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Resource Allocation and Pricing in Virtual Wireless Networks

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RESOURCE ALLOCATION AND PRICING IN VIRTUAL WIRELESS NETWORKS

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ABSTRACT

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The Internet architecture has proven its success by completely changing people's lives. However, making significant architecture improvements has become extremely difficult since it requires competing Internet Service Providers to jointly agree. Recently, network virtualization has attracted the attention of many researchers as a solution to this ossification problem. A network virtualization environment allows multiple network architectures to coexist on a shared physical resource. However, most previous research has focused on network virtualization in a wired network environment. It is well known that wireless networks have become one of the main access technologies. Due to the probabilistic nature of the wireless environment, virtualization becomes more challenging. This thesis consider virtualization in wireless networks with a focus on the challenges due to randomness. First, I apply mathematical tools from stochastic geometry on the random system model, with transport capacity as the network performance metric. Then I design an algorithm which can allow multiple virtual networks working in a distributed fashion to find a solution

such that the aggregate satisfaction of the whole network is maximized. Finally, I proposed a new method of charging new users fairly when they ask to enter the system. I measure the cost of the system when a new user with a virtual network request wants to share the resource and demonstrate a simple method for estimating this "price".

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CHAPTER 1

INTRODUCTION

1.1 Motivation

The Internet has clearly been a widely-used successful network architecture over the past three decades. However, due to conflicting goals and policies of multiple stakeholders, the improvement of the existing Internet has been limited to simple incremental updates, and adopting a new architecture is extremely difficult [20], [16]. Recently, the concept of network virtualization has been proposed as a solution. A network virtualization environment allows multiple heterogeneous network architectures to coexist on a shared physical substrate, and each virtual network (VN) in the system is a subset of the substrate network resource. Network virtualization is believed to provide flexibility, diversity and increased manageability, as surveyed in [4], [3]. Since multiple virtual networks share a physical resource, finding an efficient allocation of resources, which is termed the "virtual network embedding problem", is extremely important. A survey [6] reviews existing research on the virtual network embedding problem.

Wireless networks have become more and more popular since, compare to wired alternatives, they have many advantages such as mobility and low cost. From cellphone systems to large sensor networks, wireless networks play an important role in information communication. Hence, applying virtualization in a wireless network environment should be an important part of network virtualization research. However, the probabilistic nature is a main characteristic of the wireless environment, and thus it is necessary to focus on randomness when considering the virtual wireless network problem.

Due to the interference and fading of the wireless channel and node mobility, there are different challenges than in a wired network. Because the wireless channel has the broadcast property, the communication between two nodes also affects the transmission of other nodes, while the mobility of nodes results in locations of nodes that are random, hence resulting in a very difficult problem when we consider resource allocation. Another significant difference between wired and wireless networks is the physical link. For a wired network, links are often of nearly constant quality. However, for the wireless network, each link experiences random failing. This randomness of both the interference environment and signal propagation makes the network virtualization problem for wireless networks much different than that for wired environments. It is obvious that following the similar ideas to approaches in wired virtual networks will give limited insight into solutions for the main challenges in wireless scenarios. Indeed limited approaches for dealing with virtualization in the wireless environment have been proposed, but these approaches miss some paramount characteristics of wireless networks such as the mobility of wireless nodes and the need for distributed allocation algorithms [6]. This gap in the study of virtual wireless networks motivates this thesis, which takes a first step to consider the randomness challenges.

In the thesis, virtualization in wireless environments is considered with a focus on their probabilistic nature. First, mathematical tools from stochastic geometry are applied to derive the transport capacity on the random system model, where the locations of nodes viewed from a snapshot form a *Poisson Point Process*. Then a dynamic algorithm for resource allocation is proposed. This algorithm allows multiple virtual networks working in a distributed fashion to find a solution such that the total utility of the system is maximized. Finally, a new pricing scheme which can charge new users fairly when they enter the network is proposed. The cost to the system when a new user with a virtual network request wants to share the resource is measured, and a simple method for estimating this "price" is demonstrated.

