The discount factor beta is positive but lower than one, and thus the amount of endowments tomorrow has to be not only strictly lower, but sufficiently less than the amount today in order to let the real natural rate of interest be negative.

> fall in the price level occurs regardless of the current money supply, because any excess money will simply be hoarded, rather than added to spending." (Krugman P., 1998, pp. 147 – 148)

So far the model proposed by Krugman appears rather orthodox in his treatment of the possibility of the materialization of a liquidity trap: the real natural interest rate is determined by the preferences of the individual representative agent and the given endowments of a single consumption good. The solution of an intertemporal utility maximization problem delivers the equilibrium real interest rate. Monetary factors enter the picture only as a side element, which do not exert any influence on the evolution of output (which is equal to consumption) or the determination of the equilibrium real interest rate; the Quantity Theory of Money holds within the model. The novelty introduced by Krugman in the discussion thus rests fundamentally in two elements: the discussion of the possibility for the natural real interest rate to turn negative, and the consequence brought about by the presence of a rigidity on the nominal interest rate. Yet, with flexible prices the economy can still get the equilibrium position.

- *2.3 - The model with price rigidity*

The extension of the model discussed so far implicates the introduction of a rigidity on the current price level, which is in the section '*The Hicksian Liquidity Trap'* (pp. 148 – 150) assumed fixed. In this version we find some other novel feature: there is now a maximum capacity output y^f in period one,[11] while in period two output is still given by y^*. Again, the Euler equation for intertemporal utility maximization is exploited to get an inverse relation between consumption (and thus output, since there is no investment) and the nominal interest rate; therefore, consumption responds to interest rate variations by the standard intertemporal substitution between consumption today and consumption tomorrow.[12] Such a relation takes up the label of an "*IS curve determining real output*",[13] since a lower nominal interest rate boosts consumption today, and hence output today:

[11] It is important now to be very cautious when we deal with this 'capacity output'; it is difficult, in general, not to imagine that together with a concept involving a maximum level of output there will be the introduction of investment, capital and so on. Yet, the capacity output that Krugman mentions does not have anything to do with these other concepts. Indeed, output is again only constituted by consumption of the single good available which is now not given by endowments, but produced.

[12] Assuming a well-behaved intertemporal substitution mechanism, in which income effects are ruled out, a higher interest rate curbs consumption today since it provides an incentive to postpone consumption until tomorrow.

[13] Cfr. Krugman 1998, p. 148. Let us notice how the formulation of an IS does not require, according to Krugman, any consideration about the demand for investment and the supply of savings. On the contrary, the IS is obtained considering the amounts of consumption as given. Therefore, at the aggregate level given the two predetermined amounts of consumption and the discount factor, the real interest rate adjusts to reach intertemporal equilibrium.

$$\text{CC: } c = y = y^* \left(\frac{P^*}{\beta P}\right)^{1/\rho} (1+i)^{-1/\rho} \tag{8}$$

In it we thus find that for a given price level tomorrow, price level today, relative risk aversion, discount factor, output tomorrow, there is an inverse relationship between the nominal interest rate and the amount of consumption today, which is equal to total output today. As before, we also have a MM curve relating this time the supply of money to the output today, since now we have a level of prices today that is given by the hypothesis of rigid price for period 1:

$$\text{MM: } y = \frac{M}{p} \tag{9}$$

A higher supply of money can thus now stimulate consumption for a given output price: additional money provided to the public will foster the representative agent consumption. Graphically:

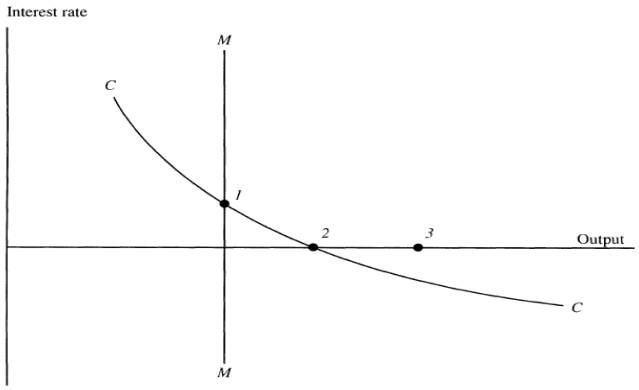

Figure 2 - Relationship between output and nominal interest rate; source Krugman (1998), p. 149

We can notice how the causal chain envisaged by Krugman starts from a decision of the Central Bank to boost output today; it can achieve such a target by supplying more money into the economy. Given the MM curve, for a given level of prices today output increases, and the MM is shifted to the right. The intersection with the CC curve delivers now the nominal interest rate, which within the Euler condition, and given the rate of inflation (both price levels, today and tomorrow, are given), ensures that intertemporal utility maximization is satisfied.

