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# Optimal Adaptive Departure Time Choices with Real-Time Traveler Information Considering Arrival Reliability

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OPTIMAL ADAPTIVE DEPARTURE TIME CHOICES WITH REAL-TIME  
TRAVELER INFORMATION CONSIDERING ARRIVAL RELIABILITY

A Thesis Presented

by  
XUAN LU

Submitted to the Graduate School of the  
University of Massachusetts Amherst in partial fulfillment  
of the requirements for the degree of

MASTER OF SCIENCE  
IN CIVIL ENGINEERING  
September 2009

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2009 September

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OPTIMAL ADAPTIVE DEPARTURE TIME CHOICES WITH REAL-TIME  
TRAVELER INFORMATION CONSIDERING ARRIVAL RELIABILITY

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Directed by: Professor Song Gao

ABSTRACT

When faced with an uncertain network, travelers adjust departure time as well as route choices in response to real-time traveler information. Previous studies on algorithm design focus on adaptive route choices and cannot model adaptive departure time choices (DTC). In this thesis, the optimal adaptive departure time and route choice problem in a stochastic time-dependent network is studied. Travelers are assumed to minimize expected generalized cost which is the sum of expected travel cost and arrival delay costs. The uncertain network is modeled by jointly distributed random travel time variables for all links at all time periods. Real-time traveler information reveals realized link travel times and thus reduces uncertainties in the network.

The adaptive departure time and route choice process is conceptualized as a routing policy, defined as a decision rule that specifies what node to take next at each decision node based on realized link travel times and the current time. Waiting at origin nodes is allowed to model DTCs that are dependent on traveler information. Departure time is a random variable rather than fixed as in previous studies. A new concept of action time is introduced, which is the time-of-day when a traveler starts the DTC decision process. Because of the efforts involved in processing information and making decisions, a cost could be associated with a departure made after the action time.

An algorithm is designed to compute the minimum expected generalized cost routing policy and the corresponding optimal action time, from all origins to a destination for a given desired arrival time window. Computational tests are carried out on a hypothetical network and randomly generated networks. It is shown that adaptive DTCs lead to less expected generalized cost than fixed DTCs do. The benefit of adaptive DTC is larger when the variance of the travel time increases. The departure time distribution is more concentrated with a larger unit cost of departure delay. A wider arrival time window leads to a more dispersed departure time distribution, when there is no departure penalty.

Key Words: Adaptive, Departure time choice, Real-time travel information

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# CHAPTER 1

## INTRODUCTION

### 1.1 Motivation

When faced with an uncertain traffic network, travelers usually shift their DTCs. Some travelers make changes based on their day-to-day experience, while others on pre-trip information, which can be received through internet or radio. We want to study the whole process of travelers' trip decision from home to destination, including when to depart and which road to take in response to online information. One of the most common assumptions is that users of the transportation network choose a route which minimizes the cost of travel. In this thesis, we only focus on the demand side of the traffic model, where we assume that individual choice will not affect the traffic network.

When uncertainties exist in traffic networks, it is necessary to choose an objective that accommodates this stochasticity. Arriving on time is always an indispensable consideration. Thus, minimizing expected travel time only as a proxy for cost is not enough, especially for those who have rigid time budget. They cannot afford being late. On the other hand, an early arrival will also result in inconvenience. Therefore, we consider applying arrival penalty as the terminal cost to capture reliability requirement.

Travelers generally implicitly place monetary values on time spent on travel and the early and late arrival delays. The value of a unit of travel time is usually different from

that of a unit of early or late arrival delay. In this thesis, the generalized cost of a trip is defined as the sum of the values a traveler puts on the travel time, early and late arrival delays. Travelers are assumed to minimize the expected generalized cost when making departure time and routing decisions.

With the growing prevalence of intelligent transportation systems (ITS) infrastructure that allows users to learn information about network conditions as they travel (for instance, from observing travel times on a variable message sign (VMS) or through an in-vehicle device), it is not only useful for transportation models to be able to account for the ability of users to update their routing decisions using information learned en route, but also the ability to make departure time adaptive. For instance, if a traveler receives information indicating that his or her intended route has a much higher cost than anticipated (such as in the case of a severe incident on that route), one would expect the traveler to choose a different route or different departure time. People can get the information by searching real-time information online at home, and then shift their departure time to avoid traffic jam or reported incident on the route. Once the traveler is en route, s/he might continue receiving real-time information based on which s/he adapts the route choices. We study how the adaptive DTC will benefit travelers compared to fixed DTC, both with succeeding adaptive routing choices.

In this thesis, we model travelers' DTC in response to real-time information, termed as adaptive DTC, in a stochastic time-dependent network. This is the first algorithmic

study on optimal adaptive DTC. We consider the whole decision process, thus action time is introduced to describe the time when travelers start to make decisions. Note that action time is not necessary the departure time.

Thus, the motivation of this work is threefold: 1) we seek a routing algorithm that introduce adaptive DTC in arrival penalty problem and that also allows for the routing decision to be altered en route as travelers take advantage of information learned regarding the future state of the network; 2) We want to study the characteristic of departure time distribution, 3) and how adaptive DTC benefit travelers under uncertain traffic network.

## 1.2 Organization

Chapter 2 surveys relevant literature in reliable routing problem and DTC model. Chapter 3 provides a framework of optimal adaptive departure time and routing choice and discusses in detail adaptive DTC in contrast to fixed DTC. An algorithm is presented to find an optimal policy minimizing the expected generalized cost. Chapter 4 carries out computational tests to study the effectiveness of designed algorithm and also the value of adaptive DTC. Chapter 5 gives the conclusion and also provides the future direction.

## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 Introduction

This thesis studies optimal adaptive departure time and route choice with real-time information considering reliable arrival time. Thus, this chapter describes existing research which is relevant to the problem at hand.

The reviews are divided into two categories. We first discuss research pertaining to DTC. It is followed by a review on routing algorithm considering travelers' reliability, including a description of routing algorithms incorporating online information access. This chapter concludes with a brief summary of the contributions of this thesis.

#### 2.2 Departure Time Choice Model

Departure time choice model is a growing interest over the years. It is due to a need to avoid increasing congestion levels. There are varied researches dealing with DTC problem, including the empirical study and evidence pertaining to departure time characteristics and travelers' response to uncertain travel time and analytical studies seeking the relationship between departure time and travel time distribution.

Some previous studies relevant to DTC focus on factors that affect trip makers' departure time, and how these factors affect those people with different socioeconomic characteristic, e.g., income, age, trip and purpose.

Small (1) develops a multinomial logit model of arrival times of car commuters in the San Francisco Bay Area. The results indicate that people are willing to shift their schedules by one or two minutes earlier if they saved some travel time. (2) employs another approach for studying the variables that affected an individual's likelihood to shift departure times. He uses a Poisson regression formulation to model how often commuters change routes and departure time per month. They model route choice and departure time separately based on the survey data conducted from morning work trip commute in the Seattle. The results indicate that the travel time on the most often used route and variable work time influenced the frequency of departure time changes. (3) also models morning commute departure time for workers in Singapore using multinomial logit and nested logit models. The results suggest that persons engaged in business were less likely to change their departure time. Commuters earning less than \$200 or greater than \$1799 per month seemed less likely to change their departure time. All the previously mentioned studies use discretized departure times; however there has been some exploration into using continuous departure time. (4) uses a joint discrete/continuous method to model the decision to delay departure to home from work in order to avoid congestion. A discrete model is used for the decision of whether or not to delay departure, and then the duration



of the delay was modeled using a continuous Weibull survival function. They come to the conclusion that traffic system characteristics dominated the delay decision while the socioeconomic characteristics and the characteristics of the area near the work location have a lesser impact. Noland and Small (5) develop an analytical model demonstrate the affect of travel time uncertainty on DTC and expected travel cost.

With the advent of advanced traveler information systems (ATIS), travelers will have access to real-time information about network conditions, which potentially change the way that travelers perceive actual travel time. For example, radio and real-time online traffic resource will have impact on trip-makers departure scheduling. Therefore the study pertaining how ATIS impact on travelers' DTC is an active research in recent years. (6) addresses departure time and route switching decisions made by commuters in response to ATIS. It is based on the data collected from an experiment using a dynamic interactive travel simulator for laboratory studies of user responses under real-time information. While travel times may be uncertain, these simulations emphasize how people learn about the shape of the congestion profile as opposed to uncertainties due to non-recurrent events. The literature on traveller response to congestion and ATIS (e.g., (7), 8),(9),(10),(11),(12),(13)) suggests that the most common response to (information about) congestion is to change departure time, although changing routes also occurs frequently. Jha (14) develops a Bayesian updating model to capture the day-to-day travel time learning mechanizing, where the information is provided by ATIS or previous experience.

This observation partly motivates the study on joint departure time and routing choices in this thesis. The propensity of travelers to switch departure time increases if more complete information is provided regarding both individual and system level and information is more specific: quantitative instead of qualitative, predictive instead of descriptive. (15) develops an analytical dynamic mixed-equilibrium model to describe the transportation system performance with routing guidance system. In this paper, we focus on deriving traveler's adaptive DTC.

Although the literature we described above modeling the DTC with the responds to real-time information, there are some significant differences in the information context we study. First of all, most of the studies are discussing DTC based on day-to-day learning. In the light of the information from experience or ATIS, travelers adjust their departure scheduling to maximize travel utilities. However, in this thesis, travel time information is provided, which is a final state. The learning process is within-day adaption. In this case, DTC could be adaptive regarding the reveled travel time information up to current time. Some studies also investigate the same information context as this thesis did. However, either they deal with the marginal travel time distribution, which unable to model adaptive choice or they study adaptive route choice on the dependency of joint link time distribution only but not pertaining to departure time problem. We will further discuss these studies in the next chapter.