1.2 Development

1.2.1 Basic concepts of a Virtual Network

Network virtualization has been propounded as one of the most promising technologies for the future Internet, because it can overcome the stagnation problems of the current Internet and make the deployment of new architectures possible. A virtual network environment allows multiple heterogeneous network architectures to coexist on the same physical substrate, with each virtual network (VN) obtaining a subset of the physical network resources (which is physical nodes and physical links, in most situations). The role of the traditional Internet Service Providers separates into two: infrastructure providers, who manage the physical infrastructure, and service providers, who create virtual networks by renting the resources from multiple infrastructure providers and offering network services according to the requests of users.

Since a substrate resource needs to support several virtual networks, how to efficiently allocate the resources is a very important problem in network virtualization area. A survey of the current research in the virtual network embedding problem [6] provides a good overview of the existing approaches, main challenges and future research directions. In most approaches to wired virtual network embedding, the authors define resource allocation as finding a algorithm for mapping the virtual nodes and links onto physical ones. However, "mapping" is only a reasonable method in wired networks since the topology of a wired network is static and previously known. In the wireless environment, simply mapping the virtual nodes and links onto the physical ones is not possible due to the randomness of the wireless environment. In this thesis, the resource allocation problem is not simply allocating sets of physical nodes and links, but instead allocating interference space for multiple VNs, since the interference is the critical factor in wireless networks. The details about the "resource" analyzed in this thesis will be discussed in Chapter 3.

1.2.2 Operating manner

In this thesis, the approach to virtual networking considers two different aspects with corresponding time scales. "User" in this thesis means the entity who has a certain virtual network request. In other words, each user in the system desires a virtual network supported by the physical resource of the wireless network.

Online. I present an algorithm for allocating resources to the current users, whose virtual networks are supported by the physical resource of the wireless system, in an optimal online manner. That is, each user with unknown virtual network request arrives to the system dynamically and can stay in the network for an arbitrary amount of time [6]. Each user in the system adjusts its own behavior according to the algorithm to maximize his/her utility, but it also maximize the total system satisfaction.

Offline. When discussing the proposed pricing scheme, an offline scenarios is considered where a new user wants to enter the system and share resources. The purpose is to provide a method to calculate how much this user needs to pay for entering the system and obtaining a certain level of network performance.

1.2.3 Challenges in Virtual Wireless Networks

Compare to wired networks, wireless networks obviously have some advantages such as mobility and low cost. However, due to the broadcast property of the wireless channel, the communication between two nodes also affects other nodes' transmission, while the mobility of nodes causes the locations of nodes to be random. This probabilistic nature of the wireless environment makes the virtualization problem more difficult. The main challenges are summarized here:

Mobility of nodes. One of the most important features of the wireless network environment is the mobility of nodes. This is an advantage compare to the wired network, but also brings a big challenge for resource allocation. In a wired network, allocating the physical nodes is to find a subset of the system's nodes and assigning them to support the corresponding virtual nodes. This idea cannot be implemented on wireless networks due to the random location of system nodes. Assigning physical nodes to a certain virtual network is not possible, since, if a node is highly mobile, it can be too far away from the other nodes to build a connection, which means the allocation fails.

Interference and fading channel. The transmission channels in wired and wireless environments are completely different. In a wired network, the transmission media are wires, which are highly reliable and almost have a constant quality compared to wireless transmission media. However, because of the broadcast property of wireless channel, the communication between two nodes also affects the transmission of other nodes, while the process of signal propagation experiences random failing. Due to the interference and fading of the wireless channel, the idea used in wired embedding of mapping virtual links onto physical links is also not applicable.

Physical resource. In order to consider resource allocation, the first problem needed to be solved is measuring the substrate (physical) resource. Most allocation approaches use the CPU capacity of physical nodes and the bandwidth of physical links as the substrate resource. This is reasonable for wired networks, where the whole topology of the system can often be known, which means the total resource capacity is known, and the remaining resources are simply the total capacity minus the resources which have been allocated to virtual networks. However, because of the two challenges discussed before, the resource for wireless networks is dynamic and has randomness. Furthermore, we assume the CPU capacity of nodes is not critical compared to the mobility of nodes.

All these challenges motive research in virtual wireless networks that focus on the defining features of wireless environment–randomness. Simply borrowing the concepts and approaches from the studies of virtual wired network will give limited insight into solutions for these challenges in wireless scenarios. In order to overcome these problem, first the system model should take mobility into account, which means the randomness of nodes' location need to be covered in the model. Then, a new way to measure the resources of the interference-limited wireless environments is necessary.