Given the downward sloping CC based only on consumption demand, and the MM curve, we see in the graph how output may be brought up until point 2. What happens when the maximum productive capacity is placed at point 3? Following Krugman, full employment output[14] would be

[14] Let us again recall that 'full employment' is an hazy concept within this model. Indeed, there is no formalization of investment, accumulation of capital, labour market etc. The model serves as a metaphor

out of reach. The reason, according to him, remains basically unchanged with respect to what we have been seeing before: the zero lower bound on the nominal interest rate prevents the attainment of equilibrium. In his words:

> "since the nominal interest rate cannot become negative, any increase in money beyond the level that drives the rate to zero will simply be substituted for bonds, with no effect on spending. And therefore no open market operation, however large, can get the economy to full employment. In short, the economy is in a classic Hicksian liquidity trap." (Krugman 1998, p. 149)

Thus, the reasoning remains close to what we have seen in the case with given endowments and a flexible price level today: if output tomorrow is sufficiently lower than full capacity output today, equilibrium requires a negative natural real interest rate. This time, and here we have the relevant difference with respect to what happened in the flexible price environment, the rigidity on the price level today prevents the economy from hitting the negative real natural interest rate by having deflation today. The consequence of the hypothesis inserting a rigidity on the price level today is thus that the inflation rate is now given: we have both the price level of today, and the one of tomorrow. As a result, when the real natural interest rate is negative the economy can be in a situation in which monetary policy is ineffective due to the zero lower bound, and inflation expectations cannot be revised. Hence, the economy may find itself stuck in a position in which it is not possible to stimulate consumption up to its capacity level. This should represent, in the idea of Krugman, a concrete real world situation in which the presence of a negative natural interest rate that is not attainable by conventional policies gives rise to underutilization of productive capacity and unemployment.

The hypothesis of price rigidity today is introduced to explain the impossibility to arrive at the negative natural real interest rate by means of a deflation today, and to show the point that Krguman wants to convey: without the needed price flexibility today, the only way to overcome the problem for monetary policy constituted by the zero lower bound is to try in any manner to remove the belief ingrained into the agents' minds about the future behaviour of the monetary authority. In other words, it is necessary to convince the public that the Central Bank will not stick to its long lasting commitment to ensure a low and stable inflation rate; from here it comes the famous policy prescription: "the Central Bank must promise to be credibly irresponsible". By freeing the future price level from being fixed, it becomes an instrument of monetary policy, which could therefore bring about the necessary inflation rate needed to get the value of the natural real interest rate despite the presence of the zero lower bound on the nominal interest rate. The ultimate issue

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which only provides some intuition upon the main elements Krugman wants to show. Then, the fact that the maximum capacity output cannot be reached in some particular case ought to demonstrate that in reality there may be cases in which an economy gets stuck below full employment.

causing a liquidity trap would thus be, according to Krugman, a credibility problem concerning the actions of the Central Bank, which has in that particular case the duty to reverse the traditional commitment to price stability in the long term.

- *2.4 - A negative rate of return on investment*

At this point we can follow Krugman into another step, which is treating investment in real capital within this framework of analysis. In the section devoted to the analysis of investment (cfr. pp. 150 – 151) Krugman dwells upon the meaning of a *negative* equilibrium interest rate in an economy with productive investment. As soon as the paragraph begins, Krugman in fact writes:

> "One way of stating the liquidity trap problem is to say that it occurs when the equilibrium real interest rate - the rate at which saving and investment would be equal at potential output - is negative. An immediate question is how this can happen in an economy in which, in contrast with the simple endowment economy described above, productive investment can take place and the marginal product of capital, while it can be low, can hardly be negative." (Krugman 1998, p. 150)