### 2.3 Reliable Routing Problem

Traffic networks are inherently uncertain with random disruptions which create significant congestion, such as crashes, vehicle breakdown, weather, special events, construction and maintenance activities. Travel reliability therefore becomes a significant determinant in travel choices, especially for trips where arrival penalties exist, e.g. an important interview or catching a flight. Abdel-Aty et al. (15) suggest that travelers are interested in not only travel time saving but also reduction of travel time variability, which is the uncertainty for exact arrival time at destination. Thus, it is considered as an added cost to a traveler making a trip. Recker et al. (16) further uncovers the contribution of travel time reliability and variability in different risk-taking behavior. The results show that the reliability is indispensable in route choice. Liu et al. (0) find that the estimated value of travel-time reliability is significantly higher than that of travel-time. However not all evidences suggest risk-averse in travel choices. For example, Avineri and Prashker (0) find that in some cases, increasing travel time variability of a less attractive route could increase its perceived attractiveness. They suggest that it might be explained by the payoff variability effect: high payoff variability seems to move choice behavior toward random choice.

With the advent of advanced traveler information systems (ATIS), travelers will have access to real-time information about network conditions, which potentially could help travelers achieve more reliable travels. For example, a variable message sign (VMS)

could inform the traveler of an incident on a downstream freeway link, and the traveler could avoid it by taking an earlier exit. In order to assess an ATIS, a comprehensive model is needed to take into consideration the demand-supply interaction under the influence of ATIS (0). This paper deals with the demand side of the problem, which describes the optimal reliable departure time and routing decisions a traveler could make with the help of real-time information. We focus on algorithms that calculate optimal or near-optimal routes in an uncertain network with reliability measures considered.

### **2.3.1 Routing Algorithm considering Reliability**

The literature on reliable routing in random networks can be roughly classified into four categories. The first category includes works from earlier days, where the focus is on the properties of shortest paths in a random network, rather than a path that optimizes some measure of reliability. For example, Frank (21) investigates the probability distribution of the cost of the shortest path in networks with stochastic arc costs, while Sigal et al. (0) examine the probability that a particular path is the shortest. In the second category, utility functions are used and the expected utility maximization criterion of von Neumann and Morgenstern is applied. Loui (0) and Eiger et al. (0) show that if and only if the utility function is affine linear or exponential, dynamic-programming type algorithms can be used. Tsaggouris and Zaroliagis (0) study a more general situation where the path cost is non-additive such that dynamic programming is not applicable. They consider the case of a non-linear convex and non-decreasing function on two attributes. The third

category deals with travel time variance directly. Sivakumar and Batta (0) solve a shortest path problem with the constraint of maximum allowable variance of travel time. Sen et al. (0) solves a series of parametric integer programs to generate efficient paths regarding both expected and variance of travel time, where arc costs are normally distributed and correlated. Gao (0) presents a heuristic for finding adaptive routes with minimum variance in a network with general discrete link travel time distributions and general information situations. The fourth category uses terminal cost to capture travel reliability requirements where usually there is a desired arrival time at the destination. de Palma et al. (0) design an algorithm for finding paths that minimize a linear combination of travel time and early and late schedule delays, which is the difference between the actual and desired arrival times. The problem is shown to be NP-hard. Bander and White (0) study a routing problem in a time-dependent random network with a terminal cost as a function of actual arrival time, and an efficient heuristic is designed. Fan et al. (0) present an approach where the probability of arriving at the destination later than a specified target arrival time is minimized. Boyles (0) uses a polynomial function for the terminal cost.

### **2.3.2 Reliable Route Choices with Information Access**

Most of the works described above assume non-adaptive route choices, i.e. travelers are assumed to follow a fixed path and do not respond to real-time information, with the exception of (0), (0) and (0). Adaptive routing algorithms in both static and dynamic

stochastic networks have been studied (see e.g., [1,2,3,4,5,6,7,8]), however only a limited number of them consider reliability. Independent link travel times and arrival-time only information are assumed in [1] and [2]. Limited spatial and temporal dependencies and information on links just traversed are assumed in [3].

## 2.4 Contributions

Based on the literature review, this paper contributes to the knowledge base of DTC in the following aspects:

- A framework is established for the optimal adaptive departure time and routing choice problem based on generic terminal costs with stochastic dependency of link travel times. The departure time is a random variable rather than fixed as in previous studies.; Computational evidences are provided to show the value of adaptive DTC in decreasing generalized travel cost.

## CHAPTER 3

### ALGORITHM DOT-REL

#### 3.1 Problem Definition

##### 3.1.1 Network

Let  $G = (N, A, T, \tilde{C})$  denote a stochastic time-dependent (STD) network.  $N$  is the set of nodes and  $A$  is the set of links, with  $|N| = n$  and  $|A| = m$ . There is one single destination node  $d$ . There is at most one directional link from node  $j$  to  $k$ , and such a link can be denoted as  $(j, k)$ . The set of all downstream nodes of node  $j$  is denoted as  $A(j)$ .  $T$  is the set of time periods  $\{0, 1, \dots, K-1\}$ . A support point is defined as a distinct value (vector of values) that a discrete random variable (vector) can take. Thus a probability mass function (PMF) of a random variable (vector) is a combination of support points and associated probabilities. In this paper, a symbol with a  $\sim$  over it is a random variable (vector), while the same symbol without the  $\sim$  is its support point. The travel time on each link  $(j, k)$  at each time period  $t$  is a random variable  $\tilde{C}_{jk,t}$  with finite number of discrete, positive and integral support points. Beyond time period  $K-1$ , travel times are static, i.e. travel times on link  $(j, k)$  at any time  $t > K-1$  is equal to that at time  $K-1$ .  $\{C^1, \dots, C^R\}$  is the set of support points of the joint probability distribution of all link travel times at all times, where  $C^r$  is a vector of time-dependent link travel times with a dimension  $K \times m$ ,  $r = 1, 2, \dots, R$ .  $C_{jk,t}^r$  is the travel time of link  $(j, k)$  at time  $t$  in the  $r$ -th support point, with a probability  $p_r$ , and

$\sum_{r=1}^R p_r = 1$ . Under any support point, there is at least one path with finite travel time connecting any node to the destination node. Assume the traveler can make the decision at each node on what node to take next. A node can be used as both an intermediate or origin node. Waiting is allowed at an origin node, while at an intermediate node, waiting is not allowed. We use  $j^o$  to denote node  $j$  used as an origin node.

### 3.1.2 Routing Policy

The information obtained during the decision process is represented by *event collection*,  $EV$ , defined as the set of support points with the same link travel time realizations as observed. In other words, the traveler cannot distinguish between support points in an  $EV$  based on the available information. As more information is obtained, the size of  $EV$  will decrease (or remains unchanged) and the traveler is more certain about the network. Ultimately the traveler might obtain a singleton, at which time the network becomes deterministic. The composition and evolution of  $EV$  generally depend on the trajectory of the traveler up to the current time and node ( $0$ ).

The routing choice is made based on the current *state*  $x = \{j, t, EV\}$ , where  $j$  is the current node (origin or intermediate). At the origin node, travelers can decide whether to depart (and go to a downstream node) or wait for one time period depending on the information on traffic conditions. If the traveler decides to wait, at the next time period



s/he could be faced with a number of different event collections, each with a certain probability. The traveler could then again decide to stay or depart based on the newly acquired information. Since the departure time depends on the network conditions which are randomly distributed across different days, the departure time itself is also a random variable.

There is a higher level decision to make: the time to start the DTC decision process. This quantity is defined as the action time. Unlike the departure time, the action time is determined a priori without real-time information. The departure cannot happen before the action time. If there is no waiting cost, it is straightforward to set the action time at 0 so that all the possible departure times can be considered. Because of the efforts involved in processing information and making decisions, a unit cost could be associated with waiting at the origin. In this case, if a traveler chooses an early action time, s/he will have a large set of departure times to choose from, yet the potential waiting cost could be high; if s/he chooses a late action time, s/he will have a small set of departure times to choose from, yet the potential waiting cost could be low.

At an intermediate node, waiting is not allowed, since it is generally not possible to wait at an intersection of a transportation network at one's free will. and a decision on what node to take next is made based on the current state. Upon the arrival at the next node  $k$ , the traveler will be in a new state  $(j, t', EV')$  based on which another decision will be made. The process will continue until the destination node is reached.

A fixed DTC situation (with adaptive routing decisions) can be modeled by forbidding waiting at the origin node. In this case the departure time is a deterministic value and non-adaptive to real-time information. The action time is then identical to the departure time. In this thesis, the adaptive departure time and route choice process together is conceptualized as a routing policy. a *routing policy*  $\mu$  is defined as a mapping from optimal action time for each node to decisions on the next node  $\mu : \{j, t, EV\} \mapsto k, k \in A(j); \{j^o, t, EV\} \mapsto k, k \in B(j)$ .

The adaptive departure time and route choice process together is conceptualized as a routing policy. A *routing policy*  $\mu$  is defined as a mapping from all states to decisions on the next node  $\mu : \{j, t, EV\} \mapsto k, k \in A(j); \{j^o, t, EV\} \mapsto k, k \in A(j) \cup \{j\}$ .

