Euplectella Aspergillum's Natural Lattice Structure for Structural Design & Stability Landscape of Thin Cylindrical Shells with Dimple Imperfections

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EUPLECTELLA ASPERGillum’S NATURAL LATTICE STRUCTURE FOR STRUCTURAL DESIGN & STABILITY LANDSCAPE OF THIN CYLINDRICAL SHELLS WITH DIMPLE IMPERFECTIONS

A Thesis Presented

by

ZOE Y. SLOANE

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EUPLECTELLA ASPERGILLUM'S NATURAL LATTICE STRUCTURE FOR STRUCTURAL DESIGN & STABILITY LANDSCAPE OF THIN CYLINDRICAL SHELLS WITH DIMPLE IMPERFECTIONS

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ABSTRACT

EUPLECTELLA ASPERGILLUM’S NATURAL LATTICE STRUCTURE FOR STRUCTURAL DESIGN & STABILITY LANDSCAPE OF THIN CYLINDRICAL SHELLS WITH DIMPLE IMPERFECTIONS

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The first portion of this thesis assesses the structural application of a bracing design inspired by the deep-sea sponge, Euplectella Aspergillum. Many studies have investigated the natural strength found in the unique skeletal structure of this species. The braced design inspired by the sponge features square frames with two sets of cross-braces that are offset from the corners of each frame, creating a pattern of open and closed cells. This study reports the results of multiple Finite Element Analysis (FEA) computations that compare the described bracing pattern to a more common bracing design used in structural design. The designs are compared in two configurations; the first is a simplified tall building design, and the second is a slender plate design. Results indicate that the sponge’s natural pattern produces considerable mechanical benefit when only considering elastic behavior. However, the same was not true when considering plastic material properties. In conclusion to these observations, the sponge-inspired lattice design is determined to be an efficient alternative to slender-solid plates but not for lateral-resisting systems intended for tall building design.

The second topic of discussion in this thesis concerns the stability of thin cylindrical shells with imperfections. The structural stability of these members is highly
sensitive to the size and shape of an imperfection. An accurate prediction of the capacity of an imperfect cylindrical shell can be determined using non-destructive testing techniques. This method does require previous knowledge of the characteristics of the imperfection, which realistically is unknown. In the hope of creating a technique to find the location of an imperfection, this study analyzes the trends in the stability landscapes of the surrounding area of an imperfection. The imperfection of interest in this study has an amplitude equivalent to the thickness of the shell. Using FEA to simulate non-destructive probing tests, it is established that there is a distinct area surrounding the imperfection where the axial load and peak probe force curves show the influence of the imperfection. This area is referred to as the zone of influence and can be used to create an efficient process to locate an imperfection on a thin cylindrical shell.
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CHAPTER 1
APPLICATION OF EUPLECTELLA ASPERGILLUM’S NATURAL LATTICE STRUCTURE TO TALL BUILDING DESIGN

1.1. Introduction

This chapter analyzes the functionality of a proposed bracing design in the application of tall building design. The following sections will provide a brief overview of the focus of this chapter, introduce the species that inspired the study, and the structural application being examined.

1.1.1. Overview and Motivation for Study

Recently, the current state of infrastructure in the United States of America has become more of a concern to the general population. The structures in use today are under more stress than they were originally designed for. Increased loading and usage, climate change, lifetime deterioration, and many other factors contribute to the decline in function. These realities have resulted in continuous maintenance, improvements, or rebuilding of structures across America. During this time of reconstruction, it is crucial to consider the environmental impact and how to improve the overall design efficiency of structures. Ideally, the new generation of infrastructure should incorporate sustainability and advanced technology to increase structural stability and lifetime use.

Sustainability is the act of trying to preserve nature and the world's natural resources. The first half of this thesis is inspired by nature in the hope of creating a more sustainable structural design option. This idea is popularly known as biomimicry, the science of creating or designing a product inspired by nature. Biomimetics can be applied
in many fields of interest including biology, chemistry, art, architecture, and engineering. Examples of successful implementations of biomimetics include airplanes and train designs inspired by birds, building facades developed from the look of honeycombs, and bird-safe glass that implements spiderweb technology (Aziz & Sherif). Biomimicry is broken down into three categories of imitation in the form of shape, a natural process, or an ecosystem (Hwang et al.). This study of *Euplectella Aspergillum* practices biomimicry by copying the natural patterns formed by the skeletal elements of the species.

The central purpose of this study is to investigate if a lattice design inspired by *Euplectella Aspergillum* could produce a more effective structural design compared to traditional configurations used in structures today. The motivation for this study is to provide a lattice design style that will reduce the material needed for construction while maintaining or even increasing structural stability. The ability to decrease material required would directly lower construction costs and the overall environmental impact. The increased strength of the design could allow for larger buildings to be constructed. To assess the efficiency of the design it is important to compare the results to alternative designs. The rest of this chapter provides insight into the background information and the results of simulation analyses.

1.1.2. Introduction to *Euplectella Aspergillum*

*Euplectella Aspergillum*, also known as Venus’s flower basket, is a living deep-sea glass sponge. Most commonly, they are found on the ocean floor in the Western Pacific. Their skeleton forms into a basket shape that can extend up to approximately 25 cm long, creating a tube-like structure. The skeletal elements are called spicules
composed of silica. Spicules give the animal its unique shape and lattice structure by being comprised of alternating silica nanospheres and thin organic layers that when combined produce fibers (Aizenberg et al. 2005). The fibers organize into flexurally rigid composite beams at the micron scale. The beams arrange into a square grid that features two intersecting sets of paired diagonal members, creating a lattice pattern of alternating open and closed frames. Figure 1 shows increasingly magnified views of the sponge’s natural skeletal structure.

The lattice configuration of spicules is essential to the survival of the species. The structure is strong and flexible allowing it to withstand harsh currents at the bottom of the sea. The alternating open and closed frames allow for water to pass through the structure, which decreases the total stresses applied to the body of the sponge. In addition to the unique pattern featured on the body of *Euplectella Aspergillum*, the species also have ridges. The ridges are formed as the sponge matures and enters the rigid phase (Fernandes et al. 2021). They can be seen in the second magnification view in Figure 1. The ridges assist the sponge in trapping food and reproduction. These components are essential to the livelihood of species and could be the key to solving real-world challenges.
1.1.3. Introduction to Tall Building Design

Tall building design has significantly progressed over the past few centuries. The first generation of skyscrapers appeared in New York City and Chicago in the late 19th century. The advancement in building technology and safety devices during this period made it possible for engineers to surpass the six-story maximum previously enforced in these cities. The Home Insurance Building in Chicago completed in 1885 stood 12 stories tall a height of 54.9 m (180 ft) and is considered to be the first tall building in America. From then on, Structural Engineers have challenged themselves to push the boundaries to design increasingly taller buildings. This has produced many significant buildings in the history of structures including the Chrysler Building, the John Hancock Center, the Empire State Building, the World Trade Center, and the Willis Tower. Today, the tallest building is the Burj Khalifa in Dubai. It stands 828 meters or 2,717 feet tall with 163 floors.
The ability to construct taller buildings requires more structural stability and design. As buildings grow in height the lateral forces become more of a concern, therefore, adequate stiffness is essential to a tall building design. The most concerning lateral loads that are applied to tall buildings are wind load and seismic load. Insufficient lateral stiffness can cause second-order P-Δ effects that can lead to failure or surpass serviceability limitations. There are three common lateral force-resisting systems: moment frames, shear walls, and braced frames. The purpose of lateral force resisting systems is to resist against the lateral forces not to carry gravity load. All tall building designs utilize at least one of these systems to decrease the total lateral deflections experienced by the. In many cases, a combination of lateral force resisting systems is
implemented in a single tall building. These systems are essential for the structural integrity of tall buildings.

One way to increase the ability to build taller structures would be to improve on the lateral force-resisting systems. If the lateral force-resisting system can withstand higher forces, it will allow for the addition of more stories (Angelucci 2020 and Mentari 2021). This chapter focuses on the analysis of an alternative braced frame design in comparison to typical braced frame designs. Figure 3 shows common braced frame patterns used in structural design (Lu et al. 2008). Braced frames can be located at any uninterrupted place in the building. In buildings such as the John Hancock Tower in Chicago and One Maritime Plaza in San Francisco the braced frame is visible on the building’s façade. Both buildings feature a cross brace design as seen in Figure 4.

![Figure 3: Commonly Used Braced Frame Designs](image)

Cross Brace | Inverted V Brace | Single Diagonal Brace | Eccentric Bracing | Two-Story Cross Brace

1.2. Existing Mechanical Studies on *Euplectella Aspergillum*

Many studies focus on the spicule structure and natural material found in the composition of *Euplectella Aspergillum* (Aizenberg et al. 2004, Arasuna et al. 2018, Mayer et al. 2004, Monn 2015, Tavangarian et al. 2021, Wang et al. 2012, Weaver et al. 2010, Woesz et al. 2006). These studies concentrate on the mechanical benefit at the microscopic level within the spicule. The inspiration for this project stemmed from research conducted by Fernandes et al. where *Euplectella Aspergillum*’s inspired lattice structure was mechanically tested at a larger scale. Rather than focusing on the spicule itself, this study centers on the configuration created by the spicules. The study experimentally tests and simulates how the natural design aids to the advanced mechanical performance of *Euplectella Aspergillum*. One conclusion of this study is that
the sponge-inspired lattice design could be applied to large-scale infrastructure based on the results from the conducted analyses (Fernandes et al. 2021). The following sections explain how the sponge-inspired lattice structure was analyzed and how this theory came to fruition.

1.2.1. Experimental Lattice Design Specifications

To demonstrate the mechanical benefits of the sponge-inspired design the study compared it to three other lattice designs. Figure 5 presents the four different designs that were investigated. Design A features the sponge-inspired alternating double cross-braced design. Design B has an alternating single cross-braced design. Design C has a single cross-brace in each frame. Design D is a simple open-frame design. Throughout the experimental testing and model analyses, all designs contained the same total volume of material and a fixed volume ratio between non-diagonal and diagonal members. The non-diagonal to diagonal member ratios were calculated for circular and rectangular cross-sections, using the respective area and volume formulas and constraints mentioned previously.

![Figure 5: Design Schematics of Lattices of Interest](https://doi.org/10.1038/s41563-020-0798-1)
Figure 5 shows the designs with members with rectangular cross-sections. The member thickness is represented as $T_{i,d}$ for the diagonal members and $T_{i,nd}$ for the non-diagonal members. The designs can be easily interchanged for members with circular cross-sections with diameters denoted as $D_{i,d}$ and $D_{i,nd}$. For simplicity, each design features square frames with a side length, $L$. Design A has a slightly more complex bracing pattern than the other designs. There are two sets of cross braces in alternating frames that are offset from the non-diagonal members. This creates a spacing ratio, $S$, of the distance from the non-diagonal member to the beginning of the diagonal member to the total side length. In testing the spacing ratio used is taken from the average spacing found in *Euplectella Aspergillum*, 0.293$L$ (Fernandes et al. 2021).

1.2.2. Three-Dimensional Model Experimentation & Finite Element Analysis

Results

The study used experimentation and finite element simulation to learn more about how the different designs’ mechanical properties compare to each other. The experimental tests were comprised of axial compression applied to 6x6 frame designs using a single axis Instron. Similar conditions were modeled and analyzed in ABAQUS/Standard. Figure 6 compares the experimental tests to the finite element simulations by plotting stress versus strain. The conclusions made from this analysis include that Design A, shown in blue, peaks at the highest load this represents the commencement of buckling. Therefore, Design A has the largest critical buckling stress compared to the other designs. Additionally, the braced designs show similar initial behavior, suggesting that the stiffness is similar between Designs A, B, and C. Design D
shows more stiffness, but less capacity compared to the alternate designs. It is also noted that the experimental and simulated results agree and validate the accuracy of the findings.

![Graph showing experimental and simulated buckling analysis results.](image)

**Figure 6: Experimental and Simulated Buckling Analysis Results.** Retrieved from “Mechanically robust lattices inspired by deep-sea glass sponges” by Fernandes et al, 2021, [https://doi.org/10.1038/s41563-020-0798-1](https://doi.org/10.1038/s41563-020-0798-1).

Further finite element analyses were conducted with different loading and boundary conditions. One set of conditions studied was a loaded cantilever shown in Figure 7. A simplified tall building design is essentially a vertical cantilever, this is the first step to test the application of the deep-sea sponge-inspired design. The graphs present the results of Riks analyses showing the buckling behavior of the designs. In both loading conditions Design A has significantly more capacity compared to Designs B, C, and D. It is also evident that in the cantilever configuration Designs B and C perform
very similarly, with Design C having slightly more stiffness and buckling capacity.

Although Design D showed the most stiffness in compression it shows the opposite behavior as a vertically loaded cantilever.

Figure 7: Buckling Analysis Results of Vertically Loaded Cantilevers (a) Cantilever with an Applied Point Load (b) Cantilever with a Distributed Load. Retrieved from “Mechanically robust lattices inspired by deep-sea glass sponges” by Fernandes et al, 2021, https://doi.org/10.1038/s41563-020-0798-1.

Two other configurations were studied; a simply supported beam with vertical loads and a cantilever with horizontal loads. Design A consistently showed increased capacity compared to the other designs. Design B performed just below or as well as
Design C, but both reported decreased critical strength than Design A. Design D had significantly lower capacity in all models except for in the configuration of a cantilever under compression. In this simulation, Design D showed more comparable results to the other designs.

### 1.2.3. Optimization Analysis

The results concluded from the mechanical testing and finite analyses demonstrate the advanced mechanical properties of Design A. The question then becomes is there a possibility to make the design even more efficient. The challenge of optimization of braced frame lateral-force resisting systems for tall buildings is a well-studied topic (Allahdadian et al. 2012, Baldock 2007, Kingman et al. 2015, Liang et al. 2000, Moon et al. 2007, Stromberg et al. 2012, and Yazdi and Sulong 2011). To further study the efficiency of the deep-sea sponge-inspired design, the research shifted to finding the optimal number of braces required to produce the most capacity. A Python implementation of the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) was used to solve the optimization problem. The variables in this problem were the number of cross braces, which in turn affect the mass ratios of non-diagonal to diagonal members and the spacing ratio. As the number of braces increases the more mass is allocated to the diagonal members causing a decrease in the non-diagonal to diagonal mass ratio. For simplicity, the non-diagonal members are assumed to be at a 45° angle in relation to the non-diagonal members, Figure 8 illustrates the designs with varying numbers of cross braces. Using the results of the CMA-ES algorithm a 6x6 frame of each design was analyzed with a finite element buckle analysis under uniform compression. This was completed for both finite-sized structures and infinite-sized structures.
Figure 8: Optimization Analysis Designs. Retrieved from “Mechanically robust lattices inspired by deep-sea glass sponges” by Fernandes et al, 2021, https://doi.org/10.1038/s41563-020-0798-1.