The rate of return, Krugman argues, is made up of two elements: the marginal product of capital and the expected rate of change of its price. Indeed, the 'expected rate of return' on an investment decision is so defined:

$$1 + r_t = \frac{R_{t+1} + q_{t+1}}{q_t} \tag{10}$$

where R_{t+1} is the marginal product of the asset, while q_t, q_{t+1} are the prices today and tomorrow of the asset on which the investment has been undertaken. In the presentation of the example proposed Krugman states that "This point is easiest to make if one considers an economy with not capital but land (which can serve as a sort of metaphor for durable capital)". The author employs an overlapping generation framework with only two generations; in it, by assumption, the young cohort work without consuming. The whole product obtained from cultivating is invested in land bought from the elders. Under these hypothesis, the price of land q in terms of the product obtained is arrived at as

$$q_t A_t = w_t L_t \rightarrow \; q_t = \frac{w_t L_t}{A_t} \tag{11}$$

where we have A_t the stock of land, w_t the marginal product of labour, L_t the labour force. The driver of the decline in the price of land would be, Krugman envisages, a decline in labour force; it is in other words determined by a demographic factor. Under the further assumption of an elastic demand for labour, a decline in population will make the price of land tomorrow in terms of

products obtained cultivating it drop.[15] Therefore, even if the marginal product of land is positive, by considering the overall rate of return, the latter turns negative when a capital loss obtains because the price drop of the asset more than outweighs its marginal productivity. Thus, as the author says, "[…]the expected return from investing in land can, in principle, be negative". The condition for the negativity of the rate of return on investing in land, that Krugman does not show, would be

$$r_t = \frac{R_{t+1} + q_{t+1}}{q_t} - 1 < 0 \rightarrow \; q_t - q_{t+1} > R_{t+1} \tag{12}$$

And it displays how, as said, in order to get a negative overall return the rate of price decrease ought to more than outweigh the positive marginal product of land. We have at this point seen how the author contends to have found an intuition to sustain the possibility of the emergence of a negative natural real interest rate even in an economy with investment.

3. Discussing the model

- *3.1 - The difference between the flexible price model and the fixed price model*

In the first place, let us question the validity of the logical step in which the author passes from the flexible price model with only endowments to the version in which there is a price rigidity. Krugman tells us that when the natural real interest rate is negative and monetary policy is constrained by the zero lower bound on the nominal interest rate, yet an equilibrium is attainable when there is price flexibility. When, in the following step, a price rigidity today is introduced, then equilibrium may not be reached, in so leaving an amount of consumption capacity today not exploited. Recalling equations 2, 4, 5, 6 we have a system of four equations involving a list of ten variables. Among those, four are endogenous $\{P^*, i, P, r\}$ and six exogenous $\{M^*, y^*, y, \rho, \beta, M\}$. In this latter list the first five are given parameters, while M can be varied according to the Central Bank decisions. Accordingly, the system runs with one degree of freedom:

$$P^* = \frac{M^*}{y^*}$$

$$1 + i = \frac{P^*}{\beta P}\left(\frac{y^*}{y}\right)^{\rho}$$

$$P = \frac{M}{y}$$

[15] As we will see below, the condition ensuring that the price level tomorrow will drop by a sufficient amount rests in the elasticity of the demand curve for labour.

$$1 + r = \frac{1}{\beta}\left(\frac{y^*}{y}\right)^{\rho}$$

With given M^* and y^* we get the value of P^*, and with given y^*, y, ρ, β we get r. We are thus left with two equations to be solved, the one for P and the one for i. By substituting equation 5 into equation 4 we get

$$1 + i = \frac{P^*}{\beta \frac{M}{y}} \frac{y^{*\rho}}{y^{\rho}} = \frac{P^*}{\beta M} \frac{y^{*\rho}}{y^{\rho-1}} \tag{13}$$

In equation 13 we have one equation in two unknowns, that are the nominal interest rate and the money supply. Once the Central Bank fixes the money supply we get the nominal interest rate. Overall, moving the money supply we get the couples $\{i, M\}$ that solve the system. Indeed, with one degree of freedom, any intersection occurring in the positive region between what in figure 1 above were the CC and MM curves is a solution. Thus, along the positive part of the CC curve, however the Central Bank modifies the money supply, the intersection with the MM curve delivers a value for the price level and the nominal interest rate that is an equilibrium solution. By hypothesis, the intersection occurring at a negatively valued nominal interest rate cannot be accepted, but we are left with the whole positive stretch to get a valid solution.