1.3 Contribution

This thesis makes three main contributions towards the research area of virtualization in wireless networks. In order to give deep insight of the problem, the approaches mainly focus on the challenges caused by the probabilistic nature of the wireless environment.

- *Virtual wireless network with randomness.* I present a system model that considers the mobility of nodes, which is an important characteristic of the wireless environment. In this model, the locations of nodes viewed from a snapshot form a Poisson Point Process (PPP). An big advantage of this model is that stochastic geometry provides tools for conveniently analyzing performance.

- *Algorithm for resource allocation.* I present a decentralized dynamic algorithm which allocates the physical resource to maximize the total network utility, which is defined as the total satisfaction of all users who seek resources in this system. Each user in the system adjusts behavior to maximize his/her utility, which I show also results in the total system utility being maximized. The resource here is not nodes' CPU capacity and links' bandwidth as in wired network; rather, the algorithm allocates interference space, which I claim is an effective method of resource allocation in the interference-limited wireless environment.

- *Pricing scheme.* I propose a novel scheme to price VN requests fairly. Supporting a new virtual network must cause worse performance of the existing virtual networks due to the interference environment. The scheme measures the cost to the network when providing the requested performance to the user who wants to enter the system and share the resources. This cost is actually the total detriment a user causes on others. In addition, an upper bound approximation to this "price" is presented which is significantly easier to implement.

CHAPTER 2

BACKGROUND

2.1 Virtual Network

The layered architecture of the existing Internet is viewed as a great success of the past three decades. However, it faces significant problems for future development. Due to its multi-provider nature, adopting a new architecture is extremely hard, because it requires the joint agreement of competing stakeholders. For this reason, the improvement of the existing Internet architecture is limited to simple incremental updates, and deployment of new network technologies is almost impossible. Many literatures identify this "ossification" problem. (see [20], [16], [4], [3]). In recent years, network virtualization has been proposed as a model for the next-generation network architecture to provide flexibility, diversity, and increased manageability. It is believed in particular, that network virtualization can overcome the ossifying forces of the Internet.

In a virtual network environment, multiple virtual networks can coexist in the same physical substrate network, which means a certain physical node or link can support several virtual nodes or links. Thus, coexistence is the defining characteristic of an network virtualization environment. It allows virtual networks with different service goals, such as high security, or low delay, to share the physical resource of a network system. Although many efforts has been put on network virtualization area, there are still several challenges that remain untouched or require further attention. In survey [4], some key research directions are listed.

One of the main research directions of network virtualization area is resource allocation, which is often referred to as the virtual network embedding problem in the existing literature. Since multiple virtual network share the physical resource in a system, finding an efficient allocation of the resource is extremely important. The surveys [6] and [2] review existing research on virtual network resource allocation or embedding problem. In particular, [2] provides a description of the main approaches of resource discovery and allocation. And in [6], a novel virtual network embedding classification scheme is presented.

2.1.1 Virtual wired networks

Nearly all efforts to this time have focus on network virtualization in the wired network environment (see, e.g. [5]). For a wired network, the physical nodes and links are almost in a static situation, which means the whole topology of the network is often previously known. In the approaches of wired network embedding, they usually model the substrate network and VN requests as a weighted undirected graph. The most commonly used substrate resources parameters are linear parameters, such as the CPU capacity of physical nodes (routers) and the bandwidth of physical links. But some other network parameters are also considered in [19], such as the memory of nodes and the propagation delay of links. The process of resource allocation is finding an efficient way of mapping a certain virtual network onto a subset of physical nodes and links, which is reduced to the $\mathcal{NP}$-hard multi-way separator problem. In [2], the author categorize the researches on resource allocation into: centralized approach, distributed approach, reconfiguration and survivability.

Centralized approach: a single entity (such as resource controller) receives virtual network requests and performs resource allocation. This entity requires complete global knowledge of the network. However, these approaches would be limited by the size of the physical network, since the communication between the central entity and the nodes will cause a certain amount of overhead.