We study the problem of finding the minimum expected generalized cost routing policy from all possible initial states to the single destination node, where the traveler has perfect online information (POI), that is, at any time  $t$ , the traveler knows the realizations of all link travel times up to  $t$ . POI is available for example from advanced in-vehicle communication systems. Results from POI also serve as benchmarks of performance for other imperfect online information situations. The reader is referred to (0) and (0) for a comprehensive discussion of information access. The output from solving the problem is an optimal routing policy, which gives the minimum expected generalized cost from each state to the destination. The optimal action time can be obtained by post-processing the optimal routing policy

### 3.1.3 Adaptive vs. Fixed DTC

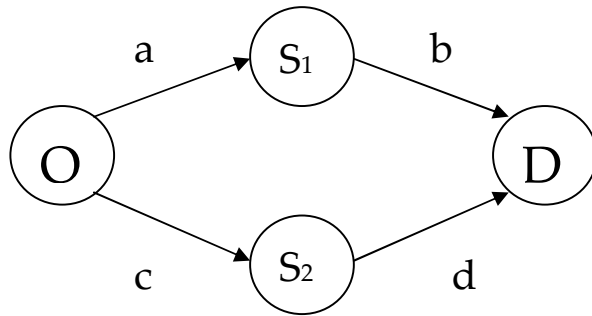
With adaptive DTC, the departure time is a random variable rather than fixed, depending on the network conditions that have been revealed during their departure making process. Therefore, travelers who follow an adaptive departure rule make decision online and start with different departure time each day. In contrast, travelers who follow a fixed departure time making decisions a priori, regardless of the network conditions before trips. A traveler can still make use of online information *en route*, and will end up with different set of links depending on revealed information. However, the performance of fixed DTC can be arbitrarily worse than the adaptive one.

In this section, we use one illustrative example (Figure 3-1) to show the difference between a fixed and an adaptive DTC. The network in Figure 3-1 has two support points of link travel time distribution, where  $M$  is a very large integer and thus larger than 10 minutes. We can view  $M$  as resulted from an incident which causes significant traffic jam. Time unit is minute. There is only one O-D pair, and we set desired arrival time at 7:30. Arrivals later than 7:30 will result in penalties. Penalty is increasing as the late minutes increase. Assume that every one minute late will result in 1\$. For instance, in the morning commute trip, getting late for work is not appreciable and could result in the loss of productivity and poor evaluation from the manager. A traveler is assumed to have POI,

and could simply observe the network to find the optimal route choice for departure time 7:00 and 7:10 respectively, because the minimum O-D travel time is 20 minutes. We present the optimal routes with adaptive and non-adaptive DTC in a time-space network (Figure 3-2 and 3-3). Time is shown along the vertical axis (the time axis), and the node symbol is shown along the horizontal axis (the space axis). Each point in this network represents a node-time pair  $(j, t)$  under support point  $C^1$  or  $C^2$ , and any link between  $(j, t_1)$  and  $(k, t_2)$  indicates that link  $(j, k)$  has a travel time of  $t_2 - t_1$ . Destination node with dotted circle indicates that  $t$  is far away from 7:30.

On departure time 7:00, the mapping between OD is to take a-b with travel time 30 minutes in support point  $C^1$  and c-d with  $20+M$  minutes in  $C^2$ . The expected travel time to depart on 7:30 is  $x(10+20)+y(10+10+M)$ . We multiply expected travel time with the value of time 0.5\$/min and plus the late penalty will get the expected generalized cost from departure time 7:00,  $0.5[x(10+20)+y(10+10+M)]+y*1*(M-10)$ . When departure time is 7:10, the optimal routing policy is to take a-b under  $C^1$  with travel time  $30+M$  and c-d under  $C^2$  with 20 minutes travel time. Expected travel time from departure time 7:10 is  $x(20+10+M)+y(10+10)$  and expected generalized cost is  $0.5[x(20+10+M)+y(10+10)]+x*1*(10+M)$ . The difference of expected generalized cost between two departure time is  $(0.5(x(10+20)+y(10+10+M))+y*1*(M-10))- (0.5(x(20+10+M)+y(10+10))-x*1*(10+M))$ . Therefore, if  $x>y$ , 7:00 is the optimal action time

and departure time for fixed DTC, otherwise when  $x < y$ , 7:10 will be the optimal action time. When  $x = y$ , there is indifferent between two departure time.

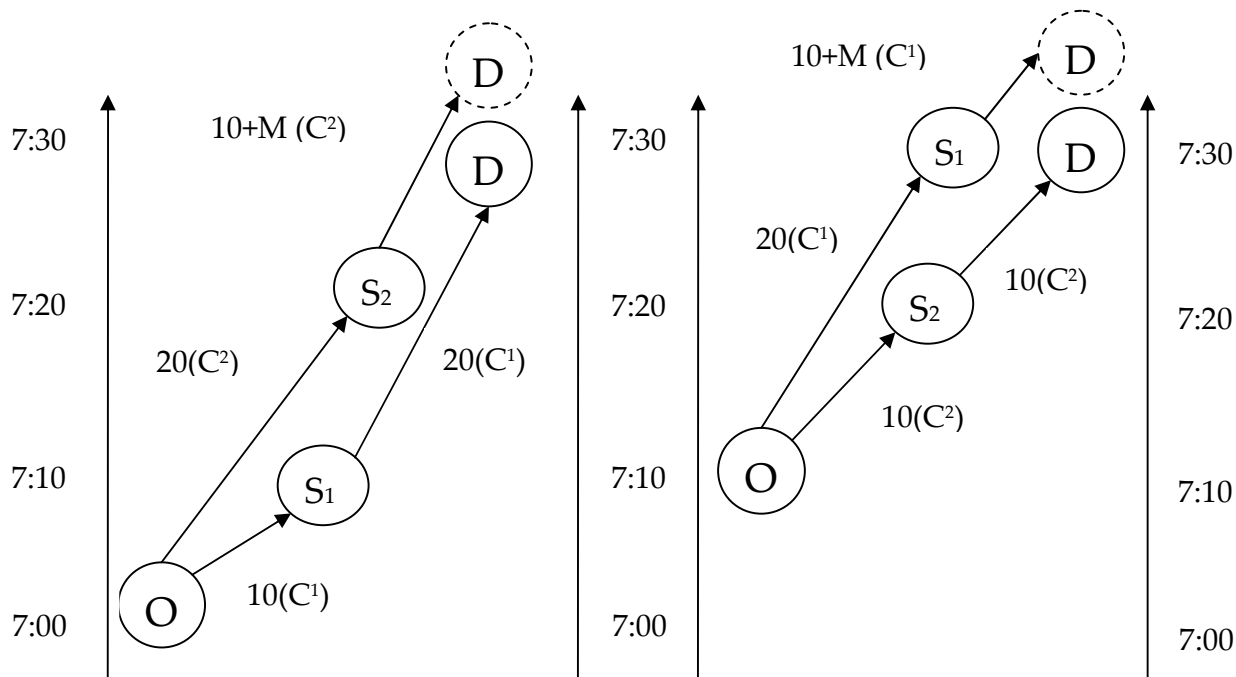


Time	Link	C <sup>1</sup>	C <sup>2</sup>
7:00	a	10	20
	b	n/a	n/a
	c	20	10
	d	n/a	n/a
7:10	a	20	10
	b	20	10+M
	c	20	10
	d	10	10+M
7:20	a	20	20
	b	10+M	30+M
	c	10	10
	d	30+M	10

Joint Distribution

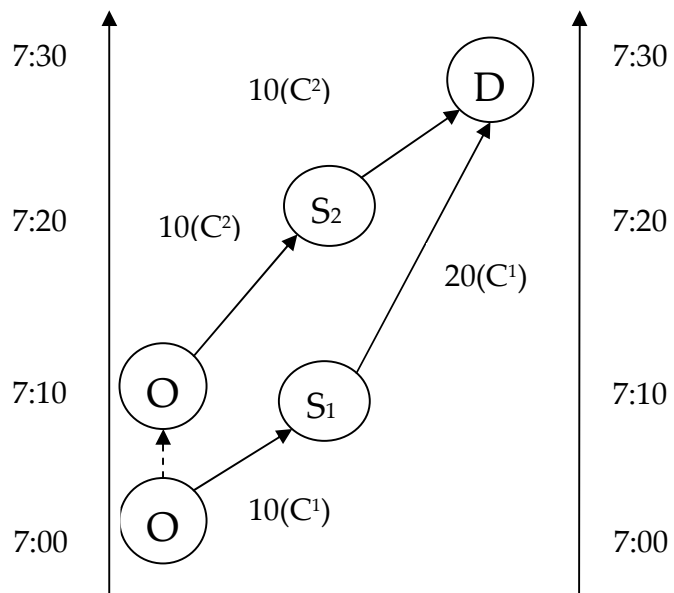
( $p_1 = x$ ,  $p_2 = y = 1 - x$ )

Figure 3-1 Adaptive vs. Fixed DTC Network



**Figure 3-2 Optimal Routing Policies with Fixed DTC in Time-Space Network**

From the time-space network, we can tell no matter which time we depart, it is always possible to encounter significant traffic jam and result large expected generalized cost. However, if we combine adaptive route choice with adaptive DTC, the performance will be always better than fixed one, because travelers can use the information to determine the departure time. Dotted line connected (O,7:00) and (O,7:10) is an imaginary link indicated traveler can stay at home for 10 minutes. In this case, we assume there is no cost associated with waiting at home. When under support point  $C^1$ , the optimal routing policy is to depart at time 7:00 and take a-b, While under  $C^2$ , it is to depart at 7:10 and take c-d.



**Figure 3-3 Optimal Routing Policy with Adaptive DTC in Time-Space Network**



The tables below summarize the travel time and expected generalized cost from adaptive and fixed DTC.