This serves to show that, even when we concede to Krugman the validity of every proposition he has made in order to get to his results, the description of his model in the terms he has used does not seem to entirely fit within the model. The author, in fact, tells us that when the natural real interest rate is negative and there is a zero lower bound, price flexibility can ensure that the disequilibrium situation will be fixed by a deflation today. When, in the case of price rigidity instead, such a deflation cannot occur, the system will remain stuck into a liquidity trap. If our view about how the model works is correct, instead, there is no need to deflate today in order to get the equilibrium; actually, there is no need either to hit the zero lower bound. The Central Bank can directly supply the amount of money that it deems necessary for the sake of getting the pair of price level today and nominal interest rate that would ensure the attainment of the natural real interest rate, be it either positive or negative. Each pair along the positive region of the CC curve would be a solution, and the model does not seem to need deflation to reach equilibrium.

- *3.2 - The role of aggregate demand*

We have seen how Krugman gauges the shrinking of the Japanese population as the main cause for the materialization of a negative natural real rate of interest. The decrease in endowments in the first model, the decrease of capacity consumption in the version with sticky prices and the decrease of population in the example with land all point to the same meaning for the metaphor that the author wants to convey. The message is actually that a decrease of the population casts shadows on the future evolution of the potential output of the economy. Even though it is an extremely stylized model, the assumption of Krugman about population decrease seems to be quite demanding because we are not speaking of a slower growth rate, but of an absolute value drop. Nevertheless, in the case of Japan the hypothesis seems not so distant from reality, as we can see from the actual data for the Japanese working age population growth rates of the last decades:

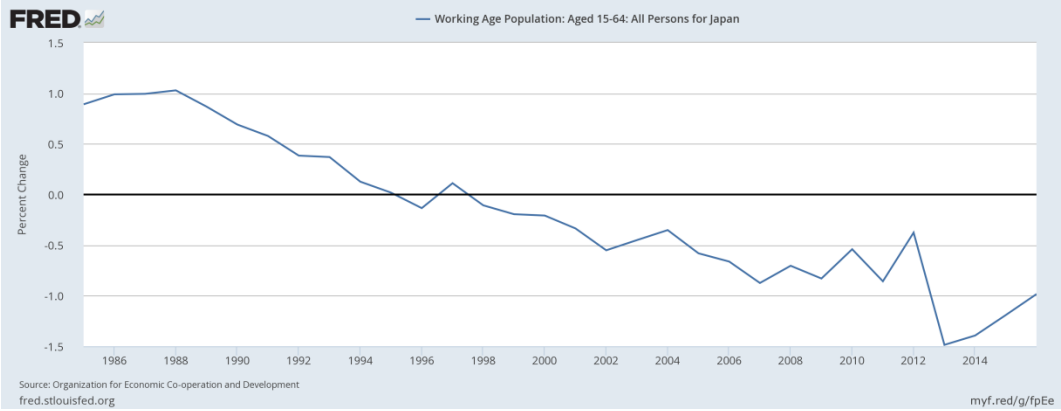

Figure 3 – Working age Japanese population pattern (age 15 – 65, years 1985 – 2015, source: FRED database)

Figure 3 shows how the working age Japanese population has been shrinking since the middle '90s. Therefore, the concern of Krugman does not seem to have been groundless. Indeed, forecasts about the population's foreseeable trend made Krugman suppose that the natural real interest rate would have turned negative, making the Bank of Japan impotent because of the zero lower bound on the nominal rate of interest. There are two main problems, though, about the preoccupation of the author.

First of all, even if one is willing to employ a neoclassical growth model in its simplest form, as the model of Solow,[16] to study the Japanese case, yet there is at least another factor to be considered along with population growth, i.e. the evolution of technical progress. Let us suppose, for the sake of discussing the point, that the neoclassical Solow growth model holds. We can thus

[16] Cfr. Solow (1956).

think about the evolution of the potential output of an economy as driven by exogenous population growth and exogenous technical change. This means that even if population decreases, there is at least another factor to be evaluated when forecasting the future growth of the economy; a downsize in population can be compatible with a growing potential output if there is technological improvement. Having a glance at some data about potential GDP growth for Japan confirms that the potential has been continuously growing throughout the years:[17]