Distributed approach: the responsibility of the resource allocation can be distributed over the physical nodes and links. They could use their local knowledge for making decisions, with a communication and cooperative protocol employed to coordinate the process. But it might be difficult to obtain optimal allocation results, and a strategy to deal with cheating behavior needs to be considered.

Reconfiguration: embedding a new virtual network can often affect the operation of the already embedded virtual network. The solution is to reconfigure the resource allocation. However, this process might require a long service disruption time which can be unacceptable for real-time and critical application. Some dynamic approaches consider the reconfiguration problem.

Survivability: dealing with the possible failure of the allocation especially for virtual networks for critical applications and services. Additional resources are required to guarantee the performance, which often means the survivability of the virtual network comes at the possible cost of reduced use of the physical resources.

2.1.2 Virtual wireless network

Since the wireless network environment has its own special constraints, some network virtualization concepts need to be redefined or modified. Few approaches have been proposed for the virtual wireless network problem. In [14], the author notice the difference between wired and wireless networks, especially in the link aspects caused by the broadcast nature of wireless environment. The basic strategy of virtualization in this approach is to divide a wireless environment into different dimensions in order to allocate the resource without interference. The typical example of dimensions could be frequency, time and so on, which can be exploited through existing multiple access methods such as TDMA, FDMA, CDMA, etc. The authors suggest a framework which allocates resource in frequency and time dimensions. The objective is to minimize the remaining resource of the system. However, the space domain is not considered in this approach because of the interference problem.

An approach to solve the virtual network embedding problem in a TDM-based wireless virtualization environment was proposed in [24]. This approach mainly focuses on the key difference between wired and wireless network inter-link interference. The authors introduce feasibility checking to examine whether an embedding solution is feasible. One way to do such is to use a conflict graph to capture the interference relation between links, which requires the complete knowledge of the wireless network topology, and the other way is to use simulation to examine the feasibility. Also a quality comparison metric for a candidate embedding is proposed based on the idea of minimizing the amount of link-interference in the path.

The approach in [11] introduces an embedding algorithm for the wireless network testbed ORBIT(Open-Access Research Testbed for Next-Generation Wireless Networks) based on FDM (Frequency Division Multiplexing) link virtualization. In [13], the authors introduce virtual network embedding for wireless mesh networks. A algorithm called WELL is proposed in this letter, which is believed to be the first work to deal with the multicast service-oriented virtual network embedding under the condition that wireless links are unreliable.

In survey [6], the authors note that all existing approaches miss some paramount characteristics of the wireless environment such as mobility and node distribution. It is clear that most research in virtual wireless networks still follows similar ideas to approaches in wired virtual networks and thus gives limited insight into solutions for the main challenges in wireless scenarios.

2.2 Stochastic Geometry

For a wireless network, the interference and thus signal quality at a receiver critically depend on the distribution of the interfering transmitters. In order to sufficiently analyze such, mathematical techniques based on stochastic geometry, including point process theory, percolation theory, and probabilistic combinatorics, have been developed in the last decade. As a consequence, stochastic geometry tools have emerged as essential to model and quantify interference, connectivity, coverage, as well as outage probability and throughput in large wireless networks. Several tutorials articles e.g. [9], [22] summarize these techniques, discuss their application to model wireless networks and presents some previous results. In this thesis, related results are used for the system model and network analysis.

2.2.1 Poisson Point Process

One of the main objects studied in stochastic geometry is a point process. A point process is a random collection of points in spaces, such as time or geographical space. More formally, a point process is a measurable mapping Φ from some probability space to the space of point measure (a point measure is a measure which is locally finite and which takes only integer values) on some space E. Each such measure can be represented as a discrete sum of Dirac measures on E:

$$\Phi = \sum\nolimits_i \delta_{X_i}. \tag{2.1}$$

The random variables $\{X_i\}$, which take their values in E are the points of Φ. The intensity measure Λ of Φ is defined as $\mathbb{E}\Phi(B)$ for Borel B, where $\Phi(B)$ denotes the number of points in $\Phi \cap B$. (From [9]).