POI with adaptive DTC

EV \	Departure Time	Route Choice	Travel Time (min)	Expected Generalized Cost (\$)
C <sup>1</sup>	t = 0	a-b	30	15
C <sup>2</sup>	t = 1	c-d	20	10

POI with fixed DTC

If $x > y$				
EV \	Departure Time	Route Choice	Travel Time (min)	Expected Generalized Cost (\$)
C <sup>1</sup>	7:00	a-b	30	15
C <sup>2</sup>		c-d	20+M	$0.5*y(10+10+M)+x*1*(M-10)$
If $x < y$				
EV \	Departure Time	Route Choice	Travel Time (min)	Expected Generalized Cost (\$)
C <sup>1</sup>	7:10	a-b	30+M	$0.5*x(20+10+M)+x*1*(10+M)$
C <sup>2</sup>		c-d	20	10

Table 3-1 Results from Adaptive DTC vs. Fixed DTC

### 3.2 Algorithm Design

#### 3.2.1 The Optimality Condition

We seek to achieve reliability of the trip to a given destination node  $d$  by introducing a general arrival penalty function  $p(t_d; \theta)$ , where  $\theta$  is a vector of parameters, including the desired arrival time  $t^*$ . The polynomial arrival penalty function studied in (0) is thus a special case. We also introduce departure delay to reflect the fact that disutility is

incurred when a traveler needs to leave later than the desired departure time. It is assumed that a traveler minimizes the expected generalized cost which is a linear combination of expected departure delay cost, travel time cost and arrival penalty (see a similar argument in (0)).

Bellman's principle of optimality applies to the minimization of travel disutility due to the additivity of the cost function. Let  $V_\mu(j, t, EV)$  and  $V_\mu(j^o, t, EV)$  be the expected generalized cost to destination node  $d$  from the state  $\{j, t, EV\}$  by following routing policy  $\mu$  when  $j$  is an origin and intermediate node respectively. and  $\omega(j^o)$  is the optimal action time at the origin node. The optimality conditions for  $\mu$  and  $\omega$  are listed as follows.

$$\begin{aligned}
(1) \quad & \left\{ \begin{aligned} V_{\mu^*}(j, t, EV) &= \min_{k \in A(j)} \{ E_{\pi_{jk,t}, \tilde{EV}'} [\alpha \pi_{jk,t} + V_{\mu^*}(k, t + \pi_{jk,t}, \tilde{EV}') | EV] \}, \\ V_{\mu^*}(j^o, t, EV) &= \min_{\tilde{EV}'} \{ \beta + E[V_\mu(j^o, t + 1, \tilde{EV}') | EV], V_{\mu^*}(j, t, EV) \}, \end{aligned} \right. \\
(2) \quad & \left\{ \begin{aligned} \mu^*(j, t, EV) &= \arg \min_{k \in A(j)} \{ E_{\tilde{C}_{jk,t}, \tilde{EV}'} [\alpha \tilde{C}_{jk,t} + V_{\mu^*}(k, t + \tilde{C}_{jk,t}, \tilde{EV}') | EV] \}, \\ \mu^*(j^o, t, EV) &= \arg \min_{\tilde{EV}'} \{ \beta + E[V_\mu(j^o, t + 1, \tilde{EV}') | EV], V_{\mu^*}(j, t, EV) \}, \end{aligned} \right. \\
(3) \quad & \omega(j^o) = \arg \min_{t \in T} \{ E_{EV} [V_{\mu^*}(j^o, t, EV)] \}
\end{aligned}$$

$\forall j \in N \setminus \{d\}, \forall t, \forall EV$ , with the boundary conditions:  $V_{\mu^*}(d, t, EV) = V_{\mu^*}(d^o, t, EV) = p(t; \theta)$ ,  $\mu^*(d, t, EV) = \mu^*(d^o, t, EV) = d$ ,  $\forall t, \forall EV$ .  $\alpha$  and  $\beta$  are nonnegative weights for travel time and departure delay respectively.

### 3.2.2 Algorithm with Perfect Online Information

Under perfect online information, the set of possible event collections at time  $t$ ,  $EV(t)$ , can be generated in an increasing order of time. At each time  $t$ , each element of  $EV(t-1)$  is partitioned into disjoint subsets based on all link travel times at time  $t$ , such that in each subset, all support points have the same link travel times at time  $t$  for all links, and any two support points from two different subsets do not have all the same link travel times at time  $t$ . Since the element of  $EV(t-1)$  is the result of successive partitions up to time  $t-1$ , the above operation will result in  $EV(t)$ . At time  $K-1$ , each event collection contains exactly one support point and the network becomes deterministic and static. The statement of the process is as follows.

#### Generate\_Event\_Collection

$$D = \{C^1, \dots, C^R\}$$

For  $t = 0$  to  $K-1$

For each arc  $(j, k) \in A$

For each disjoint set  $S \in D$

1.  $w =$  number of distinct values among  $C_{jk,t}^r, \forall r \in S$ ;
2. Divide  $S$  into disjoint sets  $S_1', S_2', \dots, S_w'$ , such that  $C_{jk,t}^r$  is constant over all  $r \in S_i', i = 1, \dots, w$  and  $\bigcup_i S_i' = S$ ;
3.  $D \leftarrow D \setminus \{S\} \cup \{S_1', S_2', \dots, S_w'\}$ ;

$EV(t) \leftarrow D$ ;

With POI,  $\tilde{C}_{jk,t}$  in Equation (1) can be replaced by a fixed value  $\pi_{jk,t}$  which is the observed travel time on link  $(j,k)$  at time  $t$ . Equation (1) can be written as

$$(4) \quad \begin{cases} V_{\mu^*}(j,t, EV) = \min_{k \in A(j)} \{ \alpha \pi_{jk,t} + \sum_{EV' \in EV(t+\pi_{jk,t})} V_{\mu^*}(k, t + \pi_{jk,t}, EV') P(EV' | EV) \}, \\ V_{\mu^*}(j^o, t, EV) = \min \{ \beta + \sum_{EV' \in EV(t+1)} V_{\mu}(j^o, t+1, EV') P(EV' | EV), V_{\mu^*}(j, t, EV) \} \end{cases}$$

where  $P(EV' | EV) = \frac{\sum_{r \in EV' \cap EV} p_r}{\sum_{r \in EV} p_r}$  is the conditional probability of the next event collection.

We choose schedule delay as the arrival penalty, defined as the difference between the desired arrival time and the actual arrival time. We distinguish between early schedule delay and late schedule delay. Let  $[t^* - \Delta, t^* + \Delta]$  be the desired arrival time range. Any

arrival times fall out of this time window will receive penalties. If the arrival time is  $t$ , early schedule delay is  $\max(0, t^* - \Delta - t)$ , and late schedule delay is  $\max(0, t - (t^* + \Delta))$ . The travel time, early and late schedule delays are converted to monetary costs by multiplying them by their respective unit cost. The unit costs for early and late schedule delays are  $\gamma$  and  $\eta$  respectively, where generally  $\gamma < \eta(0)$ .

We design an algorithm DOT-REL based on the optimality conditions. Note that the evaluation of  $V_{\mu^*}(j, t, EV)$  only depends on  $V_{\mu^*}(j, t', EV')$  from a later time  $t' > t$ , due to the positive and integral link travel time assumptions. Therefore DOT-REL is a label-setting algorithm working in a decreasing order of time, and the links can be scanned in an arbitrary order during any time period. At time  $K-1$  and beyond, we can use any deterministic static shortest path algorithm to compute  $V_{\mu^*}(j, t, EV)$ ,  $\forall j \in N$ ,  $\forall t \geq K-1$ ,  $\forall EV \in EV(K-1)$ . The statement of Algorithm DOT-REL follows, where  $e_{\mu^*}(j, t, EV)$  is the expected travel time of routing policy  $\mu^*$ ,  $es_{\mu^*}(j, t, EV)$  the expected early schedule delay,  $ls_{\mu^*}(j, t, EV)$  the expected late schedule delay.

### **Algorithm DOT-REL**

*Initialization*

Step 1: Initialization at the destination

$$\begin{aligned}
e_{\mu^*}(d, t, EV) &= 0, es_{\mu^*}(d, t, EV) = \max(0, t^* - \Delta - t), ls_{\mu^*}(d, t, EV) = \max(0, t - (t^* + \Delta)) \\
V_{\mu^*}(d, t, EV) &= V_{\mu^*}(d^o, t, EV) = \gamma es_{\mu^*}(d, t, EV) + \eta ls_{\mu^*}(d, t, EV) \\
\mu^*(d, t, I) &= \mu^*(d^o, t, I) = d, \\
&\forall t, \forall EV
\end{aligned}$$

Step 2: Compute time bound  $\hat{t} = \max(K-1, t^* - \Delta)$

Beyond this time bound, the network will become deterministic and there will be no early schedule delay, and only late schedule delay. Thus travelers will follow the shortest path to the destination and not stay at origin node.