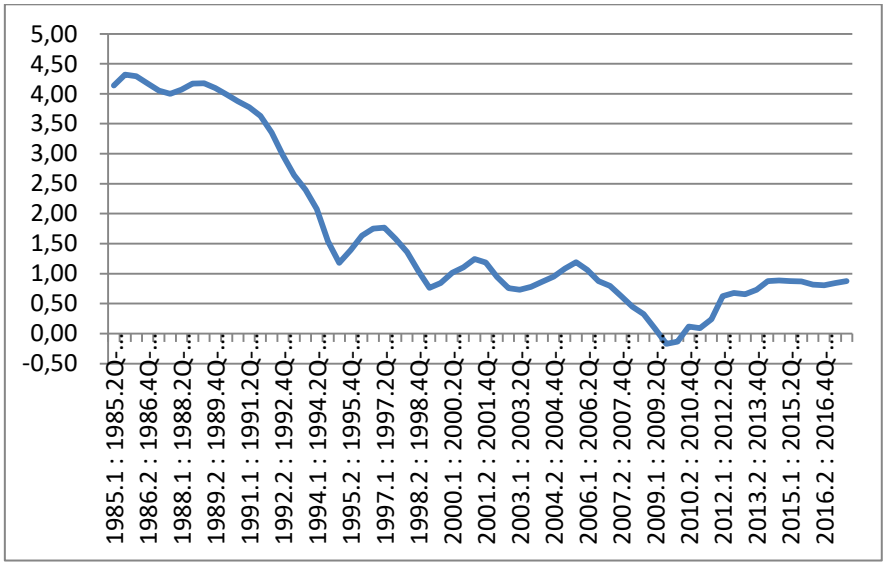

Figure 4 – Japanese potential output semi-annual growth rate estimate (1985 – 2016) (source: Bank of Japan).

Potential output growth exhibits an increasing pattern that tends to empirically disprove the theoretical apparatus of Krugman. Despite slow and even negative population growth rates, potential output has been growing all along the period taken into consideration,[18] even if its growth rate has remarkably decelerated across the years. So, even though population exhibits the trend reported by the author, potential output does not confirm a fundamental hypothesis of the model: its value tends to increase. Even remaining confined to a neoclassical explanation of growth as we have hypothesized, one may say that despite the decrease in the working age population, technical progress has more than compensated such a decrease, thereby permitting to potential output to keep on growing.

The second point that we want to highlight regards the ultimate causes of stagnation which are traceable in the model of Krugman. At first glance the model of Krugman is aimed at looking

[17] Data taken from the Bank of Japan website, which refers to the study of Kawamoto et al. "Methodology for Estimating Output Gap and Potential Growth Rate: An Update", Bank of Japan Research Paper, May 2017. The series shown is the semi-annual percentage change of the potential output estimate for the Japanese economy obtained by means of a Cobb – Douglas aggregate production function and calculated as the contribution to growth provided by the growth rate of the availability of capital and labour, and the growth rate of the total factor productivity.

[18] The only exception being the year 2009, in which there has been a slightly negative growth rate.

for explanations of the Japanese stagnation in light of the possibility of the reappearance of the old-fashioned liquidity trap. To work out such a task the author utilizes a model that in principle ought to explain a lack of investments due to the impossibility for the interest rate to sufficiently decrease. Therefore, it may seem that the problem rests in a lack of investment demand. But what does cause the equilibrium point to be characterized by a negative natural interest rate? As we have discussed right above, in the author's belief it is the pattern of the population growth the main driver of the collapse of the natural real interest rate. In this light, the logical process seems to be rather different from the framing in terms of a lack of demand. Indeed, the reconstruction of the meaning of the model appears to be reasonably described in these terms: given an exogenous drop in the main determinant of economic growth, which is population growth, the natural interest rate becomes negative; this, *per se*, would not give rise to specific troubles, were it not for the presence of a constraint on the operations of monetary policy. Being a zero lower bound on the nominal interest rate present, it is not possible to reach the negative natural real interest rate, and thus investment demand cannot be adequately stimulated. Let us for a moment assume that there is in the model the possibility for the Central Bank to set directly the nominal interest rate in the negative territory; in other words, let us temporarily remove the zero lower bound. In that instance, even though the natural rate can turn negative, a conventional monetary policy would be sufficient alone to bring the economy in an equilibrium position. If such a reconstruction is correct, the lack of demand generating the economic stagnation in this analysis seems to be a by-product of a supply side shock plus a rigidity on a policy variable.[19] The upshot of the argument is thus that the demand side concern only appears at the end of the logical process, even though the excess of savings and the lack of demand are obviously two faces of the same coin. Indeed, stating that there is an amount of savings which is not absorbed by investment is tantamount to saying that aggregate demand falls short of potential output, but the message in terms of what is the real cause for a stagnation is altogether different.