Figure 9 presents the maximum critical stress for varying numbers of diagonal braces for finite-sized structures. The color bar represents the optimal mass ratio specific to each design. The maximum stress is discovered when the design features two diagonal
sets of braces, meaning it is the most efficient design. Design A is marked on the graph as well, showing that it has a similar effective critical stress. Figure 10 details the behavior of the optimal structure in comparison to Design A. Both experimental and simulated results are reported to verify the conclusion. Although both designs have the same number of diagonal braces, the difference is the mass ratio and spacing ratio. The optimal mass ratio of non-diagonal to diagonal members was calculated to be 0.6778 with a spacing ratio of 0.180L. Similar methods were taken to approach the optimization of infinite-sized structures. The procedure also concluded that the most efficient design for infinite-sized structures features two sets of diagonal braces. However, the non-diagonal to diagonal mass ratio decreased to 0.5614 and the spacing ratio increased to 0.339L (Fernandes et al. 2021).

Figure 10: Optimization of Finite-Sized Structures Results in Relation to Experimental Results. Retrieved from “Mechanically robust lattices inspired by deep-sea glass sponges” by Fernandes et al, 2021, https://doi.org/10.1038/s41563-020-0798-1.
1.2.4. Literature Review

The research completed by Fernandes et al. documents the process in which they developed the conclusion that the bracing design inspired by *Euplectella Aspergillum* has increased mechanical capacity that could be beneficial in many applications. Capacity-wise, Design A consistently surpasses the other designs tested in this paper. This led to the conclusion that the deep-sea sponge-inspired design could be applied at larger scales to increase capacity or decrease required material. However, the simulated models are not accounting for plasticity. This is an oversight, as many structural materials are constrained by plastic rather than elastic deformations. Depending on how plasticity affects the behavior of the material, it could eliminate the benefit of the double cross-braced design featured in Design A. If the mechanical benefit is lost with the introduction of plasticity, the application of Design A would not be advantageous due to the decreased stiffness compared to Design C.

The performances of each design were consistent throughout the varying analyses. For the purpose of this thesis, the cantilever model with vertical loading is the most pertinent. From the results, Design C exhibited the closest buckling capacity to Design A. Figure 7 reinforces this conclusion. Although Design B had very similar results as Design C the initial slope of the latter was slightly greater. This signifies slightly more stiffness resulting from Design C, increased stiffness is desired in tall building design. Thus, Design C is more likely to be used in tall building design and Design B was concluded to be negligible in the following study. The same conclusion was made for Design D, due to the lack of stiffness and critical buckling strength compared to Design A and C.
The optimization portion of the research adds another level of complexity to the deep-sea sponge-inspired design. The conclusion of the analysis introduced two new alternating double cross-braced lattice designs that implement different spacing and mass ratios. The new designs were tested only in compression on a 6x6 cell configuration. For an application in structural design, it would be interesting to see the behavior of different configurations with varying loading and how it compares to the original ratios taken from *Euplectella Aspergillum*. Having a better understanding of how the lattice designs mechanically behave will help determine the proper application for the design. In addition, since the ratios do vary from the original design it should be investigated how they impact the capacity and stiffness of the structure.

1.3. Finite Element Modeling Methods

To get a better understanding of the behavior of a structure that implements the double cross-braced design, several models were created and analyzed using *ABAQUS/Standard*. The following sections will describe the properties, designs, methods, and boundary conditions applied to each model.

1.3.1. Material Properties and Lattice Design Specifications

Most tall building designs detail a steel frame as the main structural system. To assess the integrity of the double cross-braced frame design in a structural application it seemed fitting to use steel as the modeling material. The material was given a Young’s Modulus of 200 GPa and a Poisson’s Ratio of 0.3. The same material properties were used for all members of each model.
For the analyses conducted in this study, four lattice designs were examined. These consist of three variations of the double cross-braced frame design and one single cross-braced frame design. The variation of the double cross-braced frame designs comes from the ratio at which the bracing members are spaced away from the vertical members. The three ratios used in the models for this study are 0.180L, 0.293L, and 0.339L. Design 1 features the 0.180L ratio, the finite-sized structure optimal ratio. Design 2 uses the ratio closest to the natural sponge structure of 0.293L. The final double-braced lattice design uses the infinite-sized structure ratio of 0.339L (Fernandes et al. 2021). Design 4 features a single cross-brace design in each frame. Figure 11 details each design that is studied in the following analyses.

![Figure 11: Schematic of Proposed Lattice Designs](image)

The members were chosen to be solid circular rods for both the diagonal and non-diagonal members for simplicity purposes. Using similar methods as Fernandes et al. the ratio between the diameters of the diagonal and non-diagonal members was calculated. This was completed by manipulating the area and volume equations of circular diagonal and non-diagonal members. The following ratio is produced for the double cross-braced frame designs.
\[ D_{DB,ND} = 1.41\sqrt{2}D_{DB,D} \approx 2D_{DB,D} \]  
(Eq. 1)

Similarly, the same relation can be made for the single brace design, producing the following ratio

\[ \frac{D_{SB,D}}{D_{SB,ND}} = \frac{1}{2} \]  
(Eq. 2)

Overall, both designs have a 1:2 ratio of the diameter of the non-diagonal to the diagonal (Fernandes et al. 2021). Since both designs have the same diameter ratio and the same total length of diagonal members, the radius selected for non-diagonal and diagonal members is constant throughout all models. This also means the volume ratio of non-diagonal to diagonal is fixed and therefore the same amount of material is used in all designs. To determine the efficiency of all designs the volume of material used must remain consistent through all designs.

For the following analyses, the radii selected were 0.4572 m (1.5 ft) for the non-diagonal members’ radius, \( D_{ND} \), and 0.2286 m (0.375 ft) for the diagonal members’ radius, \( D_{D} \). The lattice frames in all designs are square; with side lengths, \( L \), of 4.267 m (14 ft). This dimension was chosen to represent the average floor height within a building, producing a design where each set of braces spans only one floor.

1.3.2. Modeling of the Structures

Thin tall building designs were modeled using Designs 1-4 with various numbers of floors in each model. The dimensions of the hypothetical buildings were two frames wide and 12, 24, 36, or 48 frames tall, producing a total of 16 models. The variety of models will allow the comparison between lattice designs and how height may affect the structure. Figure 12 shows each model structure that was simulated in this study.
All models were constructed in the same fashion for consistency and comparability. Due to the simplicity of the tall building designs, the models were assembled as 2D deformable wire models to negate out-of-plane deflections. Members were modeled as 1D Timoshenko beam elements (ABAQUS element type B22). As mentioned earlier all members were solid circular rods and thus modeled as circular profiles. Every structure was defined with fixed boundary conditions at the base. Additionally, all joints at which members intersect were modeled as welded connections. All models neglect the self-weight of materials and any dead or live loading. Additional loading of the structures varied based on the analysis. The individual loading conditions are defined later in this chapter based on the type of analyses conducted.

Figure 12: Schematic of Models Under Investigation
1.4. Finite Element Analysis Results

Using the described methodology of the previous sections, several analyses were conducted to investigate the application of double cross braced lattice designs in tall building design. The following sections will present the results of static, buckling, and Riks analyses.

1.4.1. Static Analysis

The first analysis completed on each model was a static analysis to investigate the stiffness of each structure. For these analyses, a concentrated force of 200 kN was applied to the top-left node of each structure in the positive x-direction. This produced the deformed shapes and the maximum displacement values of each structure. Table 1 presents the maximum displacement values from each analysis. Figure 13 displays the deformed shapes for all Design 1 models.

<table>
<thead>
<tr>
<th>Design Number</th>
<th>Number of Floors</th>
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<tbody>
<tr>
<td>Design 1</td>
<td>12 24 36 48</td>
</tr>
<tr>
<td>Design 2</td>
<td>12 24 36 48</td>
</tr>
<tr>
<td>Design 3</td>
<td>12 24 36 48</td>
</tr>
<tr>
<td>Design 4</td>
<td>12 24 36 48</td>
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</table>

Table 1: Maximum Displacement of Model in Result of 200 kN Horizontal Load (m)
Each model behaves similarly to a cantilever which results in a slight bend of the structure in the same direction of the horizontal load. The deformed shapes are consistent through all designs and floor heights but vary in magnitude. Additional figures can be found in Appendix A that present the deformed shapes of the Designs 2-4 models, Figures 55-57. Overall, all designs show an increase in deformations with the addition of more floor heights.

The displacement data presented in Table 1 shows the direct relationship between story height and maximum deflections. The values confirm that all models deform similarly despite the design used to construct them. On average the models deform 730% more when the floor height is increased from 12 floors to 24 floors, 2420% for 12 floors to 36 floors, and 5700% for 12 floors to 48 floors.
Of the deep-sea sponge-inspired lattice designs, Design 1 shows the smallest deflections while Design 3 has slightly greater maximum deflections. These results lead to the conclusion that Design 1 has slightly greater stiffness compared to Design 3. Additionally, Design 3 has the least stiffness of all designs tested. From these observations, it can be determined that the further the diagonal members are spaced away from the vertical members, results in less stiffness within the design. Overall, the smaller the spacing ratio is the better resulting stiffness.

**Figure 14: Normalized Maximum Displacement at Various Numbers of Floors**

This concept is reinforced in Figure 14, where the displacement values are normalized to the maximum displacement of Design 4. The normalized displacements for Designs 1-3 show signs of asymptotic behavior. These results show that despite the spacing ratios utilized in Designs 1-3, they display more comparable stiffness to Design 4.
at taller heights. The maximum normalized displacement values for Designs 1, 2, and 3 are respectively 1.028, 1.038, and 1.037. These values confirm that Designs 2 and 3 have almost the same total effective stiffness. The only variable that changes between the two designs is the spacing ratio of the braces. Design 2 has a spacing ratio of 0.293L which is very close to Design 3’s spacing ratio of 0.339L. Compared to the spacing ratio of 0.180L utilized in Design 1 there is a clear gap between the three designs.

As mentioned before, the common limiting factor for tall building design is stiffness. The results of the static analyses show that Designs 1-3 have slightly less overall stiffness compared to Design 4. This conclusion does cause concern for the application of Designs 1-3 in tall building design. However, the difference in the maximum stiffness between all the designs is minimal. Therefore, the double-cross braced lattice designs still may produce a greater mechanical value to offset the reduced stiffness.

1.4.2. Buckling Analysis

The second analysis completed on each model was a linear buckling analysis. Unlike the static analysis, a unit applied displacement was placed at the top-left node of each model in the positive x-direction. Tables 2 through 5 present the first five eigenvalues of each structure determined from the buckling analysis. The eigenvalues are significant as they are an indication of at what load a structure begins to buckle. The first and second eigenvalues are similar in magnitude for all models, this is also true between the third and the fourth sets of values. It stands out that Design 3 has the largest eigenvalues whereas Design 4 has the smallest eigenvalues. This is the first indication
that the sponge-inspired design has a much higher buckling capacity compared to the single braced design.

Figure 15 summarizes the first eigenvalues reported from all of the models by normalizing them to the magnitude of the first eigenvalue calculated for Design 4. The eigenvalues determined for Designs 2 and 3 are, respectively, on average 2.20 and 2.37 times more than the values found for Design 1. While the results of Design 1 are on average 1.63 times greater than those of Design 4. The ratios of Designs 1, 2, and 3 to Design 4 are consistent across floor heights, but the difference is more subtle in the 12-floor models and is much more revealing in the taller models.

Similar to the results of the static analyses, when only observing the sponge-inspired designs, Designs 2 and 3 have comparable outcomes as opposed to Design 1. Again, this could be a result of the changes in spacing ratio. Meaning, the larger the spacing ratio, the higher the buckling capacity. Inversely, the smaller the spacing ratio, the lower the buckling capacity.
### Table 2: First Eigenvalues

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### Table 3: Second Eigenvalues

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### Table 5: Fourth Eigenvalues

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### Table 6: Fifth Eigenvalues

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<td>Design 3</td>
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<td>33.6</td>
<td>67.8</td>
<td>114.7</td>
</tr>
<tr>
<td>Design 4</td>
<td>5.0</td>
<td>14.9</td>
<td>29.9</td>
<td>50.1</td>
</tr>
</tbody>
</table>
In addition to the eigenvalues, the buckling analysis also produces the mode shapes of the structure. Figures 16-20 show the first five mode shapes for Design 1 with 12 floors. As mentioned previously, the first and second eigenvalues and the third and fourth eigenvalues are similar in magnitude. The mode shapes illustrate that the first and second buckling modes are very similar in shape. The same is true for the third and fourth modes. The signs are associated with which side of the structure experiences the buckling. Negative indicates buckling occurs on the left while positive signifies the right side. The affected area of the model increases as the mode shapes progress. Similar trends are detected in the mode shapes for all other models, these can be found in Appendix A, Figures 58-132.

Figure 15: Normalized First Eigenvalues at Various Numbers of Floors
Figure 16: 12-Floor Structure with Design 1 Lattice, Mode Shape 1

Figure 17: 12-Floor Structure with Design 1 Lattice, Mode Shape 2
Figure 18: 12-Floor Structure with Design 1 Lattice, Mode Shape 3

Figure 19: 12-Floor Structure with Design 1 Lattice, Mode Shape 4
1.4.3. Riks Analysis

To further investigate the mechanical properties of Designs 1-4, a Riks analysis was performed. The Riks analysis is a non-linear analysis that will calculate the behavior of a structure after instability. The Riks analyses presented in this chapter include geometric nonlinearities. The analysis incrementally adds applied displacement while monitoring the behavior of the structure. The results of this analysis reveal the large-deformation responses of each structure. For this analysis, each design was studied with the 12-floor models, as it is the stiffest compared to the other floor heights previously investigated in this study. The increased buckling capacity of the sponge-inspired designs relative to Design 4 was consistent throughout all story levels. These two observations suggest that a smaller structure may be the more suitable application for the sponge-inspired design.