Step 3: Initialization beyond time bound  $\hat{t}$

Compute  $e_{\mu^*}(j, \hat{t}, EV)$ ,  $\forall j \in N, \forall EV \in \mathbf{EV}(K-1)$  with a static deterministic shortest path algorithm (e.g. Dijkstra's)

$$\begin{aligned}
es_{\mu^*}(j, t, EV) &= 0, ls_{\mu^*}(j, t, EV) = \max(0, t + e_{\mu^*}(j, \hat{t}, EV) - (t^* + \Delta)), \\
V_{\mu^*}(j, t, EV) &= V_{\mu^*}(j^o, t, EV) = \alpha e_{\mu^*}(j, \hat{t}, EV) + \eta ls_{\mu^*}(j, t, EV), \\
&\forall j \in N, \forall EV \in \mathbf{EV}(K-1), \forall t \geq \hat{t}
\end{aligned}$$

Step 4: Initialization before time bound  $\hat{t}$

$$\begin{aligned}
V_{\mu^*}(j, t, EV) &= V_{\mu^*}(j^o, t, EV) = +\infty \\
&\forall j \in N \setminus \{d\}, \forall t < \hat{t}, \forall EV \in \mathbf{EV}(t)
\end{aligned}$$

*Main Loop*

For  $t = \hat{t} - 1$  down to 0

For each  $EV \in EV(t)$

For each arc  $(j, k) \in A$

$$temp = \alpha \pi_{jk,t} + \sum_{EV' \in EV(t + \pi_{jk,t})} V_{\mu^*}(k, t + \pi_{jk,t}, EV') P(EV' | EV)$$

If  $temp < V_{\mu^*}(j, t, EV)$

$$V_{\mu^*}(j, t, EV) = temp$$

$$\mu^*(j, t, EV) = k$$

$$V_{\mu^*}(j^o, t, EV) = V_{\mu^*}(j, t, EV)$$

$$\mu^*(j^o, t, EV) = \mu^*(j, t, EV)$$

For each node  $j \in N$

$$temp = \beta + \sum_{EV' \in EV(t+1)} V_{\mu^*}(j^o, t+1, EV') P(EV' | EV)$$

If  $temp \leq V_{\mu^*}(j^o, t, EV)$

$$V_{\mu^*}(j^o, t, EV) = temp$$

$$\mu^*(j^o, t, EV) = j$$

For each node  $j \in N$  (To find optimal action time  $\omega$ )

$\text{min\_time} = +\infty$

For  $t = \hat{t} - 1$  down to 0

If  $V_{\mu^*}(j^o, t, EV) = \text{temp}$

$\text{min\_time} = V_{\mu^*}(j^o, t, EV)$

$\omega(j^o) = t$

Following a similar analysis as in (0), we can derive that Algorithm DOT-REL has a complexity of  $O(mKR \ln R + R \times \text{SSP})$  and  $\Omega(mKR + R \times \text{SSP})$ , where SSP is the complexity of the static deterministic shortest path algorithm. This algorithm is strongly polynomial in  $R$ .  $R$  could be an exponential function of  $mK$ . If the link travel times are highly correlated, we expect that  $R$  is much less than  $Y^{mK}$ , where  $Y$  is the maximum number of support points for a single link travel time. Algorithm DOT-REL is exact and can serve as a benchmark for checking the performances of approximation algorithms that can be used in practice.



## CHAPTER 4

### COMPUTATIONAL TESTS

#### 4.1 Objective

In this section, we plan to compare adaptive DTC and fixed DTC with perfect online information and two other adaptive and non-adaptive routing methods. The objectives of the computational tests are threefold: 1) to demonstrate the benefit of adaptive DTC over fixed DTC with POI, 2) to study how the departure time distribution change with system parameters, such as unit departure penalty and allowable arrival time window and 3) to study the effectiveness of Algorithm DOT-REL over two approximation algorithms. Based on the study of D.B Lee (45) on USA project,, the monetary value of travel time (VOTT) for passenger cars drivers in the information age ranges from 8-40\$/hr. In this paper, we choose 30\$/hr as the value of travel time. In the algorithm, we use minute as the time unit. Therefore, the unit cost for travel time is 0.5\$/min. We further assume that the unit cost for early and late schedule delays is 0.25\$/min and 1\$/min respectively. These values are consistent with those in (43) after the normalization with respect to travel cost.

#### 4.2 Computational Test Design

Our tests are designed to be carried out in two parts: the first on a hypothetical network with detailed distribution results; and the second on randomly generated

networks to obtain a high-level understanding of the performances of Algorithm DOT-REL, CE and NOI as functions of some system parameters.

First of all, we want to compare adaptive DTC verse fixed DTC. Although these two algorithms both have adaptive route choice mode, waiting at the origin node is forbidding in the fixed DTC model. The results in fixed DTC are from the optimal departure time. To find this priori value in the algorithm, we seek the minimum expected generalized cost among all departure time, and then fixed that time periods to be the departure time.

Secondly, verse two approximation algorithms, the certainty equivalent (CE) and no online information (NOI). The CE approximation replaces every link travel time random variable by its expected value. Thus the network becomes a deterministic dynamic network. Any deterministic dynamic shortest path algorithm can be used to obtain a path that minimizes the disutility function. The path is then executed in the original stochastic time-dependent network to obtain the needed summary statistics. As the complexity analysis states in last section, Algorithm DOT-REL cannot be applied to large-scale networks because of the potential high running time. CE however is very efficient. Thus, it is interesting to study the tradeoffs between effectiveness and efficiency by comparing results from Algorithm DOT-REL and CE.

The NOI approximation works with the marginal distributions of link travel time instead of joint distributions, where the current information component of any state is an empty set. Routing decisions also only depend on the current node and the current time,

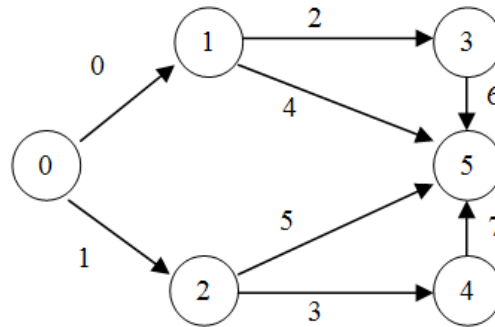
and the current information can actually be ignored in the algorithm design. Traveler's knowledge about the network remains the priori distribution of link travel time, because either the link travel time is statically independent and information obtained can't help predict future, or they got no en route information access. Theoretically, the NOI approximation is the simplest in terms of adaptive routing algorithm design with on-line information, due to the lack of current information. It is therefore the basis for the study of more complicated approximation in terms of dependence on information accessibility. Furthermore even though it is the simplest, it suffices to show some of the implications and significance of stochasticity in a dynamic context for traffic models. Practically, though the performance of NOI as an approximation can be arbitrarily worse than optimal, it does can serve as a good approximation to POI when the stochastic dependency of link travel times is weak. Computationally, the NOI variant can be solved in polynomial time. This is a very desirable result. Therefore NOI can be used as an approximation to more complicated approximations. Note that NOI performance can be either better or worse than CE for given network. An intuitive argument is that both the NOI approximation and the CE approximation are working on joint distributions different from the original one. Which one leads to a travel cost farther from the optimal solution depends on the data.

For these two approximations, we also apply DTC to the algorithm, which will provide the minimum generalized cost of all nodes to destination over all time periods. Note that, the departure time is fixed for each node, since there is no information

considered in the NOI and CE approaches, thus adaptive DTC is unable to model in these two cases.

### 4.3 A Hypothetic Network

The test network is shown in Figure 2 with 6 nodes and 8 directed links. There are diversion possibilities at nodes 0, 1 and 2. The study period is from 6:30am to 8:00am. The time resolution is 1 second for departures and arrivals at all nodes, and we have 5400 time periods in total. The link travel time distribution is generated through an exogenous simulation with the mesoscopic supply simulator of DynaMIT (0). There are random incidents in the network defined as follows: 1) There is at most one incident for any given day with probability 0.9; 2) The incident has a positive probability of occurrence on link 0, 2, 4 and 6, but zero on link 1, 3, 5 and 7; 3) If an incident occurs on a link, the start time can be every 10 minutes with equal probability. The 4 possible locations and 9 possible start times result in  $4 \times 9 + 1$  (no incident) = 37 support points. In the support point of no incident occurrence in the network, the average OD travel times over the study period are around 20.5 minutes.



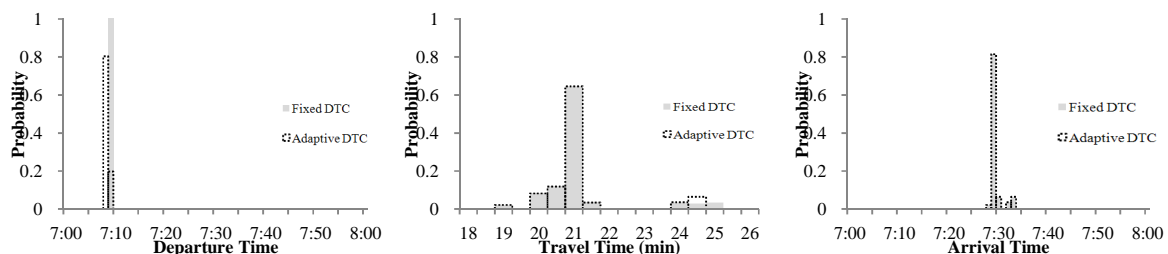
**Figure 4-1 A hypothetical Network**

For all tests, desired arrival time is 7:30. Denote the vector of unit costs as  $\lambda = \{\alpha, \beta, \gamma, \eta\}$ . The results of POI problem with  $\lambda = \{0.5, 0.15, 0.25, 1\}$ , and  $\Delta=0$  are in the first row, and the results of POI problem with  $\lambda = \{0.5, 0, 0.25, 1\}$  and  $\Delta=0$  in the second row. The gray bars in the first and second rows are the results with fixed DTC. They are repeated so that the comparison between fixed DTC and adaptive DTC can be made clearly under the two different departure penalty scenarios. In all the rows, the first column shows the travel time distribution, the second the departure time distribution and the third the arrival time distribution.