- *3.3 - How investment is introduced in the model*

In the presentation of the formal model of Krugman, we have shown how the author introduces investment once the conditions for a negative natural real interest rate had already been described. According to him, passing from a model with only consumption to a model with production does not significantly changes the results of the analysis. In what follows we are going to argue that the example presented by Krugman is based on a series of hypothesis which are so restrictive to

[19] Krugman, in the empirical section discussing the supposed halt of private investment characterizing Japan in the nineties, continuously refers to population growth as the main driver of sluggish economic performances; cfr. Krugman (1998, pp. 171 – 174).

seriously cast doubts about whether a treatment of investment in those terms can really be considered acceptable, even granting as convincing the results achieved in the model with only consumption. In the first place, it appears hardly acceptable to open a section about investment by stating that the argument will be treated using land as a metaphor for capital. Land and capital are obviously two altogether different factors of production; in other words, even within a neoclassical world in which capital is considered as a factor of production that in equilibrium earns a remuneration set by its marginal productivity, land and capital cannot be used as substitutes. Indeed, a non-producible asset in given supply cannot be compared to a producible factor which emerges from production decisions concerning capital goods. Even granting neoclassical theory that the available stock of capital is fully determined by the decision to invest the whole amount of past savings offered in the market, the treatment of capital cannot be analogous to the treatment of land. A non-reproducible factor in given supply as land can be indeed experience variations in its price determined by the demand for it. Capital is a produced factor that has a price given by its cost of production, and therefore needs a separate and different analysis. Despite the strong scepticism about the relevance of a model in which actual capital is not even mentioned, let us review the further restrictive assumptions needed to get the conclusions at which Krugman arrives.

We can see how, in order to get to the result that a negative real rate of return on investment can be an outcome of his model, Krugman needs to hypothesise that land is the only asset available for investment, consumption in period one is zero as the whole amount of product obtained by cultivating land is utilized to buy it from the old generation, the demand curve for labour is very elastic, that the negative rate of return materializes only temporarily.

The absence of any alternative for the investor permits to focus on what happens to the rate of return of the asset land, which can turn negative under the assumptions made by Krugman. But what would happen in a world in which there is only one asset and, say, money? After all, money is present in the stylized model of Krugman, and as we have seen, at the zero lower bound Krugman states that the nominal interest rate on bonds cannot go below zero since in that case money would dominate bonds. The same reasoning may be applied to this case: if there is only one investment possibility, and the forecasted rate of return on it is negative, then the presence of a store of value as money gives to the investor the possibility to preserve its purchasing power by simply keeping his savings in liquid form. Therefore, in this case the restriction difficult to accept rests in the fact that in the model with land all the amount of product obtained cultivating land has to be invested in an asset delivering a negative rate of return.[20]

[20] This point is connected also with the second restriction about the level of consumption today of the agents: they do not consume at all, but rather use the product of land and labour to buy the asset from the old cohort.

Another point necessary to get the conclusions advocated by Krugman is that the demand curve for labour ought to be fairly elastic: this assumption appears necessary in order to ensure that the price of land actually falls once the population tomorrow drops. Indeed, a decrease in the cohort of people available to cultivate the given land would render the marginal product of labour higher. Recalling equation 11 for the land price in terms of product

$$q_t = \frac{w_t L_t}{A_t}$$

we can see how for a given amount of land A_t, if labour force L_t decreases the marginal product of labour w_t increases and therefore there may the possibility that, despite the population drop, the price of land in terms of product q_t may not be affected by such an exogenous shock, or at least not to the extent needed to make the overall return on investment negative. The final effect of the price of land so defined depends indeed on the elasticity of the demand curve for labour. If the latter is steep, a drop in the available labour force may cause a remarkable rise in the real wage determined by the marginal product of labour. Such a rise may therefore prevent the necessary (for the author's conclusions) fall of the price of land. A fairly elastic labour demand curve, on the other hand, allows to say that even when the population drop is dramatic, the marginal product of labour does not rise by a degree sufficient to counteract the effect of the exogenous shock.