Similar to the buckling analysis, the Riks analysis models are loaded with an applied horizontal displacement at the top-left node of each structure. To observe the full
behavior of the structure, the applied displacement must be greater than the first eigenvalue. The value of the applied displacement is determined by multiplying the applied displacement utilized in the buckling analysis by the first eigenvalue. In this instance, a unit applied displacement was used in the initial buckling analysis. Therefore, the applied displacement required to initiate buckling in the Riks analysis is equivalent to the magnitude of the first eigenvalue.

1.4.3.1 Elastic Behavior

The initial Riks analysis completed was to verify the methods and results of Fernandes et al. discussed previously. The models completed in this thesis do vary from the previous paper, however, the general behaviors observed of the designs are very similar. Figure 21 presents the results of the elastic behavior of Designs 1-4 with a story height of 12. The values on the graph have been normalized to present normalized reaction forces versus normalized displacement. The normalized reaction forces are calculated by dividing the applied displacement being applied instantaneously to the structure by the side length of one frame. The normalized displacement is calculated by summing the reaction forces at the base of the structure and dividing it by the side length of one frame, the shear modulus (μ=14.5 MPa), and the depth of the members.
The results presented in Figure 7a are easily comparable to Figure 21. Design A is directly comparable to Design 2, while Designs 1 and 3 vary in the brace spacing ratio. Additionally, Design C and Design 4 also have the same pattern. When comparing the two graphs, it is important to consider the difference in configuration and the scale of the models. In both figures, the behavior of each design is initially very similar featuring the same linear trend. At approximately 0.8, Design 4 and Design C indicate signs of buckling. Past this point both structures can continue to withstand increasing loads but with much less overall stiffness. As for Designs 1-3 and Design A, the structures continue to show a constant slope. The similarities validate the analysis previously conducted and allow for further research.

The Riks analysis results also validate that Design 4 has more stiffness than the deep-sea sponge-inspired designs. This is determined by the slightly greater angle at
which the line representing Design 4 increases before experiencing buckling. The Riks analyses also confirm that the capacities of the Designs 1-3 are much greater than Design 4. The capacities of Designs 2 and 3 are closer in magnitude than Design 1 but all three exceed the capacity of Design 4. This is the same finding as seen in the previous buckling analysis. To get a better understanding of Designs 1, 2, and 3 data was collected past the point of buckling. The post-buckling behavior suggests Design 3 is the most unstable and cannot withstand additional loading.

There are slight variances between the two graphs. For one instance, the normalized reactions force at which the slope changes for Design 4 does differ between the two studies. The normalized reaction force is approximately $1.45 \times 10^{-3}$ compared to the $1.15 \times 10^{-3}$ presented in the work of Fernandes et al. Another variation to be noted, is that Design 2 seems to experience buckling sooner compared to the corresponding Design A from the works of Fernandes et al. Design 2 shows the onset of buckling at the normalized displacement of 1.85. Figure 7a does not illustrate the post-buckling behavior of Design A but does detail that it continues to have capacity beyond a normalized displacement of 1.85 without showing signs of buckling. These disparities could be a result of the differences in members’ cross-sectional geometry, the relationship between the diameter size and the side length of the frame, or the number of story heights being analyzed. Despite the differences, it is promising to see similar mechanical behavior for each design.

1.4.3.2 Plastic Behavior

Additional Riks analyses were conducted with the introduction of plastic material behavior. This will simulate the behavior of structural steel more realistically. Steel
material in this structural application such as the one tested in this study generally yields plastically not elastically. Each design was tested using two additional materials, one high-strength steel, and one advanced high-strength steel. The high-strength steel was defined as having a yield stress of 300 MPa and strain of 0.0 and an ultimate stress of 500 MPa and strain of 0.2. These properties were chosen to imitate the typical steel used in construction. The second steel material applied to the designs is advanced high-strength steel. Advanced high-strength steel (AHSS) is designed to exhibit high yield strength which can in return, potentially, reduce the total necessary material required (Yaghoobi et al.). The advanced high strength steel was defined with a yield stress of 1.2 GPa and strain of 0.0 and an ultimate stress of 1.3 GPa and strain of 0.2.

The same applied displacements, calculated using the eigenvalues found in the buckling analysis, are applied to the relative elastic models. Adding plasticity as a material property should in return decrease the total capacity of the structures. Therefore, the applied displacement used in the elastic modeling should be more than enough to initiate buckling in the plastic models.

Figures 22-25 present the different mechanical responses based on the design and material used. A significant loss of capacity is seen across all designs for both the high strength and advanced high strength steel. The observed normalized reaction force of Design 1 decreases 77.2% for the advanced high strength steel compared to the elastic material and 93.8% for the high strength steel. As the spacing ratio increases in the designs, the loss of capacity also increases. Design 2 experiences an 82.6% loss with the advanced high strength steel and a 95.7% loss with the high strength steel. Design 3 showed similar results, an 83.8% loss with the advanced strength steel and a 95.9% loss
with the high strength steel. Due to the sponge-inspired design’s increased elastic
capacity, Designs 1, 2, and 3 are more susceptible to capacity loss. Design 4 experiences
less overall loss with the consideration of plastic behavior. The advanced high strength
steel and the high strength steel produce a reduction of the critical normalized reaction
force by 61.8% and 89.5%, respectively.

Figure 22: Plastic and Elastic Response of Cantilevers Employing Design 1
Figure 23: Plastic and Elastic Response of Cantilevers Employing Design 2

Figure 24: Plastic and Elastic Response of Cantilevers Employing Design 3
Figure 25: Plastic and Elastic Response of Cantilevers Employing Design 4

The overall performance of each structure, with the addition of plasticity, was much more consistent compared to the results of the elastic analyses. The different designs all show signs of buckling at similar magnitudes. The advanced high strength and high strength steel indicates signs of buckling on average at approximately $0.54 \times 10^{-3}$ and $0.14 \times 10^{-3}$, respectively, across all designs. This means on average the advanced high strength steel is 3.8 times greater in capacity compared to the high strength steel.

Figures 26 and 27 detail the plastic analysis results with the high strength and advanced high strength steel for Designs 1-4. In previous analyses, it has been observed that Designs 2 and 3 behave very similarly. This remains true in the plastic analyses for both steel materials, the responses are almost identical. Design 1 performs slightly better in capacity compared to the other sponge-inspired designs. These figures indicate that
Design 4 has the greatest capacity among all of the simulated models and for both steel materials. It appears that with the inclusions of plasticity, the smaller the spacing ratio being utilized, the better the total capacity of the structure is. These findings are contrary to the observations concluded in the elastic models.

However, the relationship between lattice pattern and stiffness has stayed consistent throughout both plastic and elastic analyses. Consistently, Design 4 has slightly more overall stiffness compared to the rest of the designs. Followed by Design 1, which has the smallest spacing ratio. Proceeded by Design 2 and 3, the latter has the largest spacing ratio analyzed in this thesis.

![Figure 26: Plastic Response of Cantilevers Modeled with High Strength Steel](image-url)
1.6. Conclusion

The initial analyses described in this thesis confirm the results presented by Fernandes et al. The elastic response of the deep-sea sponge inspired design performs at a much higher capacity compared to the single cross-braced design, and therefore the alternating single cross-braced design (Design B) and open-frame design (Design D). Originally, this observation was assumed to indicate that the deep-sea sponge-inspired design was overall superior, in terms of capacity, to the comparative designs. There is concern regarding the stiffness of the design for the application of tall buildings. Despite the advanced mechanical performance of the design, Designs 1-3 showed slightly
decreased stiffness compared to Design 4. The differences in maximum displacement were minimal, less than a 4% increase at the most. Considering only the elastic analyses, the benefit seen in the capacity for Designs 1-3 outweighs the small loss of overall stiffness. A solution to combat the decreased stiffness would be to increase both the diagonal and non-diagonal member sizes. Increasing the amount of material used does, however, reduce the efficiency of the design. Overall, the results from the elastic analyses indicate that the deep-sea sponge-inspired designs do present significant mechanical benefits and could apply to tall building design.

After introducing plasticity to the material properties, this conclusion was proven false. The increased capacity observed in the elastic analyses for Designs 1, 2, and 3 was not seen in the models with plastic material properties. Design 4 shows signs of buckling at higher forces in relation to the deep-sea sponge-inspired lattices. Additionally, Design 4 continues to exhibit the greatest overall stiffness, making it the most efficient bracing design in capacity and stiffness. From these observations, it can be concluded that the sponge inspired bracing system is not efficient for the design of tall buildings. Overall, in the application of structures, this suggests that the deep-sea sponge-inspired design is only beneficial in structures that are governed by elastic deformations.

Comparing only the varying deep-sea sponge-inspired designs, certain trends began to form. The simulated models of each bracing design consist of the same member dimensions and diagonal to non-diagonal volume ratios. The varying factor between Designs 1, 2, and 3 is the spacing ratio. One trend that was consistent in all of the analyses is that the spacing ratio directly impacted the total stiffness of the structure. Increasing the spacing ratio resulted in the structure experiencing larger maximum
deflections. The relationship between spacing ratio and capacity was more dependent on the material properties. Elastically, the deep-sea sponge-inspired designs with larger spacing ratios showed a greater ability to take on lateral loading. The opposite was true for the structural models considering plasticity. The designs with larger spacing ratios presented as having decreased capacity.
CHAPTER 2

APPLICATION OF *EUPLECTELLA ASPERGILLUM*’S NATURAL LATTICE STRUCTURE TO PLATE DESIGN

2.1. Introduction

Plate design is the second application of *Euplectella Aspergillum*’s lattice structure investigated in this thesis. The findings from the tall building models have made it apparent that although there are definitive mechanical benefits in the deep-sea sponge-inspired design, this is only true if the structure is limited in its elastic state. Therefore, in theory, the double-crossed braced pattern is potentially a very efficient substitute for slender plates.

Slender plates are defined as having a width, $b$, to thickness, $t$, ratio greater than $\lambda_r$. The equation for $\lambda_r$ varies on the boundary conditions assigned to the edges of the plate (Ziemian 2010). The capacity of slender plates is controlled by elastic buckling. For this study, the plates modeled and analyzed in this chapter all fall within the slender region based on their width to thickness ratios. Constraining the geometry of the plates avoids initiating plastic deformations within the structure. This characteristic is favorable to the deep-sea sponge-inspired lattices since the benefits of the design are only valid in structures that are constrained elastically. It also ensures comparability among the plate structures.

As mentioned in the previous chapter, the deep-sea sponge-inspired bracing system should allow for reduced material volume while maintaining capacity during elastic deformations. In theory, the slender plate structure should be able to gain the most
mechanical benefit with the application of the deep-sea sponge-inspired bracing system. This would allow for either higher loading conditions or reduced member sizes. Another positive attribute of slender plates for this application is that, unlike tall building design, in-plane lateral displacement is less of a concern in plate design. With these ideas in mind, slender plates are the most promising application of the deep-sea sponge-inspired bracing system. The subsequent sections in this chapter present the methods of analyses in which the critical buckling stress and material efficiency are evaluated of various plate designs and sizes.

2.2. Finite Element Modeling Methods

Similar to the tall building application several models for the plate application were created and analyzed using ABAQUS/Standard. The following sections will describe the properties, designs, methods, and boundary conditions applied to each model.

2.2.1. Material Properties and Lattice Design Specifications

The same steel material properties that were used in the previous set of models for the tall building analyses are also used in the following plate simulations. This details a Young’s Modulus of 200 GPa and a Poisson’s Ratio of 0.3. The same material properties were used for all members of each model for consistency.

Designs 1 and 4 from the previous chapter are continued to be examined in the application of plate design. To simulate a plate the lattice frames were arranged into square structures. From the previous study of tall buildings, the results revealed that
Design 1 has elastic benefit while also having the greatest structural stiffness and plastic behavior of the deep-sea sponge-inspired brace design. Designs 2 and 3 are not examined in this chapter due to their decreased stiffness and plastic capacity. To study the efficiency of the designs more closely, Designs 1 and 4 are compared to solid square plates of equal volume.

The lattice designs for the tall building model were scaled to span across an entire floor height. Similarly, the bracing designs for the plate models were scaled down to have a frame side length equal to 1 meter. To keep the ratio between frame side length and the member diameters consistent, the diagonal and non-diagonal diameters were scaled to 0.05358 m and 0.10714 m, respectively. The diagonal to non-diagonal diameter ratio remained the same using Equations 1 and 2. This also allowed the diagonal to non-diagonal member volume to remain constant for plates of comparable sizes. The same diameters were used for both Design 1 and 4 at all plate sizes.

The dimensions of the solid plate models were calculated to have the equivalent total volume as Designs 1 and 4. The solid plate models are square, with side lengths equal to the number frames in the lattice designs multiplied by the frame side length. The thickness of the plate was determined by summing the volume of material used for Designs 1 and 4, which are equivalent, divided by the side length of the plate squared. Table 7 details the lattice plate dimensions and the equivalent measurements of the solid plate for various sizes. Based on the width to thicknesses ratio of the equivalent solid plates, all plates being analyzed in this chapter are considered slender. Meaning, that the elastic behavior controls the buckling capacity of the plate. This chapter will only explore the elastic behavior of the models.
Table 7: Plate Model Dimensions

<table>
<thead>
<tr>
<th>Lattice Plate Dimensions (m)</th>
<th>Total Volume of Diagonal Members (m³)</th>
<th>Total Volume of Non-Diagonal Members (m³)</th>
<th>Total Volume of Lattice Plate (m³)</th>
<th>Equivalent Solid Plate Dimensions (m)</th>
<th>Solid Plate Thickness (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x2</td>
<td>0.013</td>
<td>0.108</td>
<td>0.121</td>
<td>2x2</td>
<td>0.0302</td>
</tr>
<tr>
<td>3x3</td>
<td>0.028</td>
<td>0.216</td>
<td>0.244</td>
<td>3x3</td>
<td>0.0271</td>
</tr>
<tr>
<td>4x4</td>
<td>0.051</td>
<td>0.361</td>
<td>0.412</td>
<td>4x4</td>
<td>0.0257</td>
</tr>
<tr>
<td>5x5</td>
<td>0.079</td>
<td>0.541</td>
<td>0.620</td>
<td>5x5</td>
<td>0.0248</td>
</tr>
<tr>
<td>6x6</td>
<td>0.115</td>
<td>0.757</td>
<td>0.872</td>
<td>6x6</td>
<td>0.0242</td>
</tr>
<tr>
<td>7x7</td>
<td>0.155</td>
<td>1.010</td>
<td>1.165</td>
<td>7x7</td>
<td>0.0238</td>
</tr>
<tr>
<td>8x8</td>
<td>0.204</td>
<td>1.298</td>
<td>1.502</td>
<td>8x8</td>
<td>0.0235</td>
</tr>
<tr>
<td>9x9</td>
<td>0.257</td>
<td>1.623</td>
<td>1.880</td>
<td>9x9</td>
<td>0.0232</td>
</tr>
<tr>
<td>10x10</td>
<td>0.319</td>
<td>1.983</td>
<td>2.302</td>
<td>10x10</td>
<td>0.0230</td>
</tr>
</tbody>
</table>

2.2.2. Modeling of the Structures

Several sizes of square plates and braced frame structures are analyzed in the following sections. They vary from side lengths of 2 meters to 10 meters in increments of one meter or in other words one frame length. The purpose of the multiple sizes is to study whether the size of the structure impacts the efficiency of the design. Figure 28 provides the schematics of each model with equivalent total side lengths of 6 meters or 6 frames. This is representative of the relationship between the three models for all sizes.