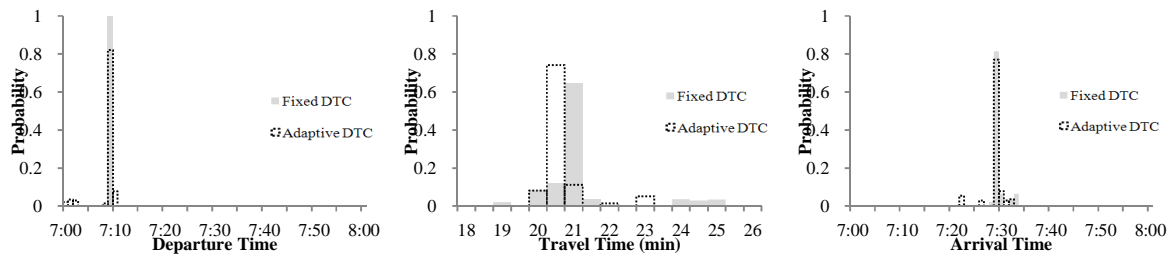
Comparing the results of POI with fixed DTC and adaptive DTC with 0.15 unit cost departure delay penalty (first row), we find that the arrival time distribution almost remain unchanged, while the travel time slightly decrease from fixed DTC. Adaptive DTC with departure penalty provide DTC from 7:09 to 7:10, while the non-adaptive departure time is fixed at 7:10, which lead to the difference in the travel time distribution. Optimal

action time for fixed DTC equals to the departure time at 7:10, while for adaptive one is the earliest departure time over all support point, 7:09.

Comparing the results of POI with fixed DTC and adaptive DTC without departure delay penalty (second row), we find that the travel time is more desirable. The departure time distribution spreads from 7:00 to 7:11. The optimal action time can take any value from the beginning of the study period (6:30) up to the earliest departure time 7:00. As there is no waiting cost at the origin, any action time before the earliest departure time will result in the same and minimum generalized cost. The arrival time distribution from the adaptive DTC is more dispersed than that from the fixed DTC, with more early schedule delay and less late schedule delay. Comparing the results of adaptive DTC (no fill) with (row 1) and without (row 2) departure penalty, we observe that the departure time distribution is more dispersed without departure penalty, because the unit cost on departure delay restrict the DTC close to the action time. More weight on departure delay will result in a more concentrated departure time distribution. Fixed DTC can be also viewed as the extreme case of very large penalty on departure delay.



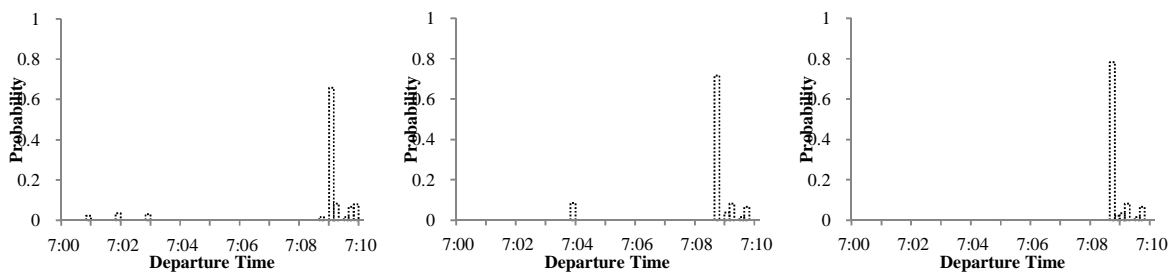
Results with departure penalty,  $\lambda = (0.5, 0.15, 0.25, 1)$



Results without departure penalty,  $\lambda = (0.5, 0, 0.25, 1)$

**Figure 4-2 Results of POI With and Without Departure Penalty**

We plot departure time distributions with different departure penalties, 0, 0.05 and 0.15 in Figure 4-3. We can see the departure time distribution becomes more concentrated when a larger unit cost is imposed on departure delay. This is because departure delay penalty restricts the range of DTC, and the departure time will not spread too far away from the action time.



Departure Penalty = 0

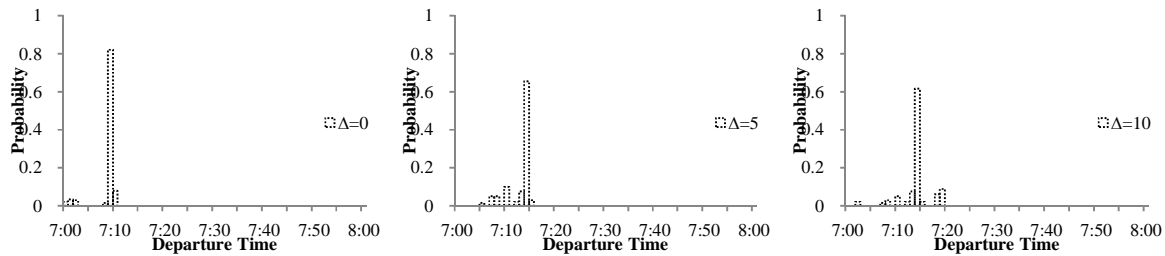
Departure Penalty = 0.05

Departure Penalty = 0.15

**Figure 4-3 Departure Time Distribution with Different Departure Penalty**

Figure 4-4 shows the departure time distribution change with different arrival time window. Arrival at destination within  $[7:30-\Delta, 7:30+\Delta]$  will not result in penalty. The time unit of  $\Delta$  is minute. The results of adaptive DTC with  $\lambda = (0.5, 0.15, 0.25, 1)$  and with  $\lambda = (0.5, 0, 0.25, 1)$  are in the first and second row respectively. In both rows, allowable time window of 0, 5 and 10 minutes are shown in the first, second and third columns respectively.

In these two rows we find the same trend in both without departure delay penalty, which with wider arrival time window the departure time distribution is more dispersed. Because we relax the arrival requirement, traveler seek more desirable route policy in a wider range, which will results in more desirable solutions. However, the trend of departure distribution is not predictable when with departure penalty.



Departure Distribution from POI without departure penalty,  $\lambda = \{0.5, 0, 0.25, 1\}$

**Figure 4-4 Departure Distribution with Different Arrival Time Window**

#### 4.4 Random Networks

Multivariate normal distribution is assumed for the joint distribution of travel times of all links at all time periods. The random network generator takes as input: 1) the number of nodes, 2) the number of links, 3) the number of time periods, 4) the number of support points, 5) the uniform link travel time mean, 6) the uniform standard deviation of link travel times, 7) the uniform correlation coefficient of link travel times, 8) the maximum in-degree, and 9) the maximum out-degree. A uniform correlation coefficient is used for every pair of random variables. The topology of the network is randomly generated. An in-tree rooted at the destination node is generated to ensure the



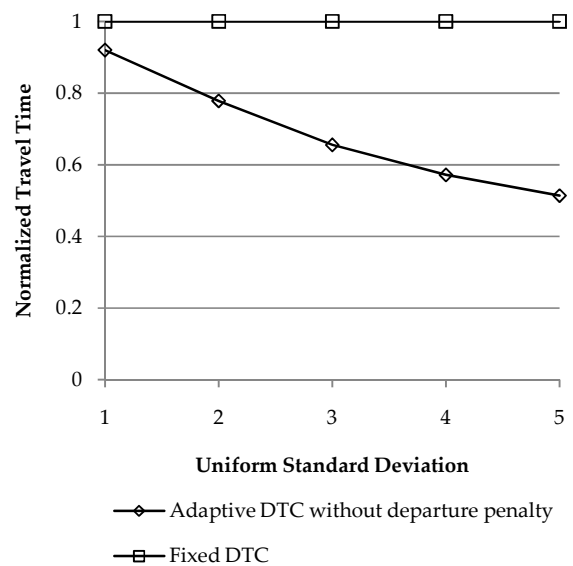
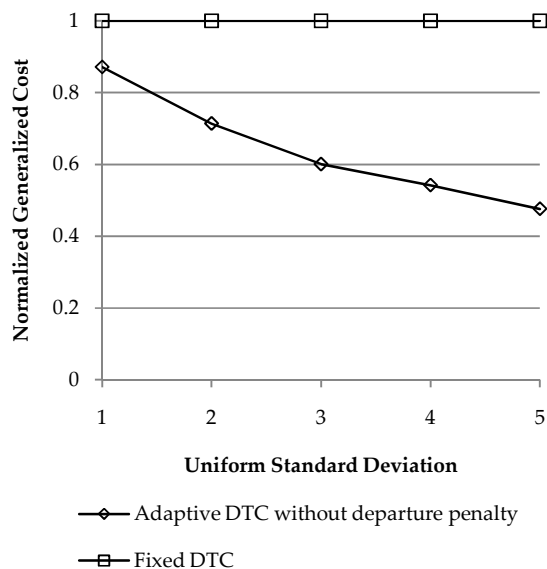
connectivity to the destination. The remaining links are generated randomly, respecting the maximum in-degree and out-degree. More details on the random network generation can be found in (0). In all the tests, the desired arrival time is 30 (min) with  $\Delta=0$ ,  $\lambda = \{1,0,0.5,2\}$ , and DTCs and routing choices are combined.

For a given network setting, 10 random networks are generated. For a given network and a given scenario (ADTC with POI, FDTC with POI, DTC under CE or DTC under NOI), at each origin node we calculate the optimal generalized cost and the corresponding expected travel time, early and late schedule delays at the corresponding optimal action time. We then take expectations of these quantities over all event collections and take averages over all nodes. For each network, results are normalized by that from fixed DTC with POI, and averages are taken over the 10 random networks. This creates one data point in each of the sub-figures in Figures 4-5 and 4-6.