The simplest and widely used example of a point process is the *Poisson Point Process* (PPP), which is a spatial generalization of the Poisson Process. Mathematically, the PPP is a point process for which Φ is Poisson on E. A formal definition of the Poisson point process given in [9] is:

Let Λ be a locally finite measure on some metric space E. A point processes Φ is Poisson on E if

- For all disjoint subsets $A_1, \cdots, A_n$ of E, the random variables $\Phi(A_i)$ are independent;
- For all sets A of E, the random variables $\Phi(A)$ are Poisson.

A PPP provides a computational framework for different network performance. The PPP used in this thesis is homogeneous and stationary; that is, the density of the points is constant across space, and the law of the point process is invariant by translation. Moreover, [9] also summarizes some useful properties of PPP. I list two of them which are used in this thesis:

- The superposition of two or more independent PPPs is again a PPP; this can be extended to denumerable sums under some conditions.
- The independent thinning of a PPP is again a PPP.

2.2.2 Interference representation

The interference of wireless system is a function of the network geometry. Also the path loss and the fading characteristics are all dependent on the geometry. In [22], the authors introduce a mathematical framework based on stochastic geometry to characterize the network interference in wireless system which are modelled as spatial Poisson process. In this thesis, several results from [22] are used for the analysis.

In most cases of wireless networks analysis, the power relationship between the transmitted signal and that received is due to the propagation characteristics of the environment. Usually, one assumption is the transmitted signal is affected by path loss and fading. So the power P_{rx} received at a distance R from the transmitter is given by:

$$P_{rx} = \frac{P_{tx} Z_k}{R^\alpha}, \tag{2.2}$$

where P_{tx} is the transmitting power, $\{Z_k\}$ are independent random variables, which account for the multipath fading and shadowing. The term $1/R^\alpha$ represents the path loss with distance R, where the power path loss exponent α is environment-dependent and can approximately range from 0.8 (e.g. hallway inside building) to 4 (e.g. dense urban environment). This model is general enough to capture the propagation characteristics of various scenarios from path loss only channel ($Z_k = 1$) to a channel with different kinds of fading (e.g. Rayleigh fading, or Nakagami-m fading).

In section III, C of [22], the aggregate interference power generated by all the nodes in the system are studied. According to the propagation analysis above, each interference node contributes the term P_i/R^α, where P_i represent an arbitrary quantity associated with interferer i, which characterize the propagation effects such as multipath fading or shadowing.

Let $\{R_i\}_{i=1}^{\infty}$ denote the sequence of distances between the reference receiver and a random points of a two-dimensional Poisson Process with density λ. Let $\{P_i\}_{i=1}^{\infty}$ be a sequence of i.i.d. real nonnegative random variables and independent of the sequence $\{R_i\}$. Let I denote the aggregate interference power at the reference receiver by all the nodes in the infinite plane, such that

$$I = \sum_{i=1}^{\infty} \frac{P_i}{R_i^\alpha} \tag{2.3}$$

for $\alpha > 2$. So I is a random variable, whose characteristic function is:

$$\mathit{\Phi}_I(w) = exp\left(-\gamma |w|^{\frac{2}{\alpha}} \left[1 - j\beta sign(w) tan\left(\frac{\pi}{\alpha}\right)\right]\right), \tag{2.4}$$

where

$$\beta = 1 \tag{2.5}$$

$$\gamma = \pi\lambda C_{2/\alpha}^{-1}\mathbb{E}\left\{P_i^{2/\alpha}\right\}, \tag{2.6}$$

and C_α is defined as

$$C_\alpha \equiv \begin{cases} \frac{1-\alpha}{\Gamma(2-\alpha)\cos(\pi\alpha/2)}, & \alpha \neq 1 \\ \frac{2}{\pi}, & \alpha = 1, \end{cases} \tag{2.7}$$

with $\Gamma(\cdot)$ denoting the gamma function.

The random variable I is a special random variable called a stable random variable. The stable distribution is a class of probability distributions that allows skewness and heavy tails and has many intriguing mathematical properties. Paul Lévy characterized this class of distribution in his study of sums of independent identically distributed terms, so it is also called Lévy stable. The book [15] introduce the properties and application of this class of distribution. In this thesis, only a basic property is used in the analysis in section 1.6 of [15], which is that sums of stable random variables produce a stable random variable.

CHAPTER 3

NETWORK ANALYSIS

In this chapter, a system model which capture the randomness of nodes is presented. The mathematical foundation of this model is the stochastic geometry that introduced in the background section.