The models analyzed in this chapter were all modeled as 3D deformable wire or shell features. The structures that showcase Designs 1 and 4 were modeled with wire base features and the solid plates were modeled with the shell base feature. The models with a brace design were constructed with the same methods that were used for the cantilever models. The solid plates were modeled as thin shell elements (ABAQUS element type...
S8R5) and defined with a homogeneous shell section type. All structures were fully fixed along the bottom plane and constrained along the top plane with ties to simulate rigidity. Again, the models neglect self-weight and loading conditions are dependent on the analysis being performed. These conditions are defined further in this chapter.

![Figure 28: Example Schematics of Plate Designs. (left) Design 1 (middle) Design 4 (right) Solid Plate.](image)

2.3. Finite Element Analysis Results

Using the described construction procedure of the previous sections, several analyses were performed to further study the efficiency of the deep-sea sponge-inspired bracing designs. There are many studies that examine the buckling and post-buckling behavior of flat plates under compression, shear, and a combination of the two (Cook and Rockey 1963, Cox 1933, Dima 2015, Featherston and Ruiz 1998, Koiter 1963, Rhodes and Harvey 1971, Shufrin and Eisenberger 2006). The plate models in this chapter are simulated with two loading conditions: axial compression and shear loading. The following sections will present the results of a series of buckling analyses for each loading case.
2.3.1. Compression Loading

To simulate a plate under compression, an applied unit displacement was placed along the top plane of the plate in the negative y-direction. Table 8 presents the first eigenvalues computed from the linear buckling analyses for all models. This analysis was completed to observe the relationship between the size of the structure and the elastic buckling capacity. For all of the models under compression, the first eigenvalue decreases as the size of the plate increases. In relation to each other, Design 1 exhibits the largest critical buckling stress in compression followed closely by Design 4 and, finally, with the smallest reported eigenvalues, the solid plate design. This hierarchy is consistent for all plate sizes and is a promising indication of the efficiency of Design 1. The rate of loss in the magnitude of the eigenvalues decreases as the plate size increases.

Table 8: First Eigenvalues Computed of Plate Models Under Compression Loading

<table>
<thead>
<tr>
<th>Plate Dimensions (m)</th>
<th>Design of Plate Model</th>
<th>Design 1</th>
<th>Design 4</th>
<th>Solid Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x2</td>
<td>8.49E-04</td>
<td>8.38E-04</td>
<td>9.75E-05</td>
<td></td>
</tr>
<tr>
<td>3x3</td>
<td>5.64E-04</td>
<td>5.57E-04</td>
<td>5.23E-05</td>
<td></td>
</tr>
<tr>
<td>4x4</td>
<td>4.20E-04</td>
<td>4.17E-04</td>
<td>3.53E-05</td>
<td></td>
</tr>
<tr>
<td>5x5</td>
<td>3.35E-04</td>
<td>3.33E-04</td>
<td>2.62E-05</td>
<td></td>
</tr>
<tr>
<td>6x6</td>
<td>2.78E-04</td>
<td>2.77E-04</td>
<td>2.09E-05</td>
<td></td>
</tr>
<tr>
<td>7x7</td>
<td>2.38E-04</td>
<td>2.37E-04</td>
<td>1.73E-05</td>
<td></td>
</tr>
<tr>
<td>8x8</td>
<td>2.08E-04</td>
<td>2.08E-04</td>
<td>1.47E-05</td>
<td></td>
</tr>
<tr>
<td>9x9</td>
<td>1.85E-04</td>
<td>1.84E-04</td>
<td>1.28E-05</td>
<td></td>
</tr>
<tr>
<td>10x10</td>
<td>1.66E-04</td>
<td>1.66E-04</td>
<td>1.13E-05</td>
<td></td>
</tr>
</tbody>
</table>
Figure 29: First Eigenmode Shapes of 2-Meter Plates with Compression Loading
(top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 29 presents the first eigenmode shapes for each design at the 2-meter plate size under axial compression. The shapes are uniform across each design and the magnitude of displacement is consistent. The first eigenmode shapes of the remaining sized plates loaded with axial compression can be found in Appendix B presented as Figures 133 to 140.

![Figure 30: The Relationship Between the Braced Frame Designs and Solid Plates Under Compression](image)

Figure 30 depicts the relationship between the braced designs’ first eigenvalues and the solid plate’s first eigenvalues at increasing side lengths. In compression, Design 1 has slightly increased buckling capacity compared to Design 4. Therefore, the ratio of eigenvalues of Design 1 to the solid plate is also slightly larger than the ratio between Design 4 and the solid plate. The normalized eigenvalues also appear to start converging as the plate increases in size. At a side length of 10 meters, the model using Design 1
demonstrates a critical buckling stress 14.68 times greater than that of a solid plate with equal volume. This indicates that, in comparison, a larger solid plate is significantly less effective in compression versus a braced plate with the same total volume of material. Looking at the smaller Design 1 configurations, there is still substantial mechanical benefit. The model that implements Design 1 bracing with a side length of 2 meters exhibits 8.71 times the capacity compared to the solid plate. These results show clear evidence of the increased capacity and material volume efficiency contributed to the bracing configuration of Design 1.

Table 9: Initial and Required Solid Plate Thicknesses for Compression Loading

<table>
<thead>
<tr>
<th>Plate Dimensions (m)</th>
<th>Original Thickness, t_{sp,o} (m)</th>
<th>Required Equivalent Thickness, t_{sp,e} (m)</th>
<th>Ratio t_{sp,e} / t_{sp,o}</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x2</td>
<td>0.0302</td>
<td>0.0893</td>
<td>2.95</td>
</tr>
<tr>
<td>3x3</td>
<td>0.0271</td>
<td>0.0891</td>
<td>3.28</td>
</tr>
<tr>
<td>4x4</td>
<td>0.0257</td>
<td>0.0888</td>
<td>3.45</td>
</tr>
<tr>
<td>5x5</td>
<td>0.0248</td>
<td>0.0886</td>
<td>3.57</td>
</tr>
<tr>
<td>6x6</td>
<td>0.0242</td>
<td>0.0884</td>
<td>3.65</td>
</tr>
<tr>
<td>7x7</td>
<td>0.0238</td>
<td>0.0883</td>
<td>3.71</td>
</tr>
<tr>
<td>8x8</td>
<td>0.0235</td>
<td>0.0882</td>
<td>3.76</td>
</tr>
<tr>
<td>9x9</td>
<td>0.0232</td>
<td>0.0882</td>
<td>3.80</td>
</tr>
<tr>
<td>10x10</td>
<td>0.0230</td>
<td>0.0881</td>
<td>3.83</td>
</tr>
</tbody>
</table>
To analyze the material volume efficiency further, the thickness of the solid plate was increased. Additional material was added to the point where the first eigenvalue of the solid plate was equivalent to the first eigenvalue of the corresponding sized Design 1 braced frame structure. Table 9 presents the original thickness of the solid plate compared to the thickness required to achieve the same critical buckling stress. The results of these comparisons show that even for the smallest structure the solid plate would require almost three times the volume to possess the same buckling behavior as the 2-meter Design 1 braced-frame plate. As the plate dimensions increase, the ratio of required thickness, $t_{sp,e}$, to original thickness, $t_{sp,o}$, increases. Implied that Design 1 provides increased material efficiency as the structures get larger compared to the solid plate models.

### 2.3.2. Shear Loading

A similar series of buckling analyses were conducted for plates with shear loading. An applied unit displacement was placed along the top plane of the plate in the positive x-direction. Table 10 presents the first eigenvalues computed from the linear buckling analyses for all sized plates. Similar to the compression analysis results, the models with shear loading also experience a decrease in the magnitude of the first eigenvalue as the plate size increases. However, there is a substantial difference in the magnitude of the eigenvalues when comparing the shear to the compression results. Observing the relationship between the different designs in the configuration of a plate, Design 1 shows higher critical buckling stresses. As the plate size increases Design 4 becomes closer in proximity to the magnitude of the first eigenvalue of the respective
Design 1 sized plate. The results collected for the solid plate models indicate the design initially loses capacity at a much higher rate compared to the braced frame plates. Increasing the plate size length from 2 meters to 3 meters, it is noted that the first eigenvalue of the solid plate almost loses half of its magnitude. This indicates a large loss of material efficiency. Brace Designs 1 and 4 experience less overall loss compared to the solid plates.

Table 10: First Eigenvalues Computed of Plate Models Under Shear Loading

<table>
<thead>
<tr>
<th>Plate Dimensions (m)</th>
<th>Design of Plate Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design 1</td>
</tr>
<tr>
<td>2x2</td>
<td>2.53E-02</td>
</tr>
<tr>
<td>3x3</td>
<td>1.85E-02</td>
</tr>
<tr>
<td>4x4</td>
<td>1.53E-02</td>
</tr>
<tr>
<td>5x5</td>
<td>9.23E-03</td>
</tr>
<tr>
<td>6x6</td>
<td>7.23E-03</td>
</tr>
<tr>
<td>7x7</td>
<td>6.20E-03</td>
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<td>9x9</td>
<td>4.68E-03</td>
</tr>
<tr>
<td>10x10</td>
<td>4.15E-03</td>
</tr>
</tbody>
</table>
Figure 31: First Eigenmode Shapes of 2-Meter Plates with Shear Loading (top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 31 presents the first eigenmode shapes for each design at the 2-meter plate size with applied shear loading. The eigenmode shapes are similar between the three plate designs, however, the shape of Design 1 is reflected over the y-axis. Design 1 also experiences slightly less resulting displacement in the 2-meter sized plate. This is not true for the larger-sized plates. The first eigenmode shapes of the remaining sized plates with applied shear force can be found in Appendix B presented as Figures 141 to 148.

![Normalized First Eigenvalue vs Side Length of Plate](image)

**Figure 32: The Relationship Between the Braced Frame Designs and Solid Plates with Applied Shear Force**

Figure 32 illustrates the relationship between Design 1 and 4’s first eigenvalues and the solid plate’s first eigenvalues at increasing side lengths. The trend line of the normalized eigenvalues for Design 4 with applied shear force is very similar to its respective compression trend line. The difference can be seen in the magnitudes, the
normalized eigenvalues from the shear analyses are more than two times the magnitude of the compression results presented in Figure 32. The results of Design 1’s shear and compression loading analyses do not have matching trend lines. The shear results of Design 1 produce a maximum normalized eigenvalue of approximately 36 for the 4-meter sized plate. Past the peak, the normalized first eigenvalues alternate between increasing and decreasing in magnitude as the size of the plate is incrementally increased. It appears that the trend line is converging to 30.5, which happens to be the maximum ratio for Design 4. The alternating positive and negative slopes observed from the shear analyses are unlike the behavior seen in the rest of the results presented in this chapter. It would be interesting to investigate this behavior in the future.

Overall, the eigenvalue ratio is the largest at all plate sizes when utilizing Design 1. This suggests that the braced design has the highest material efficiency. This is the same conclusion drawn from the compression analyses results. The results of the simulations involving shear forces only; reveal that there is more mechanical benefit at smaller-sized structures compared to both Design 4 and the solid plate.

The shear analysis results produced higher normalized eigenvalues compared to the compression analyses. This will in return increase the required volume of material needed to match the first eigenvalue of the Design 1 plates. The same method of analysis was used for the shear models as the compression simulations. Table 11 presents the original thickness of the solid plate compared to the thickness required to achieve the same magnitude. The ratio of the required equivalent thickness to the original thickness has the same cyclic trend as seen in Figure 32. The ratio increases for the first three sizes of plates and then decreases only to increase again. There is an overall trend of the ratio
increasing despite the results of the 5-meter, 6-meter, and 8-meter sized plates. The results lead to the same conclusion of the compression analysis, that Design 1 provides increased material efficiency as the structures get larger compared to the solid plate models.

**Table 11: Initial and Required Solid Plate Thicknesses for Shear Loading**

<table>
<thead>
<tr>
<th>Plate Dimensions (m)</th>
<th>Original Thickness, $t_{sp,o}$ (m)</th>
<th>Required Equivalent Thickness, $t_{sp,e}$ (m)</th>
<th>Ratio $t_{sp,e} / t_{sp,o}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x2</td>
<td>0.0302</td>
<td>0.1432</td>
<td>4.74</td>
</tr>
<tr>
<td>3x3</td>
<td>0.0271</td>
<td>0.1486</td>
<td>5.48</td>
</tr>
<tr>
<td>4x4</td>
<td>0.0257</td>
<td>0.1554</td>
<td>6.04</td>
</tr>
<tr>
<td>5x5</td>
<td>0.0248</td>
<td>0.1344</td>
<td>5.42</td>
</tr>
<tr>
<td>6x6</td>
<td>0.0242</td>
<td>0.1302</td>
<td>5.37</td>
</tr>
<tr>
<td>7x7</td>
<td>0.0238</td>
<td>0.1301</td>
<td>5.47</td>
</tr>
<tr>
<td>8x8</td>
<td>0.0235</td>
<td>0.1281</td>
<td>5.46</td>
</tr>
<tr>
<td>9x9</td>
<td>0.0232</td>
<td>0.1280</td>
<td>5.51</td>
</tr>
<tr>
<td>10x10</td>
<td>0.0230</td>
<td>0.1270</td>
<td>5.52</td>
</tr>
</tbody>
</table>

**2.4. Conclusion**

The plate simulation analyses presented in this chapter were completed as an extension of the study of the structural application of the braced frame design inspired by *Euplectella Aspergillum*. Considering the conclusions deducted in Chapter 1 the deep-sea sponge-inspired design is most efficient for structures limited by elastic buckling.
Therefore, the application of Design 1 to a slender plate design should be highly beneficial to the structure’s capacity.