Lastly, Krugman states that the liquidity trap can actually be caused by a negative rate of return on investing in a specific asset, but it would seem that the result of his analysis depends in this case upon the willingness to study the value of what ought to represent a negative real natural interest rate outside a steady state position of equilibrium, in which relative prices would not change. Let us recall equation 10 and 12: in a transitory situation the supposed price drop assumed by Krugman can surely make the overall rate of return negative; but if we want to study an equilibrium position in which the natural real interest rate is negative, then we can see that it would be difficult to suppose that such a situation may occur. Indeed, in a steady state position in which relative prices do not change we would have that:

$$1 + r_{ss} = \frac{R_{ss} + q_{ss}}{q_{ss}} \rightarrow r_{ss} = \frac{R_{ss}}{q_{ss}} < 0 \qquad (14)$$

Equation 14 displays that in a steady state the rate of return on the investment, which ought to represent the natural real interest rate, can be negative in two cases: either the marginal productivity

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Krugman himself labels it a "special assumption"; it may be stated that the intertemporal decisions about consumption should at least ensure the possibility for the young generation to become old by consuming a portion of what has been produced. It would obviously be possible to redefine the argument in terms of 'consumption above subsistence', but in the case we are handling this would encounter the difficulty constituted by the fact that the price of land is defined in terms of the whole product obtainable on it.

of the asset is positive and its price is negative, or the marginal productivity is negative while the price is positive. Neither case would be acceptable, since a negative price for the asset would deprive the reasoning of economic meaning, while showing the coexistence of a positive marginal productivity of the asset with a negative real return on investing was the initial target of the author.

Generally speaking, the list of hypothesis that are necessary to get the conclusions of Krugman may perhaps be accepted singularly. Actually, it may be interesting to analyse an economy with only labour and land, an economy with only one asset, an economy with an elastic demand curve for labour, an economy operating outside a steady state equilibrium position. They are all particular cases that may provide interesting insights about some specific argument of research. What appears doubtful is their relevance when they are taken all together to answer a quite demanding research question. If such a question is indeed: is it possible to state that the natural real rate of interest in an economy with investment can be negative while the marginal product of capital is positive? Then it appears that a result obtained under a full list of *ad hoc* assumptions as the ones we have been reviewing is deprived of relevance, or at least of the possibility to convincingly answer the initial question.

4. Conclusions

Krugman in the late '90s formalized a new version of the old-fashioned liquidity trap, featuring a negative natural real interest rate, for the sake of analysing the long lasting Japanese stagnation. Such a seminal work is grounded on a stylized model that starts without capital and production. Even in such a basic case, we have been asking ourselves whether the description of the process leading to an equilibrium is sound and keeps consistent when different cases are analysed, as for instance in the passage from the scenario without price rigidity to the one with sticky prices. We have seen how even in his basic version the model does not seem to be coherent with its premises. When capital is subsequently introduced, the example featuring only labour and land did not seem to be particularly compelling for various reasons. The introduction of a rate of return on investment that separates the marginal productivity of capital from the real interest rate was the solution envisaged by Krugman, but the extremely a*d hoc* nature of several different assumptions casts doubts about the general reliability of that result. In addition, we have also seen how the entire reasoning has been based on a pure supply side shock, described in the form of a decrease in population. Then, the negativity of the natural real rate of interest and the presence of the zero lower bound on the nominal rate of interest resulted in an excess of savings over investment. Therefore, even though the final result of the analysis is that actually there can be a persistent lack of private investment, the role of aggregate demand in this explanation of stagnation appears of no relevance

when compared to a supply side factor, which is the real responsible for the slack in the economic performances. In conclusion, therefore, our claim is that the 1998 model of Krugman cannot be relied upon to provide a rationale for the Japanese stagnation. Moreover, the recent attempt of Summers to base his Secular Stagnation theory upon a way of reasoning analogous to Krugman's for what concerns the role of a liquidity trap and of weak aggregate demand in determining a prolonged economic slump ought to more satisfyingly deal with these issues.

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