3.1 System model

As mentioned in the introduction, a proper system model should reflect the randomness of the wireless network. We choose a similar system model as in [21], which can fit highly mobile nodes.

This thesis considers an ad hoc wireless network with a large number of nodes spread over a large area. The transmitters in the network do not coordinate with each other in making transmission decisions. That is, nodes employ Aloha as the medium access control protocol, which means in each slot each node decides whether to transmit or listen independently. This model views the network at a snapshot in time, where the locations of the transmitting nodes at that snapshot are assumed to form a stationary Poisson Point Process (PPP) of density λ on the plane, denoted $\Pi(\lambda) = \{X_i\}$, where $X_i \in \mathbb{R}^2$ is the location of node i. The PPP system model is accurate only with uncoordinated transmitters independently and uniformly distributed over the network area. Some of the results derived from this model have been used for multi-hop problems such as in [1].

The transmitted signal is assumed to be affected by pathloss and frequency-nonselective fading. That is, the instantaneous received power at distance d away from the transmitter is $PHd^{-\alpha}$, where P is the transmitting power, $\alpha > 2$ is the pathloss exponent, and H is the reduction in the received power due to fading.

The success of a transmission is determined by the signal-to-interference ratio lying above a specified threshold β. Here, because wireless networks are interference-limited [8], an assumption that the thermal noise is negligible should be made. In this model, each transmitter which transmits at the same time with the reference transmitter obviously generates interference at the reference receiver. Each transmitter (node) is assumed to have an assigned receiver located at a distance r away. The *outage probability*, denoted by q, is the probability that the signal-to-interference ratio (SIR) at the reference receiver is below a specified threshold β required for successful reception:

$$\begin{aligned} q(\lambda) &= \mathbb{P}\left(\mathrm{SIR} < \beta\right) && (3.1) \\ &= \mathbb{P}\left(\frac{PHr^{-\alpha}}{\sum_{X_i \in \Pi(\lambda)} P_i H_i d_i^{-\alpha}} < \beta\right), && (3.2) \end{aligned}$$

where d_i is the distance between X_i and the reference receiver, and P_i and H_i are the transmitting power and fading coefficient of the i^{th} interfering transmitter, respectively.

The performance metric used here is the *transport capacity* (TC) which is defined as the total bit-meters per second a network can reliably support [8]. Numerous prior works have considered the transport capacity (e.g. [7], [12], [10], [12]). With the concept of outage probability, the transport capacity in this network model assuming communication at the Shannon rate $\log_2(1+\beta)$ is easy to write as:

$$TC = \lambda(1-\epsilon)\log_2(1+\beta)r, \tag{3.3}$$

where $\epsilon \in (0, 1)$ is the outage probability constraint. The ϵ is a quality of service measure, which means transmission will succeed with probability $1 - \epsilon$ and $\lambda(1 - \epsilon)$ is the number of successful transmissions.

Because interference is the critical limiting factor in wireless networks, the allocation of physical "resources" considered here is not the allocation of physical hardware, but rather a license to cause a certain amount of interference in the network. Since each virtual network is a subset of the physical resource, the locations of nodes in each VN should also form a PPP with corresponding spatial density according to the property, that an independent thinning of a PPP is still a PPP, mentioned in chapter 2. Notice that the density λ is the only parameter for the PPP, so through setting the density and power parameter for each virtual network, we provide a certain level of performance, which is the transport capacity in our model.

Although this PPP random system model captures the mobility of nodes in the wireless environment and provides a chance to study the virtualization challenges of wireless networks, it has some limitations (see [21]), in particular, for practical moderate-sized networks. Most importantly, this model is only accurate for uncoordinated transmitters independently and uniformly distributed over the network area. It is well-known that centralized scheduling mechanisms provide remarkable gains (e.g., [1]). However, the results in the PPP model are still valuable for more general study, as explained well in [21].

3.2 Network analysis

In this section, the transport capacity will be analyzed under our system model. Note that there have been a large number of works that employ stochastic geometry to model or quantify network performance, such as interference, connectivity, and throughput. A number of these previous results are used for deriving the outage probability and transport capacity, and we then extend these results to consider the virtual wireless network problem.