The results of the buckling analyses in both loading conditions confirm the mechanical benefit generated from the structure of Design 1 in the form of a plate. The analyses of the plates that feature Design 1 repeatedly present results indicating increased capacities in relation to plates constructed with Design 4 and solid geometry. This was true for both loading conditions applied during the extent of this study. Of the two loading conditions investigated in this study, the plate models with shear loading indicate higher required forces to initiate buckling compared to the plate models under compression. On average the plate modeled with Design 1 with shear loading produced eigenvalues 28 times the magnitude of the compression-loaded models. This speaks to Design 1’s increased ability to withstand shear forces.

The initial buckling analyses were simulated in a way so that each plate of the same side length used the same total volume of material. Since the volume of material remained consistent and Design 1 showed improved capacity, it can be concluded that Design 1 allocates material more efficiently compared to Design 4 or a solid plate. To develop a better understanding of the material volume efficiency, the solid plate geometry was altered to match the eigenvalues retrieved from the plates with Design 1. The results from this computation provided the ratio of the required thickness for equivalent eigenvalues to the required thickness for equivalent volume. This is equivalent to the ratio of the volume of material used in the updated solid plates to the volume of material used in the Design 1 plates. To acquire approximately the same eigenvalues under compression loading, the solid plate requires about three times more material.
volume compared to Design 1 at a minimum. As the plate sizes increase the ratio increases, meaning the material efficiency in Design 1 rises in direct relation to the size of the plate. This is the same behavior seen with shear loading but to a greater extent. To obtain equivalent eigenvalues, the solid plate must utilize 4.74 times the volume of material than Design 1.

In all, the main conclusions of this chapter are the following. The use of Design 1 in slender plates is more effective in shear and compression than Design 4 and solid plates. However, the deep-sea sponge-inspired design is overall more beneficial under shear forces. The solid plate models require significant increases in the material volume in order to compete with the capacity of the braced designs. This suggests that Design 1 requires considerably less material to achieve the same capacity as a solid plate.

### 2.5. Recommendations for Future Work

Based on the results of this research, the following work would be the most beneficial to the research of the structural application of the Euplectella Aspergillum’s natural lattice structure.

- Perform an analysis on a short building model that incorporates earthquake loading. Generally, shorter buildings are very stiff structures that are prone to failure as a result of smaller lateral deflections caused by earthquakes. The decreased stiffness found in Designs 1-3 could be beneficial to structural earthquake design.

- Additional optimization studies of the deep-sea sponge-inspired bracing system. The volume ratio of non-diagonal to diagonal members used for this
study remained consistent in all designs and models. The ratio used was
determined by the natural structure of Euplectella Aspergillum. Increasing the
volume of non-diagonal members would increase the stiffness of the structure
and may indicate more desirable overall performance compared to typical
braced frame designs.

- It is visible that naturally, the double cross braced design is not perfect
throughout the entire body of the species. There are signs of damage and
imperfections. Based on this observation it would be beneficial to incorporate
research concerning progressive collapse resistance of framed structures
completed by a research group at UMass Amherst (Gerasimidis and
Baniotopoulos 2011, Gerasimidis et al. 2014, Gerasimidis and Baniotopoulos
2015, Gerasimidis et al. 2015, Gerasimidis et al. 2016, Gerasimidis and Sideri
2016, Sideri et al. 2015)

- Further investigation of the post-buckling behavior of the plates under
compression and shear. Plate structures are capable to continue to withstand
loading past the initial sign of buckling. It would be interesting to continue to
observe the change of behavior as the plate with Design 1 bracing increases in
size.

- Simulations of biaxial compression and shear loading. Understanding the
behavior under a common loading pattern, such as this, will provide a better
idea of how to apply the bracing system more effectively.
• Conduct plate analysis with the consideration of plastic material properties and compare them to the results found for the plastic-elastic tall building models.
CHAPTER 3

STABILITY LANDSCAPE OF THIN-WALLED CYLINDRICAL SHELLS WITH
DIMPLE IMPERFECTIONS

3.1. Introduction

The second topic of consideration studied in this thesis concerns the stability
landscape of thin-walled cylindrical shells. The following sections will provide a brief
overview of the central research topic and introduce the supporting concepts that are
essential to the study.

3.1.1. Overview and Motivation for Study

Thin cylindrical shells are very versatile structures that are featured in many
ingenerated designs ranging from a simple Coke can to an aerospace structural design.
The structural efficiency of thin cylindrical shells is very appealing in the design process;
however, these structures are highly sensitive to imperfections. The introduction of a very
small imperfection on the shell’s surface results in a significant loss of axial buckling
capacity (Gerasimidis et al. 2018, Hutchinson et al. 1970, Jiao et al. 2018, Tsien 2012,
Von Kármán and Tsien 2003, Yadav and Gerasimidis 2020, and Wagner and Hühne
2017). The size, geometry, and location of the imperfection are vital characteristics that
affect the sensitivity of the structure. Understanding the stability sensitivity of thin
cylindrical shells in relation to an imperfection is critical to accurately calculate the
capacity of the structure.

Design codes take into consideration the imperfection sensitivity using the
knockdown factor approach. This factor conservatively estimates the capacity of a thin
cylinder shell with imperfections. Currently, the only way to obtain the exact capacity of a thin cylindrical shell with an imperfection is to already know the specifics of the imperfection. Realistically, this information is not typically known and cannot be obtained easily. Therefore, it is essentially impossible to accurately predict the buckling behavior of thin cylindrical shells.

Many studies previously completed have alluded to the idea of being able to locate an imperfection based on non-destructive probing methods (Abramian et al. 2020, Ankalhope and Jose 2021, Yadav et al. 2021, Virot 2017). The central procedure of the method would be to probe randomly along the surface of a cylinder under axial compression. Based on the results of the probing analysis, the probe is relocated, and the process is repeated until the probe is within the proximity of the imperfection. This methodology requires a more advanced understanding of the stability landscape of the thin cylindrical shell.

This chapter examines the buckling behavior of the area surrounding an imperfection on a thin cylindrical shell. This is accomplished through finite element analysis software, ABAQUS. The purpose of the analyses conducted in this chapter is to identify the boundaries of the zone of influence. The zone of influence refers to the area surrounding the imperfection that shows signs of irregular buckling behavior. Defining the zone of influence in relation to the amplitude of an imperfection is the first step to creating an efficient method that can locate an unknown imperfection on a thin cylindrical shell. The capability to locate imperfections would aid the ability to accurately predict the capacity of the structure.
3.1.2. Introduction to Non-Destructive Prediction Methods for Thin-Walled Cylinders

Many studies have detailed the behavior of thin cylindrical shells under axial compression (Bisagni 2000, Calladine 2018, Kreilos and Schneider 2017, NASA 1965, Ricardo 1967, and Rotter 2004). Understanding the behavior of these structures is essential in the ability to predict their capacities. In addition, to predict the capacity of a thin cylindrical shell, the testing methods must be non-destructive. The objective of the analysis is to be able to obtain an accurate buckling capacity to be implemented in an engineered design. Any damage incurred during testing would decrease the effective capacity of the structure and therefore null the testing results.

Non-destructive testing methods have been implemented experimentally and computationally (Yadav et al. 2021). The central elements of the testing techniques remain the same and are completed as follows. The cylindrical shell is loaded with uniform axial compression, $F_a$, at a magnitude much less than the initially predicted buckling capacity. Applying reduced amounts of compression ensures that the cylinder structure does not undergo buckling failure. After the shells have been axially loaded, they are probed in the radial direction. This process is repeated, at minimum, five times at increasing levels of compression. The data collected for each analysis is graphed to create a stability landscape and from this, the capacity of the cylindrical shell can be predicted. It has been determined that the final axial capacity prediction is more accurate with the addition of more computations or experiments (Yadav et al. 2021).

The non-destructive prediction methods described above are used in this study and have been adapted from the works of Yadav et al. (Yadav et al. 2021). Other studies
present similar methodology as well (Ankalhope and Jose 2021 and Fan 2019). Experimental and computation analyses have been completed to assess the accuracy of these methods. The results presented in this chapter are from simulated models, the specific modeling and analysis methods used are described in Sections 3.3 and 3.4.

3.1.3. Introduction to the Stability Landscape

The stability landscape is an alternative way to view the stability of a thin cylindrical shell. They are presented in the three-dimensional phase space comprised of the axial force, probe displacement \( D_p \), and probe force \( F_p \). Using the non-destructive prediction methods described in the previous section, a stability landscape can be constructed. Figure 33 is the stability landscape of a perfect cylinder with the same geometry detailed in Section 3.3.1. Each line represents the probing displacement and force data associated with a single applied axial load. For each applied axial load, the probe displacement is increased past the point of instability this creates a ridge-like shape. The point at which instability occurs corresponds to a maximum or peak probe force, \( F_p^{max} \). This moment is marked with a red star for each applied axial load in Figure 33. The stability landscape is dependent on the geometry of the imperfection, but typically the inverse relationship between applied axial compression and peak probe force is constant for all scenarios (Virot 2017).
Using the data points located at the top of each ridge, a two-dimensional graph can be created by plotting axial load versus peak probe force. This is a complementary view of the stability landscape. The data is fitted with a quadratic line of best fit, where the y-intercept is equivalent to the predicted capacity. This method is also referred to as ridge tracking and has been verified numerically and experimentally (Ankalhope and Jose 2021, Arbocz and Babcock 1968, Fan 2019, Lozano et al. 2019, and Yadav et al. 2021). The results presented in this thesis will follow this graphical method of ridge tracking.

### 3.2. Existing Studies on Thin Cylindrical Shells

It has been proven that the introduction of even a minor imperfection can have significant negative effects on the capacity of thin cylindrical shells. Many extensive studies investigate the buckling behavior and stability landscapes of thin cylindrical shells to get a better understanding of this phenomenon (Ankalhope and Jose 2021, Budiansky and Hutchinson 1972, Fan 2019, Gerasimidis et al. 2018, Haynie et al. 2012, ...
These studies point to the ultimate challenge that is associated with imperfect thin cylindrical shells: to create an accurate predictive methodology with no knowledge of the geometry or location of an imperfection. To create a solution to this problem much more information about the stability landscape of thin cylindrical shells must be uncovered.

The study presented in this chapter is an extension of previous works completed by Yadav et al. (Yadav et al. 2021) and Ankalhope and Jose (Ankalhope and Jose 2021). These studies introduce the concept of probing away from the dimple and using non-destructive testing methods to predict the axial capacities of imperfect thin cylindrical shells. The following sections discuss the existing research on these topics and how they relate to the computational study presented in this chapter.

3.2.1. Probing Away from the Dimple Imperfection

The work of Yadav et al. evaluated the accuracy of the non-destructive prediction methods, discussed in Section 3.1.2 (Yadav et al. 2021). First, this was assessed through a computational analysis of a perfect and an imperfect thin cylindrical shell. The probe was located at mid-height of both cylinders and at the center of the imperfection for the imperfect shell. For the perfect cylindrical shell, the results showed a 2.5% difference between the predicted and the numerically obtained capacities. The imperfect cylinder being analyzed featured an imperfection with an amplitude of 0.1t, t being the thickness of the shell. The percent difference found between the predicted and the numerically obtained capacities of this model was 0.11%. These results support the accuracy and
future use of the non-destructive analysis methods to predict the capacity of thin cylindrical shells. The accuracy of the prediction method was also tested experimentally, with the same probe locations as the computational study. The results of the experimental tests reveal that the method is more accurate for shells with large imperfections that are greater than or equal to $1.5t$ in amplitude. The predicted capacities were found to be within 0.5% of the actual critical load for cylinders with large imperfections. For smaller imperfections, amplitudes less than $1.5t$, the method of testing consistently over-predicted the capacity.


Based on the conclusion that the non-destructive prediction methods are accurate in simulation, it is proposed that the method could be used to accurately predict the
capacity at locations away from the center of the imperfection. To test this theory a computational study was completed where the location of the probe was moved multiple times in the axial and circumferential directions on cylinders with imperfections of amplitudes $1t$ and $2t$. Figure 34 presents the axial load and peak probe force curves for probing locations along the mid-height circumference of a cylinder with an imperfection amplitude of $1t$. Of the seven chosen probing locations, only the first three locations ($0^\circ$, $3.7^\circ$, $10^\circ$) showed signs of decreased capacity caused by the imperfection. The curves for probing locations of $30^\circ$ and larger follow the curve of the perfect cylinder, meaning the imperfection is not being accounted for. The same results were observed for the $2t$ imperfection.

The data collected from the probe moving axially away from a 1t amplitude imperfection, is presented in Figure 35. The distance between the probe location and the center of the imperfection is measured by the half-wavelength of the classical axisymmetric buckling mode, $\lambda$. In the axial direction, the probe was placed in five locations. The four locations above the center of the imperfection all show similar behavior. At lower axial loads the probing data is unable to detect the imperfection and has similar patterns to the perfect cylinder curve. However, at high axial loads, all of the probing locations detect the influence of the imperfection. The same behavior was observed for the 2t amplitude imperfection. These results also support the theory that the axial load and peak probe force curves can indicate the influence of the imperfection. The axial loads that show the influence of the imperfection may be too close to the capacity of the cylinder and could cause buckling if not careful.

The experimental and computational studies completed confirm the accuracy of the non-destructive prediction methods. It can also be concluded that axial load and peak probe force curves at probing locations away from the imperfection can indicate the effects of an imperfection. These are important steps toward creating a technique to locate an imperfection and accurately predict the cylinder’s capacity.