We also check the standard deviation of generalized cost from each node to the destination from adaptive and non-adaptive DTC. The result is normalized by that from fixed DTC. Then an average over 10 same setting networks is taken. This create one dot in Figure 4-7.

Figure 4-5 shows the normalized expected generalized cost, travel time, early and late schedule delays from POI with adaptive and fixed DTC as functions of the uniform standard deviation (STD) of link travel times. We find that normalized generalized cost

and travel time decrease as a function of uniform standard deviation, while normalized early and late schedule delay are relatively flat but within the range of 0.4 to 0.6 and 0.1 to 0.3 of that from fixed DTC respectively. This is because POI with fixed DTC can take advantage of STD by making adaptive route choice along trip, while adaptive DTC can seek lower generalized cost through time dimension. Larger STD provides more opportunities of lower link travel time along time and space dimensions. Adaptive route choice can take the advantage on the space dimension, while adaptive DTC can take such opportunity along time axis. Therefore as the STD increases, the difference from POI with adaptive DTC to POI with fixed DTC in generalized cost also increases.



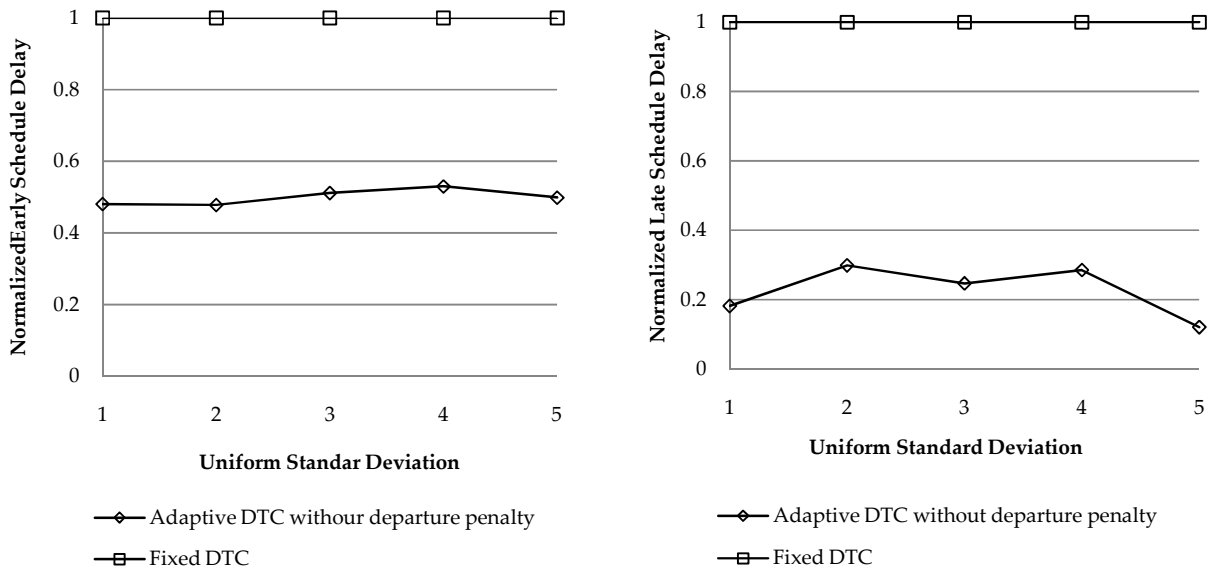
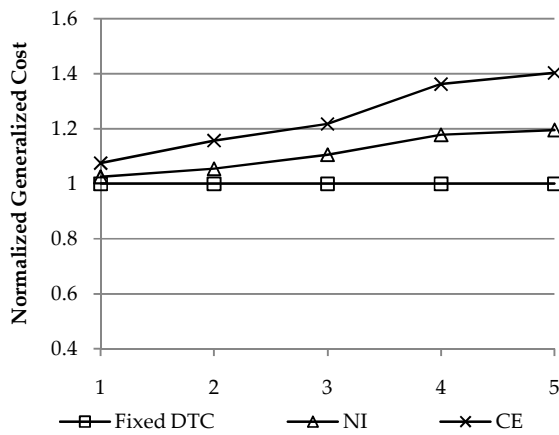


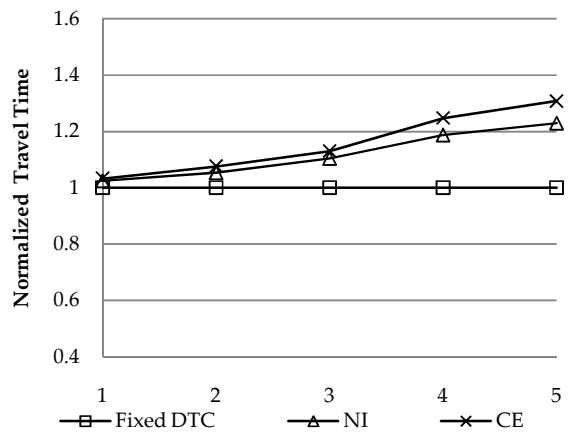
Figure 4-5 POI with adaptive DTC compared to POI with fixed DTC as functions of the uniform standard deviation of link travel times (with 15 nodes, 30 links, 40 time periods, 20 support points, 6 as the uniform mean link travel time, 0.5 as the uniform correlation coefficient of link travel times, 30 as desired arrival time, 0 time tolerance, and  $\lambda = \{0.5, 0, 0.25, 1\}$ )

Figure 4-6 shows the normalized expected generalized cost, travel time, early and late schedule delays from POI fixed DTC versus NI and CE approximation as functions of the uniform standard deviation (STD) of link travel times. When the standard deviation is large, the link travel times are more dispersed, and thus the expected travel times of different paths (routing policies) are more likely to differ. Although they are all combined with DTC, none of them are adaptive. We fix the departure time POI manually, while NOI and CE approaches are unable to model adaptive DTC due to their inherent algorithm, which ignore the information and work with marginal distribution of joint distributed travel time distribution. Even though they are all with non-adaptive DTC, we can see the normalized generalized cost, travel time and late schedule delay of NI and CE increases as

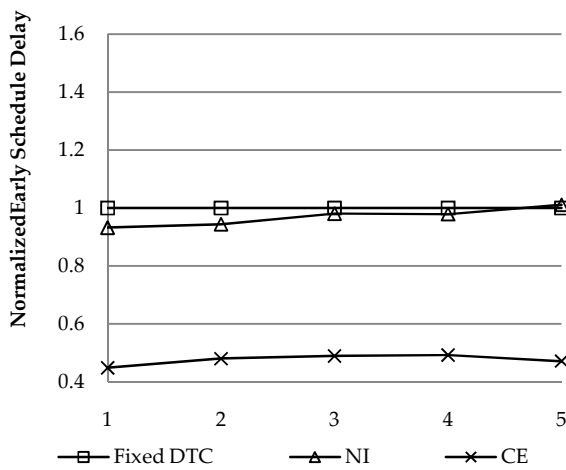
a function of STD, while the early schedule delay less than that from POI fixed DTC. This can be explained by the fact that the travel time and late schedule delay have much larger weight than the early schedule delay, and in making the tradeoffs the adaptive routing algorithm could sacrifice early schedule delay a little bit to make the overall disutility function minimized. This also shows that a more variable network (with the same mean link travel time) provides more risk of incurring high travel disutility, but meanwhile also more opportunity of exploiting low disutility. As for the results of CE and NOI, we see the lines in all four sub-figures are almost overlapped. This is because the two marginal distribution CE and NOI work on are no better than the other. We see the approximation lines in all four sub-figures are relatively in similar trend. This is because CE and NOI works respectively with mean link travel times and marginal distribution of link travel times only, thus not sensitive to STD changes. An adaptive traveler can take the opportunity of exploiting lower travel disutilities by adapting to actual network conditions. A non-adaptive traveler, on the other hand, assumes an averaged network and thus cannot exploit the lower travel times. Also a traveler without help from current information predicting future, in another word, take expected value of link travel times at any state and also cannot find the lower travel times. Therefore as the STD increases, the difference from CE and NOI to POI fixed DTC in generalized cost also increases.



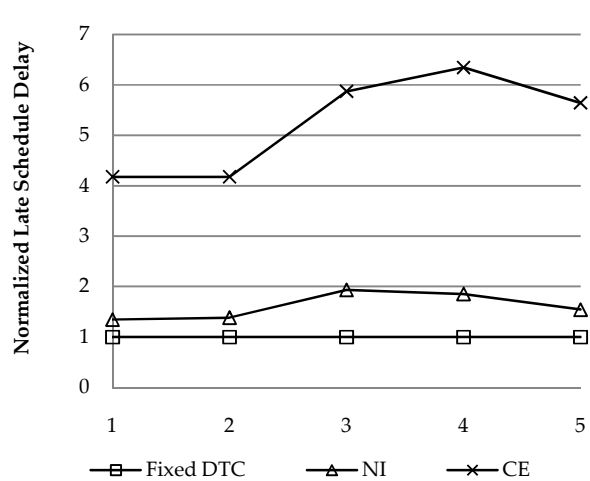
Uniform Standard Deviation



Uniform Standard Deviation



Uniform Standard Deviation



Uniform Standard Deviation

Figure 4-6 CE and NOI approximations compared to POI as functions of the uniform standard deviation of link travel times (with 15 nodes, 30 links, 40 time periods, 20 support points, 6 as the uniform mean link travel time, 0.5 as the uniform correlation coefficient of link travel times, 30 as desired arrival time, 0 time tolerance, and  $\lambda = \{0.5, 0, 0.25, 1\}$ )

Figure 4-7 shows the normalized standard deviation of expected generalized cost from POI with adaptive and fixed DTC as functions of the uniform standard deviation

(STD) of link travel times. We find that as the standard deviation of link travel time increases, the normalized standard deviation of generalized cost from POI with adaptive DTC decreases. This is a byproduct of minimizing generalized cost. Decreasing generalized cost variance could be viewed as a benefit of adaptive DTC. Adaptive DTC can provide less varied generalized cost than fixed DTC with the same POI. The benefit also increases when the link travel time is more fluctuated. It is because adaptive DTC works better in taking advantage of large variance network to find lower trip disutility. Although as the STD of link travel time increase, the absolute value of the generalized cost from FDTC and ADTC also increase, the gap between these two are magnified. Therefore, the generalized cost of trips are less varied with ADTC than FDTC, because they ranged relatively lower and concentrated.