3.2.1 Outage probability

Pathloss-only Channel: For networks affected only by pathloss (i.e. $H = 1$), the outage probability defined in (3.2) is:

$$q(\lambda) = \mathbb{P}\left(\frac{r^{-\alpha}}{\sum_{X_i \in \Pi(\lambda)} d_i^{-\alpha}} < \beta\right) \tag{3.4}$$

$$= \mathbb{P}\left(\frac{r^{-\alpha}}{Z_\alpha} < \beta\right) \tag{3.5}$$

$$= 1 - F_{Z_\alpha}\left(\frac{r^{-\alpha}}{\beta}\right), \tag{3.6}$$

where $F_{Z_\alpha}(.)$ is the cumulative distribution function (CDF) of $Z_\alpha \equiv \sum_{X_i \in \Pi(\lambda)} d_i^{-\alpha}$. From [18], Z_α is a Lévy stable random variable. Here each node in a given network is assumed to have the same transmission power, but we will consider different transmission powers for nodes in different virtual networks.

Rayleigh fading Channel: For the channel with Rayleigh fading, each fading coefficient H_i is exponentially distributed. With each node transmitting at the same power, the exact outage probability expression is presented in [21]:

$$q(\lambda) = 1 - \exp\{-\lambda \pi r^2 \beta^{\frac{2}{\alpha}} \frac{2\pi}{\alpha} \csc(\frac{2\pi}{\alpha})\}. \tag{3.7}$$

In fact, only Rayleigh and Nakagami fading channels have exact results for the outage probability and transport capacity, but for general fading, approximations for the outage probability are available in previous research [21].

3.2.2 Optimization of the SIR Threshold and distance r

Since the transport capacity is treated as the performance metric for allocating resources, it is useful to change previous results into an expression only in terms of the network density λ and the outage probability constraint ϵ. Generally, the SIR threshold β and the assigned distance between transmitters and receivers should be chosen reasonably by the system designer to maximize the network performance, i.e. transport capacity. So in this system model, the objective is:

$$\max_{\beta,r} \lambda(1-\epsilon)\log_2(1+\beta)r. \tag{3.8}$$

Here the network density λ and the outage constraint ϵ are fixed, and the goal is to find the expression of β and r in terms of λ, ϵ. For the network with pathloss only, the exact outage probability is set equal to ϵ:

$$\epsilon = q(\lambda) = 1 - F_{Z_\alpha}\left(\frac{r^{-\alpha}}{\beta}\right). \tag{3.9}$$

Then, the distance r can be written as a function of β:

$$r = \left(\beta F_{Z_\alpha}^{-1}(1-\epsilon)\right)^{-\frac{1}{\alpha}}. \tag{3.10}$$

Since λ and ϵ are fixed, the optimized β^* is:

$$\beta^* = \arg\max_{\beta} \beta^{-\frac{1}{\alpha}} \log_2(1+\beta). \tag{3.11}$$

A closed-form (but complicated) solution for β^* can be derived from the related result in [21]:

$$\beta^* = e^{\alpha+\mathcal{W}(-\alpha e^{-\alpha})} - 1, \tag{3.12}$$

where $\mathcal{W}(z)$ is the principle branch of the Lambert function, such that $z = \mathcal{W}(z)e^{\mathcal{W}(z)}$.

3.2.3 Multiple virtual networks with different transmission powers

With the help of analysis above, it is possible to employ the PPP system model to consider the virtualization problem. For the results in previous sections, it is assumed that all nodes in the system transmitted with the same power. Here, each user can set their own transmission power, which means the nodes in user m's VN transmit with power P_m. Recall from the system model that the node locations of each virtual network form a PPP. Here X_m is the location of the nodes, and $\Pi(\lambda_m)$ is the corresponding PPP formed by user m's VN.

3.2.3.1 Two users sharing the resource

The first step is to study the simplest scenario: only two users, user 1 and user 2, sharing resources in the system, with corresponding transmission powers P_1 and P_2. Here, only the channel with pathloss is considered. For the fading channel, it is not possible to get such clean closed-form expressions, but a similar method is still applicable.