### 3.2.2. The Least Resistance Path to Probing

The work of Ankalhope and Jose presents an alternative way to analyze the data recorded from the non-destructive prediction methods previously described (Ankalhope and Jose 2021). The authors propose a method to predict the capacity of a cylindrical shell through the means of the Least Resistance Path to probing (LRP). The curve
corresponding to the LRP on a probe force versus probe displacement graph has the lowest energy barrier to buckling compared to alternate probing curves. The energy barrier is the additional energy required to initiate buckling, for a particular axial compressive load (Ankalhope and Jose 2021 and Thompson 2015). The magnitude of the energy barrier is equivalent to the area beneath the probe force and probe displacement curve. Applying axial loads close to the actual capacity of the cylindrical shell reduces the required additional energy to initiate buckling, therefore the energy barrier is lower.


The prediction method of this study implements non-destructive testing analyses to predict the capacity of an imperfect thin cylindrical shell with no knowledge of the imperfection. Rather than focusing on the relationship between the axial load and the peak probe force, this study investigates the relationship between probe force and probe
displacement. From this relationship, the associated energy barriers for each probing location can be tracked and the LRP can be identified.

The proposed testing methodology is the following. A consistent axial load, of about half the magnitude of the expected capacity, is applied to the cylindrical shell. After being axially loaded, the cylinders are probed at several locations that are equally spaced along the circumference. For this study, they chose to use eight probing locations spaced 45° away from each other. Figure 36 is a schematic of the probe locations and experimental setup used in this study. The data collected from each probing location is plotted on a probe force versus probe displacement graph. Figure 37 is the data from test Specimen D1. The LRP is associated with the probing location at which the energy barrier to buckling is the smallest. Figure 38 shows that on Specimen 1D probe location L7 has the least area under the curve, therefore has the smallest energy barrier. After identifying the LRP, a ridge tracking analysis of the stability landscape at the accompanying probing location will determine the predicted capacity. This experimental process was repeated on seven specimens, the results concluded that this method had an average percent difference of 10.4%.

The main conclusion of this study is that the method of identifying the LRP is a somewhat accurate way to predict the axial capacity of an imperfect thin cylindrical shell. However, the accuracy of this method does increase with the addition of more probing locations. The authors also note that the structure is very sensitive to external perturbations and the methods presented should be considered with the addition of statistical analysis.

Figure 38: Graphical Representation of the Energy Barrier Associated with L1, L5, and L7 Probing Locations. Retrieved from “Non-destructive prediction of buckling load of axially compressed cylindrical shells using Least Resistance Path to Probing” by Ankalhope and Jose, 2021, https://doi.org/10.1016/j.tws.2021.108497
3.2.3. Literature Review

The findings presented by Yadav et al. (Yadav et al. 2021) and Ankalhope and Jose (Ankalhope and Jose 2021) are very important additions to the understanding of thin cylindrical shells. Both studies acknowledge that the applied axial capacity for non-destructive testing methods needs to be limited to avoid instability of the structure. This is important to keep in mind when completing further studies.

The work of Yadav et al. confirms the accuracy of the non-destructive prediction methods if the location and geometry of the imperfection are known. This leads to the hypothesis that non-destructive prediction techniques can be used to locate an imperfection. Yadav et al. initiated the investigation of this proposition by probing away from the center of the imperfection in the axial and circumferential directions. Trends were observed in the axial force and peak probe force curves for the circumferential probing that indicated signs of being near the imperfection. The axial probing analyses were less telling. In the circumferential direction, the probe was moved from $10^\circ$ to $30^\circ$. This leaves a large gap of the circumference unreported. The same could be said for the probing locations in the axial direction. Understanding the zone of influence of an imperfection would aid the ability to utilize the non-destructive prediction techniques to locate an imperfection. Mapping the zone of influence would require probing locations to be spaced at much smaller increments in both the axial and circumferential directions.

The study completed by Ankalhope and Jose builds onto the non-destructive prediction methods by introducing the LRP (Ankalhope and Jose 2021). This technique to predict the capacity of an unknown imperfect cylinder would also benefit from the identification of the zone of influence. The source of error in the predicted capacity
values presented in this paper is the spacing between probing locations. This could be avoided if the probes were spaced within the boundary of one zone of influence, placing them at the edges of a zone of influence would be the most efficient. Decreasing the spacing between probing locations would guarantee that at least one of the axial force and peak probe force curves would show signs of the imperfection. However, if the boundaries were identified and defining characteristics of the axial force and peak probe force curves were detectable this would dissolve the need to identify the LRP. The LRP could be an alternative way to define the boundaries of the zone of influence.

It would have also been interesting to investigate the accuracy of the method probing in the axial direction. Based on the results from Yadav et al. it seems as though the imperfection influence in the axial direction is harder to identify compared to the results of the circumferential testing (Yadav et al. 2021). Since the LRP method is tested at lower compressions, there may be even less identifiable changes in the energy barrier when the probe is moved axially.

3.3. Finite Element Modeling Methods

Thin-walled cylindrical shell models were created and analyzed using ABAQUS/Standard. The following sections will describe the material properties, dimensions, methods of construction, loading, and boundary conditions applied to the models.

3.3.1. Material Properties and Cylinder Dimensions

The cylinders modeled for analysis in this chapter are scale models of mini aluminum Coke cans (7.5 fl oz.). The material properties applied to the cylinder models
represent those of aluminum. This includes a Young’s Modulus of 68.95 GPa and a Poisson’s Ratio of 0.3. The same material properties were used for all models.

The geometry of the cylindrical model does not include a top or bottom surface. The model only represents the curved face of a Coke can. The dimensions of the cylinder are consistent for all models and feature a radius, $R$, of 28.6 mm, length, $L$, of 104.1 mm, and a wall thickness, $t$, of 0.10 mm. For simplicity, the cylinders were modeled as having perfect circular cross-sections throughout the full length of the model. This characteristic of the geometry is not an accurate representation of an actual Coke can, as it would most likely have imperfections. Figure 39 is a schematic of the cylinder without any imperfections included. The simulated cylinder is composed of 22,540 nodes, with 196 nodes along the circumference and 115 nodes axially. The base geometry of the cylinder is uniform for all analyses.

![Figure 39: Schematic Geometry and Variable Conventions of the Thin Cylindrical Shell](image)
3.3.2. Modeling of the Thin-Walled Cylinder

The modeling methods for this study are similar to those described in the works of Haynie et al. (Haynie et al. 2012) and Yadav et al. (Yadav et al. 2021). The cylinder was modeled as a 3D deformable shell. The mesh was created using the ABAQUS element type S4R with an approximate size of 0.91 mm, equaling about $0.54\sqrt{Rt}$, in the axial and radial direction. The section is defined as a homogeneous shell type with a thickness, $t$, of 0.10 mm.

Figure 40 shows the orientation of the cylinder simulated for analysis. At both ends of the cylinder, the edge nodes are constrained to a central node within the same plane by rigid links. This allows for the consistent application of boundary and loading conditions. One end of the cylinder is fully fixed, while the opposite end only has translational freedom in the Z-direction. This is to allow for displacement when compression is applied to the cylinder.

![Figure 40: Orientation of Cylinder in ABAQUS Simulations](image-url)
To complete the non-destructive prediction analyses a concentrated force in the negative z-direction is applied to the central node located on the non-fully fixed end of the cylinder. This produces uniformly distributed axial compression along the edge of the cylinder. The magnitude of the load is defined in the user-written code and does not exceed the axial buckling capacity. In addition to the applied axial compression, the analysis also requires a probing force. A probe is simulated by assigning an applied displacement in the radial direction. The location of the probe is moved incrementally away from the dimple for each analysis. The magnitude of the applied probe displacement is restricted by five times the thickness of the shell. This value is determined by experimentation, peak probe force is unlikely to appear past the magnitude of 5t. Additionally, applied displacements larger than 5t may induce plastic deformations (Yadav et al. 2021).

### 3.3.3. Modeling of the Dimple Imperfection

The cylinder analyzed in this chapter features a local dimple imperfection located in the middle of the cylinder. The probing results from the imperfect cylinder will be compared to the results of a perfect cylinder later in this chapter. To introduce a dimple imperfection to the shell of the cylinder, it is modeled as a two-dimensional normal distribution function described in Gerasimidis et al. (Gerasimidis et al. 2018), Gerasimidis and Hutchinson (Gerasimidis and Hutchinson 2020), and Yadav and Gerasimidis (Yadav and Gerasimidis 2019). This method of modeling the imperfection is also utilized in the computational analyses presented in Yadav et al. (Yadav et al. 2021). The following formula describes the dimple imperfection:
\[ w = -\delta e^{-\left(x-x_0/L_1\right)^2} e^{-\left(\theta-\theta_0/\theta_1\right)^2} \]  
(Eq. 3)

where \( w \) represents the change in position in the radial direction, \( \delta \) is the amplitude of the imperfection, \( x \) and \( \theta \) are the axial and circumferential coordinates, and \( L_1 \) and \( \theta_1 \) are the parameters that dictate the length in the axial direction and the width in the circumferential direction of the dimple. The variables \( x_0 \) and \( \theta_0 \) represent the coordinates of the center of the dimple imperfection, for this study the dimple will remain located on the axial center plane for the entirety of the analyses. The \( L_1 \) and \( \theta_1 \) parameters were defined as 0.55 \( \lambda \) and 0.55 \( \lambda/R \), respectively, this is consistent with the works of Yadav et al. (Yadav et al. 2021). The variable \( \lambda \) is the half-wavelength of classical axisymmetric buckling mode and is defined by Equation 4.

\[ \lambda = \pi \sqrt{\frac{Rt}{\sqrt{12(1-v^2)}}} \]  
(Eq. 4)

For this study, all variables remain constant in Equations 3 and 4 to keep the dimple imperfection consistent. The thin cylindrical shell featured in this thesis contains an imperfection amplitude, \( \delta \), of 1t. Based on the geometry of the perfect cylindrical shell, the value of \( \lambda \) is 2.92 mm. Figure 41 shows the location of the dimple imperfection, as well as the applied loads that will be simulated in the computational analysis. The probe force illustrated in Figure 41 indicates it is being applied at the center of the dimple, the analyses completed in this chapter will move the probe incrementally away from the center of the imperfection.
3.4. Finite Element Modeling Results

The finite element analyses that were completed during this study simulate the non-destructive prediction methodology. The initial step of the analysis applies the axial load onto the cylinder and performs a static analysis with the inclusion of non-linear geometry. The second step initiates the probing force on the previously axially loaded model during a Riks analysis. This procedure is repeated for multiple axial loads per probing location.

The goal of this investigation is to map the zone of influence of a 1\textit{t} amplitude dimple imperfection on a thin cylindrical shell based on the behavior observed at different probing locations. The location of the probe is moved incrementally by a single
node. The nodes on the surface of the cylinder are spaced approximately 0.91 mm away from one another in both the axial and circumferential directions. The data from each probing location will be compared to the extreme cases: probing at the center of the imperfection and probing at the center of a perfect cylinder.

In Figure 42 the bolded blue line represents the analysis of a perfect cylinder being probed on the circumference at the center axial plane. The bolded red line shows the axial load and peak probe force curve when the probe is located at the center of the dimple imperfection. Focusing on the comparison of these two lines, there is a clear difference in slope. Both analyses result in an almost linear trend line, however, the perfect cylinder has a much steeper slope compared to the imperfect cylinder. Based on these curves the y-intercept for the perfect cylinder is significantly greater than the latter. Therefore, the capacity predicted for the perfect cylinder would be much greater. Using Equation 5 the capacity of the perfect cylinder is calculated to be 2622 N (Weingarten 1968).

\[ P_c = \frac{2\pi El_t^2}{\sqrt{3(1-v^2)}} \]  

(Eq. 5)

The capacity of the imperfect thin cylindrical shell was computed to be approximately 1362 N by means of simulation under axial compression. This reduction in capacity is as expected based on the previous knowledge that with the introduction of imperfections the capacity has a significant decrease in magnitude.
**Figure 42: Axial Compression Load Versus Peak Probe Force Curves for Perfect and Imperfect Thin Cylindrical Shells**

For consistency Figures 43, 45, and 47-52 are presented with the same color scheme. The bolded blue and red lines are the same as presented in Figure 42. As the probe moves away from the center of the imperfection while staying within the boundary of the zone of influence the line color transitions from red to yellow. The bolded black line is applied to the axial load and peak probe force curve of the location at which the boundary of the zone of influence is identified. Past this boundary, the colors of the curve transition from green to blue as the probe moves further away.
3.4.1. Probe Moving Circumferentially

The first set of analyses was completed to study the buckling behavior of the cylinder as the probe moved around the circumference, away from the center of the imperfection. The central angle relative to the center of the imperfection is used to describe the location of the probe. For the first analysis, the probe was located one node to the right of the center of the dimple imperfection, this is equivalent to a central angle of 1.84°. Sequentially, the probe is moved incrementally by one node in the counterclockwise direction, thus increasing the central angle by 1.84° each time. Based on the work of Yadav et al. the behavior of the cylinder past a central angle of 30° does not indicate decreased buckling capacity (Yadav et al. 2021). This study will probe at locations up to a central angle of 30° to get a better understanding of the zone of influence.

At each probing location, the cylinder was analyzed at 12 different magnitudes of axial load ranging from 790 N to 1290 N. This ensures accurate axial force and peak probe force curves and prevents the initiation of buckling by not overloading the cylinder. The objective of these analyses is not to obtain the capacity of the cylinder but to compare the behavior at different probing locations, therefore it is not essential to test the cylinders at higher compression loads.
Figure 43: Axial Compression Load Versus Peak Probe Force at Probing Locations Moving Incrementally Away from the Imperfection in the Counter-Clockwise Circumferential Direction. Note: the amplitude of the dimple is scaled for visibility.

Figure 43 presents the two-dimensional representation of the stability landscape for each probing location. The extreme cases discussed previously are included to aid the distinction of the zone of influence. Moving away from the imperfection, the axial force and peak probe force lines increase in slope magnitude in relation to the imperfect curve. Probing at 1.84° and 3.68°, there appears to be a slight increasing convex curve in the lines. At 5.52° the results return to a linear trend, with a larger slope than imperfect results. Past 5.52° the axial force and peak probe force lines begin to showcase a concave curvature. The concavity intensifies until 11.04°. At 12.88°, the axial force and peak probe force curve instantly reverts back to featuring a slight convexity that slowly
reduces to a linear line as the probe moves further away from the imperfection. This instantaneous change in concavity is unlike the gradual changes observed at probing locations less than 9.20° and above 12.88°. Due to this behavior, the boundary of the zone of influence is defined at the 11.04° probing location and is shown in the bolded black line in Figure 43. The illustration included in the top right-hand corner of Figure 43 shows the cross-section of the cylinder at center height and the total assumed circumferential zone of influence, shown in green.