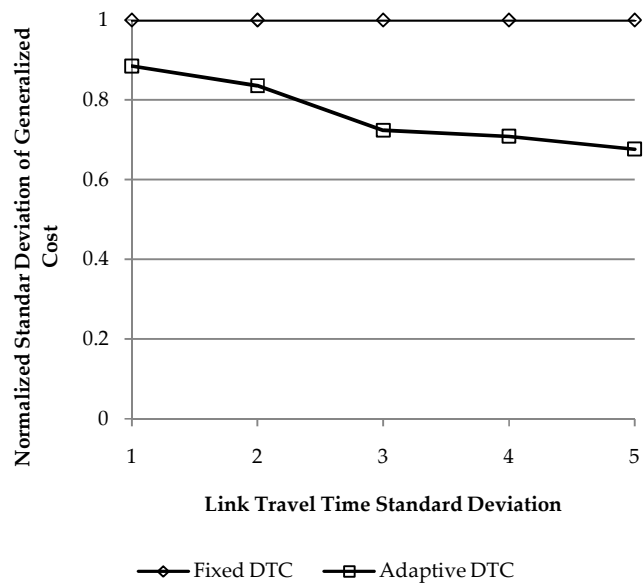


Figure 4-7 Variance of POI with adaptive DTC compared to POI with fixed DTC as functions of the uniform standard deviation of link travel times (with 15 nodes, 30 links, 40 time periods, 20 support points, 6 as the uniform mean link travel time, 0.5 as the uniform correlation coefficient of link travel times, 30 as desired arrival time, 0 time tolerance, and  $\lambda = \{0.5, 0, 0.25, 1\}$ )

## CHAPTER 5

### CONCLUSIONS AND FUTURE DIRECTIONS

In this paper we study optimal adaptive DTC in travel reliability problem in stochastic time-dependent network. Adaptive DTCs routing combined with route choice under perfect online information is designed as a routing policy to achieve more reliable travel in terms of less arrival penalty. The optimal routing policy problem is defined to seek minimized generalized cost which is the linear combination of expected travel time, schedule delays and departure delay multiplying the value of time. Generic optimality conditions are given with generic information access, general link travel time stochastic dependencies and generic arrival penalty. An exact algorithm (Algorithm DOT-REL) is designed to solve a special case of the problem with perfect online information (POI) and schedule delay as arrival penalty. In this algorithm we first model the adaptive DTC, which is a random variable depend on network condition and arrival requirements. A new concept, action time is introduced indicated the start time of people making trip decisions. Computational test are carried out. It is shown that adaptive DTCs with perfect online information result in more reliable travel than fixed DTC with POI and conventional non-adaptive choices in terms of less travel disutility, especially less expected schedule delay. The benefit of adaptive DTCs is an increasing function of network variability in decreasing generalized cost. Another benefit of adaptive departure time is decrease the generalized cost variance, which is a byproduct of minimizing expected generalized cost. We also



study the unit cost departure delay penalty restrict the DTC variability and arrival time window wider leads to more dispersed departure time distribution.

Future directions on reliable adaptive routing policy in stochastic time-dependent network can be as follows:

- It is generally believed that with more information one can make more informed decision to get more reliable route choice. There are however many different types of traveler information situations. Gao and Huang (0) study three different partial online information situations for the minimum expected travel time routing policy problem. The same approach could be applied to the reliable routing problem.
- We assume travelers' decisions will not affect the flow on the network in this paper. This will eventually be embedded in an equilibrium traffic assignment model where the demand-supply interaction is taken into account. Such a model is needed to design and evaluate a traveler information system when the penetration of information is high enough so that choices made with the help of information affect traffic conditions.

## APPENDIX

### AN ILLUSTRATIVE EXAMPLE FOR ALGORITHM DOT-REL

We use an example to illustrate how Algorithm DOT-REL works. The network in Figure 1 has 3 nodes, 3 links and the number of time periods is 2. The travel time support points are also shown, each of which has a probability of 1/2. We solve the minimum expected generalized cost routing policy problem from node  $a$  at a desired departure time 0 to the destination node  $c$ . Assume  $\alpha=1, \beta=0.3, \gamma=0.5, \eta=2$ , desired arrival time  $t^*=2, \Delta=0$ .

Step 1: Construct  $EV(t), t = 0, 1$

A summary of the results of constructing event collections is as follows.

$$EV(0) = \{C^1, C^2\}$$

$$EV(2) = EV(1) = \{C^1, C^2\}$$

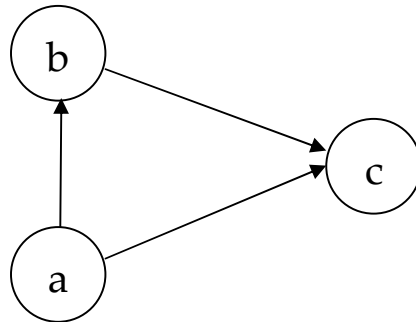
Step 2: Compute the time bound  $\hat{t} = \max(K-1, t^*-\Delta) = 2$

Step 3: Compute  $V_{\mu^*}(j, 2, EV), \forall j \in N, \forall EV \in EV(2)$

$$V_{\mu^*}(j^0, 2, EV) = V_{\mu^*}(j, 2, EV)$$

This step involves solving deterministic static shortest path problems with each single support point  $v_r, r = 1, 2$ . Any classical shortest path algorithm can be used. In our small network, this can be done by observation. For each node, the minimum disutility, and the corresponding expected travel time, early (ESD) and late schedule delay (LSD) and the next node are given. The optimal next nodes at time 1 are the same as time 2, since all link travel times are at least 1 and thus no early schedule delay will occur. It is

therefore beneficial to leave the node without waiting and follow the shortest path to node  $c$ . The results at time 1 are also listed in Table 1. Note that on both time periods, results for origin nodes and intermediates nodes are the same.



Time	Link	$C^1$	$C^2$
0	a-b	1	1
	b-c	1	1
	a-c	3	3
1	a-b	1	1
	b-c	1	2
	a-c	1	1

**Figure A-1 The Illustrative Network**

**Table A-1 Results at t=2 and t=0**

<i>t = 2, EV = {C<sup>1</sup>}</i>					
Node	Disutility	Travel Time	ES D	LS D	Next Node
<i>a</i>	3	1	0	1	<i>c</i>
<i>b</i>	3	1	0	1	<i>c</i>
<i>c</i>	0	0	0	0	<i>c</i>
<i>t = 2, EV = {C<sup>2</sup>}</i>					
<i>a</i>	3	1	0	1	<i>c</i>
<i>b</i>	6	2	0	2	<i>c</i>
<i>c</i>	0	0	0	0	<i>c</i>
<i>t = 1, EV = {C<sup>1</sup>}</i>					
<i>a</i>	1	1	0	0	<i>c</i>
<i>b</i>	1	1	0	0	<i>c</i>
<i>c</i>	0.5	0	1	0	<i>c</i>
<i>t = 1, EV = {C<sup>2</sup>}</i>					
<i>a</i>	1	1	0	0	<i>c</i>
<i>b</i>	4	2	0	1	<i>c</i>
<i>c</i>	0.5	0	1	0	<i>c</i>

### **Main Loop**

$t = 0$

$$EV = \{C^1, C^2\}$$

$$EV_1' = \{C^1\}, \Pr(EV_1' \mid EV) = 0.5$$

$$EV_2' = \{C^2\}, \Pr(EV_2' \mid EV) = 0.5$$

$$(j, k) = 1$$

$$temp = 1 + V_{\mu^*}(b, 0+1, EV_1') P(EV_1' \mid EV) + V_{\mu^*}(b, 0+1, EV_2') P(EV_2' \mid EV)$$

$$= 1 + 1 \times 0.5 + 4 \times 0.5 = 3.5$$

$$V_{\mu^*}(a, 0, \{C^1, C^2\}) = 3.5, \mu^*(a, 0, \{C^1, C^2\}) = \text{node } b$$

$$(j, k) = 3$$

$$temp = 3 + V_{\mu^*}(c, 0+3, EV_1') P(EV_1' \mid EV) + V_{\mu^*}(c, 0+3, EV_2') P(EV_2' \mid EV)$$

$$= 3 + 2 \times 0.5 + 2 \times 0.5 = 5 > 3.5$$

//We do not consider link 2 because we are only interested in decisions at

node  $a$

$$temp = \beta + V_{\mu^*}(a^0, 0+1, \{C^1\}) P(EV_1' \mid EV) + V_{\mu^*}(a^0, 0+1, \{C^2\}) P(EV_2' \mid EV)$$

$$= 0.3 + 1 \times 0.5 + 1 \times 0.5 = 1.3 < 3.5$$

$$V_{\mu^*}(a^0, 0, \{C^1, C^2\}) = 1.3, \mu^*(a^0, 0, \{C^1, C^2\}) = \text{node } a \text{ (waiting at the origin)}$$

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