In addition to the trend line behavior, another indication of the 11.04° probing location being the boundary of the zone of influence is the y-intercepts. The axial force and peak probe force curves for probing at the central angles of 0° to 7.36° have similar end behavior that indicates y-intercept values close to 1350 N. This behavior is in agreement with the imperfect cylindrical results. It is expected to see a decrease in accuracy as the probe moves away from the imperfection. This is observed at 9.20° and 11.04°, the curves indicate slightly higher y-intercept values. Probing locations further than these show more similar behavior to the perfect cylinder rather than the imperfect. The axial force and peak probe force curves reported for central angles greater than 12.88° are almost parallel to each other and the perfect cylinder line, indicating higher capacities than are true to the imperfect cylinder and even the perfect cylinder. These observations support the conclusion that the boundary of the zone of influence in the form of a central angle is approximately 11.04°. This is equivalent to 6 nodes away from the imperfection in the circumferential direction.
Figure 44 is a contour plot of the surface of the cylinder with an imperfection of amplitude $1t$ in relation to a perfect cylinder. The center of the imperfection is located at the center of the plot with coordinates $(0,0)$. To get a better understanding of the size of the imperfection, the figure shows that the affected area of the dimple is approximately 5.6 nodes circumferentially and axially. This is equivalent to a circular area with a diameter less than 5.46 mm. Figure 44 also illustrates the first defined boundary of the zone of influence in relation to the amplitude and size of the dimple. The red ‘x’ marker...
on the figure represents the circumferential boundary of the zone of influence determined by the observations drawn from Figure 43. Based on the symmetry of the dimple imperfection modeled in these analyses, it is assumed that the zone of influence would also exhibit a symmetrical landscape. The white ‘x’ marker included in Figure 44 represents the second circumferential boundary, which was deducted by reflecting the red ‘x’ marker across the axial centerline of the dimple. This indicates that the total assumed circumferential zone of influence has a central angle of 22.08°.

3.4.2. Probe Moving Axially

A similar method of analysis was completed to study the axial boundaries of the zone of influence. For the following analyses, the probe is incrementally moved in the positive axial direction by one node. An increment of one node is equivalent to 0.31λ. The probe for the first analysis was located one node above the center of the imperfection. The previous study completed by Yadav et al. indicates that the axial force and peak probe force curves are relatively unchanged at probing locations equal to and greater than 4λ, see Figure 35. This study will probe at locations up to twelve nodes away from the center of the imperfection, in the positive axial direction to get a better understanding of the zone of influence. The range of probing locations is equivalent to the value of 3.72λ. The same loading conditions described in Section 3.4.1 were applied to simulations completed to investigate the axial boundaries of the zone of influence.
Figure 45 presents the axial force and peak probe force curves for each probing location. Again, the extreme cases discussed previously are included to aid the distinction of the zone of influence. Similar to the circumferential probing analysis results, the first two probing location curves (1 Node and 2 Nodes) are comparable to the imperfect curve but feature a slight convex curvature and a larger slope magnitude. The results of probing at three nodes above the center of the imperfection show that the axial force and peak probe force curve becomes concave. The concave behavior incrementally increases for the probe locations of four and five nodes above the imperfection. At the sixth node, the axial force and peak probe force curve completely changes shape. At low axial loads,
below 1160 N, the line for probing at the sixth node has a slight convex curvature that is almost parallel to the perfect cylinder axial force and peak probe force curve. Above compression loads of 1160 N, the curve bends and becomes linear. The upper-end behavior of the curve points just above the y-intercept of the imperfect axial force and peak probe force curves.

This axial force and peak probe force curve shape just described is accurate for the rest of the nodes analyzed. The compression at which the line changes shape increases as the probe gets further from the imperfection. Additionally, the further the probe is located from the imperfection the curve becomes more similar to the perfect axial force and peak probe force curve. Based on the line shapes shown in Figure 45, it is determined that the axial boundary of the zone of influence is at the fifth node or 1.55𝜆 from the dimple imperfection. The axial force and peak probe force curve for this location is graphed in the bolded black line in Figure 45. The illustration included in Figure 45 shows the axial cross-section of the cylinder and the total assumed axial zone of influence, shown in green, in the top right-hand corner.

Unlike the circumferential probing results, the end behavior of the axial probing curves at high compression loads does not reveal the zone of influence. The axial force and peak probe force curves for nodes one through five exhibits consistent line shapes that point towards the same y-intercept value as the imperfect curve. The data extracted from probing at nodes six through twelve also lead closer to the y-intercept value determined for the imperfect curve, however, each locations’ respective curve features the change of line shape previously discussed. If the testing was limited to 1160 N, the end behaviors of the axial force and peak probe force curves would be more informative.
Decreasing the applied axial force would result in the curves for nodes six to twelve to have almost parallel line shapes to the perfect curve, completely changing the end behavior of the line. While the curves for nodes 1 to 5 will remain pointing towards the same y-intercept. These observations support the conclusion that the boundary of the zone of influence is five nodes axially away from the center of the imperfection. This suggests that at high axial loads it becomes more difficult to define the zone of influence.

Figure 46: Axial Boundaries of Zone of Influence in Relation to the Amplitude of the Imperfection
Figure 46 introduces the axial boundaries of the zone of influence on the contour plot of the surface of the cylinder. The red ‘x’ markers on the figure represent the boundaries of the zone of influence determined by the computational analyses. The white ‘x’ markers included in Figure 46 represent the boundaries assumed based on symmetry. Comparing the circumferential boundary to the axial boundary, the former spans a total of twelve nodes compared to the latter’s ten nodes. This indicates that the cylinder is slightly more affected by the imperfection in the circumferential direction.

3.4.3. Probe Moving Circumferentially and Axially

The purpose of the final set of analyses is to map the full shape of the zone of influence. This was completed by moving the probe circumferentially and axially away from the center of the imperfection. The circumferential and axial boundaries defined in Sections 3.4.1 and 3.4.2 limit the zone of influence by 11.04° and five nodes, respectively. To capture any irregularities that may be present in the cylinder’s zone of influence, the cylinder is probed at all of the nodes within a central angle of 11.04° and six nodes axially. Probing up to six nodes axially rather than five nodes was chosen so that at least one axial force and peak probe force curve would exhibit characteristics of being outside of the boundary of the zone of influence.

The methodology of this analysis is the following. Starting at the center of the imperfection, the probe is moved one node in the counterclockwise circumferential direction. Looking back at Figure 46, this would place the probe at coordinates (1.84,0). An analysis is completed at this location, creating the first axial force and peak probe force curve denoted at the 0 nodes line in Figure 47. The probe is then incrementally
moved in the positive axial direction by one node, making the new location of the probe to be (1.84,1). Another ridge tracking analysis is completed before the probe is moved up another node. This is repeated until reaching the sixth node, (1.84,6). After analyzing the probe location at the sixth node, the probe returns to the center circumferential plane of the cylinder and restarts the process at (3.68,0). This procedure is continued until reaching the probe location of (11.04,6), making a total of 36 analyzed probing locations.

In addition to the increase of probing locations, the results presented in this section include a wider range of axial loads. The analyses of the probe moving purely in the axial direction indicates that there may be a maximum limit to the magnitude of the applied load. As discussed previously, the zone of influence was more distinct at compression loads less than 1160 N in the two-dimensional view of the stability landscape. This suggests that there is a certain range of axial loads that create a more distinct indication of the boundaries of the zone of influence. To explore this theory, each probing location is analyzed at 19 axial loads ranging from 340 N to 1290 N. Figures 47 through 52 present the two-dimensional view of the stability landscape for each probing location, organized by the central angle coordinate.
Figure 47: Axial Load Versus Peak Probe Force at Probing Locations Moving Incrementally Away from the Imperfection in the Positive Axial Direction with a Central Angle of 1.84°

Figure 47 shows the axial force and peak probe force curves for the axial nodes zero through six at a central angle of 1.84°. Above 790 N the graphed lines look very similar to the ones pictured in Figure 45. As the probe moves further away from the center of the imperfection, the curve becomes more concave until at the sixth node the curve features two line-shapes. Again, at approximately 1160 N the axial force and peak probe force curve changes from a linear line to a concave quadratic line. Using the same criteria as the previous sections, this graph indicates that Node 5 is the boundary of the zone of influence at a central angle of 1.84°. Otherwise stated as the probing location of (1.84,5). Looking at the line behaviors below 790 N, the definition of the zone of
influence is more difficult to distinguish. At very low axial forces, below 550 N, all probing locations have parallel linear line shapes including the axial force and peak probe force lines for the perfect and imperfect cylinders. This portion of each curve points in the direction of the imperfect cylinder capacity, not the perfect cylinder. In previous analyses, the only probing locations that experience a change of line shape were outside of the zone of influence. This is not true when taking into account lower axial forces.

Figure 47 shows that at a central angle of 1.84º and axial nodes 3, 4, and 5 the axial force and peak probe force curves change from a concave quadratic line to a decreasing linear line. This transformation occurs between 550 N and 790 N.

Although the distinct changes of the line shape at nodes 3 through 6 may seem to add complexity to the definition of the zone of influence, there is a sign of the boundary. As defined previously, when an axial force and peak probe force curve transitions from a decreasing linear line to a concave line, this signifies the probe location is outside of the zone of influence. Based on Figure 47, the boundary can be defined as the furthest probing location at which the axial force and peak probe force curve that changes from a concave quadratic line to a linear line.

To avoid the possibility that the line-shape may change, for probing at a central angle of 1.84º the range of axial loads should be reduced to 790 N to 1160 N. This range is shown by the dashed purple lines. These values were chosen based on the points at which line shapes change for nodes 3 and 6. The zone of influence can still be determined in this range of compressions based on the line shape. An axial force and peak probe force curve that is parallel to the perfect curve indicated the probing location to be outside of the zone of influence.
Figure 48: Axial Load Versus Peak Probe Force at Probing Locations Moving Incrementally Away from the Imperfection in the Positive Axial Direction with a Central Angle of 3.68°

Similar observations and conclusions can be made for the successive central angles. Figure 48 presents the axial load and peak probe force curves for nodes located along the axial plane located at a central angle of 3.68°. The results are very similar to the results of the 1.84° analyses. Again, the boundary is found at the fifth axial node based on the axial load and peak probe force curve shapes. There are some variations that should be noted. Moving circumferentially away from the center of the imperfection causes the x-intercepts to decrease in magnitude and move away from the x-intercept resulting from the imperfect line. Additionally, the range of axial loads that results in consistent axial
load and peak probe force curve shapes for all probing locations is altered. For the central angle of 3.68°, the range of axial loads should be 870 N to 1200 N.

Figure 49: Axial Load Versus Peak Probe Force at Probing Locations Moving Incrementally Away from the Imperfection in the Positive Axial Direction with a Central Angle of 5.52°

Figure 49 is the graphical representation of the ridge tracking method for nodes located along the axial plane located at a central angle of 5.52°. The same patterns are present in this graph as well. The boundary of the zone of influence is defined to be at the probing location of (5.52,4). This is the first graph that shows an axial load and peak probe force curve to have three distinct portions of the curve. The curve for node 6 shows an initial change of slope at approximately 1250 N and another at 780 N. This suggests
an even more complex stability landscape as the probe moves further away from the zone of influence. The range of axial loads that keep the axial load and peak probe force curves steady in shape is approximately 830 N to 1160 N.

![Figure 50: Axial Load Versus Peak Probe Force at Probing Locations Moving Incrementally Away from the Imperfection in the Positive Axial Direction with a Central Angle of 7.36°](image)

Figure 50 presents the axial load and peak probe force curves for probing locations located along the axial plane located at a central angle of 7.36°. The boundary of the zone of influence is defined to be at the probing location of (7.36,4). Unlike the previous central angles, the axial load range at which the line shapes remain constant is
bounded by the behavior of the probing locations outside of the zone of influence for both the maximum and minimum. The axial load range for this central angle is 780 N to 1250 N. The axial load and probing force curves for probing at locations (7.36,6) and (5.52,6) are very similar in shape, they display changes of slope at the same magnitudes of load.

Figure 51: Axial Load Versus Peak Probe Force at Probing Locations Moving Incrementally Away from the Imperfection in the Positive Axial Direction with a Central Angle of 9.20°
The axial load and probe force curves for a central angle of 9.20° have the same general shapes recorded in the smaller central angle analyses. The curves are presented in Figure 51. The axial boundary is assumed to be at three nodes above the central circumference of the cylinder. This is equivalent to a probing location of (9.20, 3). Similar to the central angle of 7.36°, the axial load range for consistent line shape is constrained by the axial load and probe force curve that features three slopes. This is seen in the axial load and probe force curve for the fourth node. Probing further away from the center of the imperfection than the fourth node does not have the same line shape characteristic of three slopes. The fifth and sixth nodes indicate no sign of the imperfection and display results similar to probing a perfect cylinder. This suggests that the curves that feature three slopes are right outside of the zone of influence. They are not considered within the zone because the line shape is parallel to the perfect cylinder’s curve at loads between 780 N to 1250 N.

The final central angle analyzed was 11.04°. From the circumferential probing study, the probe location of (11.04,0) was already determined to be a boundary of the zone of influence. The purpose of moving the probe axially from this location is to verify this boundary. Figure 52 presents the results of these computations. The results collected from these probe locations produced very interesting results. Unlike the previous graphs presented for smaller central angles, the axial load and probe force curves did not gradually transition from the imperfect line shape to the boundary of the zone of influence line shape. The line corresponding to the probe location of (11.04,0) does represent a boundary of the zone of influence. However, so does the probe location of (11.04,2). Since it is the furthest probing location from the center of the imperfection and
exhibits a concave quadratic that transitions to a linear decreasing line. Based on this observation and the geometry of the axial load and peak probe force curve this also classifies the probe location (11.04,3) as a boundary of the zone of influence. Another difference that should be noted is that there is no clear upper boundary for axial force. The concavity present in the curves for nodes 0, 1, and 2 appears at high axial forces, greater than 1200 N. This indicates that it may be very hard to experimentally observe this line behavior at these probing locations.

Figure 52: Axial Load Versus Peak Probe Force at Probing Locations Moving Incrementally Away from the Imperfection in the Positive Axial Direction with a Central Angle of 11.04º
3.4.4. Definition of the Zone of Influence

Currently, there is no mathematical definition of the zone of influence. The boundaries of the zone of influence presented in this chapter are deducted based on the geometric behavior of axial force and peak probe force curves. Figure 53 plots the probe locations of the bolded black curves from Figures 47 to 52 in relation to the geometry of the dimple imperfection. The red markers represent the analyses completed in this study and the white are the boundaries assumed by symmetry. This presents the zone of influence in the shape of an oval, spanning 12 nodes wide and 10 nodes tall. This is equivalent to $22.08^\circ$ circumferentially and $3.1\lambda$.

The probe locations marked in this figure can be found by examining the axial force and peak probe force curves at axial loads between 790 N and 1160 N. The range of axial loads corresponds to 60% and 88% of the computed capacity of the imperfect cylindrical shell. Reducing the applied axial force not only makes the curves more uniform it will also decrease the chances of spontaneous buckling in an experimental setting. Based on the geometry of the cylinder investigated in this thesis and the defined zone of influence shown in Figure 53, it would require 17 probe locations circumferentially and 12 axially to map the entire cylinder most efficiently. This is a total of 204 probing locations and analyses.
There is axial force and peak probe force curve behavior that is only exhibited at high compression loads. When the cylinder is studied under higher applied axial loads close in proximity to the shell’s capacity, the zone of influence can be expanded. The curves that feature three linear sections can be interpreted as being part of the zone of influence. The right end behavior of these curves agrees with the curve that represents probing at the center of the imperfect cylinder. Figure 54 includes these probing locations in magenta. Based on the initial analyses and assumptions of the zone of influence, the boundaries presented are limited by the area of probing done in this study. Completing
additional analyses with high axial forces at further probing locations is recommended. From the data collected, the alternative zone of influence spans 12 nodes wide and 12 nodes tall. This is equivalent to 22.08° circumferentially and 3.72λ. Based on this alternative zone of influence, the boundaries would reduce the total amount of required probing locations to 170. This defined expanded zone of influence is only applicable to the simulated analysis. Applying axial loads close to the capacity of the shell can cause instantaneous buckling in the experimental setting. This would cause permanent deformations in the testing specimens and should be avoided for any prediction method.

Figure 54: Map of the Boundaries of the Zone of Influence of a Dimple Imperfection with an Amplitude of 1t at High Axial Forces
3.5. Conclusion

There is still not a clear mathematical definition of the zone of influence. However, there are trends in the axial force and peak probe force curves defined in this chapter that indicate if the probing location is within the zone of influence of the imperfection. The zone of influence defined in this study for an imperfection with an amplitude of $1t$ is slightly more than double the size of the imperfection itself in both the axial and circumferential directions.

For the geometry of the cylinder being analyzed in this study, the recommended applied axial forces should be within the range of 790 N to 1160 N. This will produce consistent line shapes and avoid instability in the structure. The generalized defining characteristic of a probing location being within the zone of influence is if the associated axial force and peak probe force curve has a $y$-intercept much less than the magnitude of the perfect cylindrical shell’s capacity. If the probing location is outside of the zone of influence the axial force and peak probe force curve will demonstrate parallel behavior to the perfect cylindrical shell. The axial force and peak probe force curve gradually change in line shape as the probing location moves further away from the imperfection. This is an indication that the zone of influence is not random and most likely directly relates to the size and geometry of the imperfection.

Applying more axial force enlarges the zone of influence, making it easier to detect imperfections further away from its center. This can be done computationally, as there are no consequences for overloading the cylinder. Although this is not true experimentally, the cylindrical shell may experience instantaneous buckling due to the probing at high compression. This is essential to avoid in the experimental setting. When
applying high compression loads, another concern is the effect of multiple probing analyses on the overall stability of the cylinder. Applying high compression loads increases the chance of damaging a specimen, thus causing more loss in capacity. Therefore, any experimental or computational analyses should limit the applied axial load.

Overall, it is proven that the zone of influence can be determined through the two-dimensional view of the stability landscape. The analysis methodology presented in this chapter can be applied to any imperfection with any amplitude. Knowing the relationship between amplitude and zone of influence will aid in the task of locating an imperfection. It does so by providing the most efficient circumferential and axial spacing distances for probing locations that will minimize the total required probing analyses to locate the imperfection. If this information was combined with the LRP predictive methods, the accuracy and consistency of the technique would improve greatly. Incorporating the zone of influence into any non-destructive prediction testing methods is the best way to map the stability of a thin cylindrical shell most efficiently.

3.6. Recommendations for Future Work

Based on the results of this research, the following work would be the most beneficial to the development of a method to locate an imperfection of a thin cylindrical shell.

- Expand the area of investigation and probe at further locations with high axial compression. Taking into consideration the axial force and peak probe force curve
trends reported in this chapter this will provide a full map of the alternative zone of influence.

- Repeat the analyses conducted in this thesis at multiple different imperfection amplitudes to find the relationship between the boundaries of the zone of influence and amplitude of imperfection. Understanding this relationship will help determine the spacing between probe locations when trying to find an imperfection.

- Define zone of influence using the LRP methods by moving the probe in smaller increments, radially and circumferentially. The energy barrier should be consistent at far probing locations from the center of the imperfection, so in theory the smaller the energy barrier the closer in proximity the probe is to the imperfection.

- Investigate why probing past the boundary of the zone of influence results in axial force and peak probe force curves that point above the perfect cylinder’s capacity. This could potentially be an indication of the presence of an imperfection.

- Conduct an experimental study to find the maximum axial capacity that can be applied to a cylinder while undergoing a non-destructive prediction method analysis.

- Examine the effect that multiple probing analyses can have on the structural stability of a thin cylindrical shell through an experimental study.
APPENDIX A

Figure 55: Design 2 Deformed Shapes (top right) 12 floors, (top bottom) 24 floors, (bottom right) 36 floors, (bottom left) 48 floors.

Figure 56: Design 3 Deformed Shapes (top right) 12 floors, (top bottom) 24 floors, (bottom right) 36 floors, (bottom left) 48 floors.
Figure 57: Design 4 Deformed Shapes (top right) 12 floors, (top bottom) 24 floors, (bottom right) 36 floors, (bottom left) 48 floors.

Figure 58: 24-Floor Structure with Design 1 Lattice, Mode Shape 1
Figure 59: 24-Floor Structure with Design 1 Lattice, Mode Shape 2

Figure 60: 24-Floor Structure with Design 1 Lattice, Mode Shape 3

Figure 61: 24-Floor Structure with Design 1 Lattice, Mode Shape 4
Figure 62: 24-Floor Structure with Design 1 Lattice, Mode Shape 5

Figure 63: 36-Floor Structure with Design 1 Lattice, Mode Shape 1

Figure 64: 36-Floor Structure with Design 1 Lattice, Mode Shape 2
Figure 65: 36-Floor Structure with Design 1 Lattice, Mode Shape 3

Figure 66: 36-Floor Structure with Design 1 Lattice, Mode Shape 4

Figure 67: 36-Floor Structure with Design 1 Lattice, Mode Shape 5
Figure 68: 48-Floor Structure with Design 1 Lattice, Mode Shape 1

Figure 69: 48-Floor Structure with Design 1 Lattice, Mode Shape 2

Figure 70: 48-Floor Structure with Design 1 Lattice, Mode Shape 3
Figure 71: 48-Floor Structure with Design 1 Lattice, Mode Shape 5

Figure 72: 48-Floor Structure with Design 1 Lattice, Mode Shape 5

Figure 73: 12-Floor Structure with Design 2 Lattice, Mode Shape 1
Figure 74: 12-Floor Structure with Design 2 Lattice, Mode Shape 2

Figure 75: 12-Floor Structure with Design 2 Lattice, Mode Shape 3

Figure 76: 12-Floor Structure with Design 2 Lattice, Mode Shape 4
Figure 77: 12-Floor Structure with Design 2 Lattice, Mode Shape 5

Figure 78: 24-Floor Structure with Design 2 Lattice, Mode Shape 1

Figure 79: 24-Floor Structure with Design 2 Lattice, Mode Shape 2
Figure 80: 24-Floor Structure with Design 2 Lattice, Mode Shape 3

Figure 81: 24-Floor Structure with Design 2 Lattice, Mode Shape 4

Figure 82: 24-Floor Structure with Design 2 Lattice, Mode Shape 5
Figure 83: 36-Floor Structure with Design 2 Lattice, Mode Shape 1

Figure 84: 36-Floor Structure with Design 2 Lattice, Mode Shape 2

Figure 85: 36-Floor Structure with Design 2 Lattice, Mode Shape 3
Figure 86: 36-Floor Structure with Design 2 Lattice, Mode Shape 4

Figure 87: 36-Floor Structure with Design 2 Lattice, Mode Shape 5

Figure 88: 48-Floor Structure with Design 2 Lattice, Mode Shape 1
Figure 89: 48-Floor Structure with Design 2 Lattice, Mode Shape 2

Figure 90: 48-Floor Structure with Design 2 Lattice, Mode Shape 3

Figure 91: 48-Floor Structure with Design 2 Lattice, Mode Shape 4
Figure 92: 48-Floor Structure with Design 2 Lattice, Mode Shape 5

Figure 93: 12-Floor Structure with Design 3 Lattice, Mode Shape 1

Figure 94: 12-Floor Structure with Design 3 Lattice, Mode Shape 2
Figure 95: 12-Floor Structure with Design 3 Lattice, Mode Shape 3

Figure 96: 12-Floor Structure with Design 3 Lattice, Mode Shape 4

Figure 97: 12-Floor Structure with Design 3 Lattice, Mode Shape 5
Figure 98: 24-Floor Structure with Design 3 Lattice, Mode Shape 1

Figure 99: 24-Floor Structure with Design 3 Lattice, Mode Shape 2

Figure 100: 24-Floor Structure with Design 3 Lattice, Mode Shape 3
Figure 101: 24-Floor Structure with Design 3 Lattice, Mode Shape 4

Figure 102: 24-Floor Structure with Design 3 Lattice, Mode Shape 5

Figure 103: 36-Floor Structure with Design 3 Lattice, Mode Shape 1
Figure 104: 36-Floor Structure with Design 3 Lattice, Mode Shape 2

Figure 105: 36-Floor Structure with Design 3 Lattice, Mode Shape 3

Figure 106: 36-Floor Structure with Design 3 Lattice, Mode Shape 4
Figure 107: 36-Floor Structure with Design 3 Lattice, Mode Shape 5

Figure 108: 48-Floor Structure with Design 3 Lattice, Mode Shape 1

Figure 109: 48-Floor Structure with Design 3 Lattice, Mode Shape 2
Figure 110: 48-Floor Structure with Design 3 Lattice, Mode Shape 3

Figure 111: 48-Floor Structure with Design 3 Lattice, Mode Shape 4

Figure 112: 48-Floor Structure with Design 3 Lattice, Mode Shape 5
Figure 113: 12-Floor Structure with Design 4 Lattice, Mode Shape 1

Figure 114: 12-Floor Structure with Design 4 Lattice, Mode Shape 2

Figure 115: 12-Floor Structure with Design 4 Lattice, Mode Shape 3
Figure 116: 12-Floor Structure with Design 4 Lattice, Mode Shape 4

Figure 117: 12-Floor Structure with Design 4 Lattice, Mode Shape 5

Figure 118: 24-Floor Structure with Design 4 Lattice, Mode Shape 1
**Figure 119:** 24-Floor Structure with Design 4 Lattice, Mode Shape 2

**Figure 120:** 24-Floor Structure with Design 4 Lattice, Mode Shape 3

**Figure 121:** 24-Floor Structure with Design 4 Lattice, Mode Shape 4
Figure 122: 24-Floor Structure with Design 4 Lattice, Mode Shape 5

Figure 123: 36-Floor Structure with Design 4 Lattice, Mode Shape 1

Figure 124: 36-Floor Structure with Design 4 Lattice, Mode Shape 2
Figure 125: 36-Floor Structure with Design 4 Lattice, Mode Shape 3

Figure 126: 36-Floor Structure with Design 4 Lattice, Mode Shape 4

Figure 127: 36-Floor Structure with Design 4 Lattice, Mode Shape 5
Figure 128: 48-Floor Structure with Design 4 Lattice, Mode Shape 1

Figure 129: 48-Floor Structure with Design 4 Lattice, Mode Shape 2

Figure 130: 48-Floor Structure with Design 4 Lattice, Mode Shape 3
Figure 131: 48-Floor Structure with Design 4 Lattice, Mode Shape 4

Figure 132: 48-Floor Structure with Design 4 Lattice, Mode Shape 5
Figure 133: First Eigenmode Shapes of 3-Meter Plates with Compression Loading (top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 134: First Eigenmode Shapes of 4-Meter Plates with Compression Loading
(top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 135: First Eigenmode Shapes of 5-Meter Plates with Compression Loading (top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 136: First Eigenmode Shapes of 6-Meter Plates with Compression Loading
(top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 137: First Eigenmode Shapes of 7-Meter Plates with Compression Loading (top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 138: First Eigenmode Shapes of 8-Meter Plates with Compression Loading (top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 139: First Eigenmode Shapes of 9-Meter Plates with Compression Loading (top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 140: First Eigenmode Shapes of 10-Meter Plates with Compression Loading
(top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 141: First Eigenmode Shapes of 3-Meter Plates with Shear Loading (top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 142: First Eigenmode Shapes of 4-Meter Plates with Shear Loading (top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 143: First Eigenmode Shapes of 5-Meter Plates with Shear Loading (top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 144: First Eigenmode Shapes of 6-Meter Plates with Shear Loading (top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 145: First Eigenmode Shapes of 7-Meter Plates with Shear Loading (top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 146: First Eigenmode Shapes of 8-Meter Plates with Shear Loading (top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 147: First Eigenmode Shapes of 9-Meter Plates with Shear Loading (top) Design 1, (middle) Design 4, (Bottom) Solid Plate
Figure 148: First Eigenmode Shapes of 10-Meter Plates with Shear Loading (top) Design 1, (middle) Design 4, (Bottom) Solid Plate


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