For user 1 having virtual network VN_1, the transmissions of nodes in user 2's virtual network VN_2 are treated as interference. So the outage probability defined in (3.2) is:

$$q(\lambda_1) = \mathbb{P}\left(\frac{P_1 r^{-\alpha}}{\sum\limits_{X_m \in \Pi(\lambda_1)} P_1 d_m^{-\alpha} + \sum\limits_{X_n \in \Pi(\lambda_2)} P_2 d_n^{-\alpha}} < \beta_1\right) \tag{3.13}$$

$$= \mathbb{P}\left(\frac{r^{-\alpha}}{I_1 + I_2} < \beta_1\right), \tag{3.14}$$

where $I_1 \equiv \sum_{X_m \in \Pi(\lambda_1)} d_m^{-\alpha}$, $I_2 \equiv \sum_{X_n \in \Pi(\lambda_2)} \frac{P_2}{P_1} d_n^{-\alpha}$ are two independent stable random variables.

From the properties of stable random variables, $I \equiv I_1 + I_2$ is a stable random variable [15]. $I \sim \sum_{X_l \in \Pi(\lambda')} d_l^{-\alpha}$ is called the interference variable, where λ' is the density parameter. In order to use previous results, λ' needs to be found.

In [22], the characteristic function of stable random variables is studied. If $Y = \sum_{X_k \in \Pi(\lambda)} P d_k^{-\alpha}$, where P is constant, the characteristic function of Y is:

$$\phi_Y(w) = \exp\left(-\gamma|w|^{\frac{2}{\alpha}}\left[1 - jsign(w)\tan\left(\frac{\pi}{\alpha}\right)\right]\right), \tag{3.15}$$

where $\gamma = \lambda\pi C_{2/\alpha}^{-1} P^{2/\alpha}$, and C_α is a constant given the fading environment (i.e. it only depends on α). The parameter γ in the characteristic functions of stable random variable I_1, I_2 and I are:

$$\gamma_{I_1} = \lambda_1 \pi C_{2/\alpha}^{-1} \tag{3.16}$$

$$\gamma_{I_2} = \lambda_2 \pi C_{2/\alpha}^{-1}\left(\frac{P_2}{P_1}\right)^{2/\alpha} \tag{3.17}$$

$$\gamma_I = \lambda' \pi C_{2/\alpha}^{-1}. \tag{3.18}$$

In (3.15), it is clear that, for a specific α, only the parameter γ depends on the power and density. Recall that the characteristic function of the product of two independent random variables is: $\phi_{I_1+I_2}(w) = \phi_{I_1}(w)\phi_{I_2}(w)$. So

$$\begin{aligned} \Phi_I &= \Phi_{I_1+I_2}(w) \\ &= \Phi_{I_1}(w)\Phi_{I_2}(w) \\ &= \exp\left(-\gamma_{I_1}|w|^{\frac{2}{\alpha}}\left[1 - jsign(w)\tan\left(\frac{\pi}{\alpha}\right)\right]\right)\exp\left(-\gamma_{I_2}|w|^{\frac{2}{\alpha}}\left[1 - jsign(w)\tan\left(\frac{\pi}{\alpha}\right)\right]\right) \\ &= \exp\left(-(\gamma_{I_1}+\gamma_{I_2})|w|^{\frac{2}{\alpha}}\left[1 - jsign(w)\tan\left(\frac{\pi}{\alpha}\right)\right]\right). \end{aligned} \tag{3.19}$$

Hence,

$$\gamma_I = \gamma_{I_1} + \gamma_{I_2} \tag{3.20}$$

$$= \left[\lambda_1 + \lambda_2\left(\frac{P_2}{P_1}\right)^{2/\alpha}\right]\pi C_{2/\alpha}^{-1}. \tag{3.21}$$

and thus $\lambda' = \lambda_1 + \lambda_2\left(\frac{P_2}{p_1}\right)^{2/\alpha}$, and

$$I \sim \sum\nolimits_{X_l \in \Pi\left(\lambda_1 + \lambda_2\left(\frac{P_2}{P_1}\right)^{2/\alpha}\right)} d_l^{-\alpha}; \tag{3.22}$$

that is, the stable random variable I has equivalent distribution to $\sum_{X_l\in\Pi\left(\lambda_1+\lambda_2\left(\frac{P_2}{P_1}\right)^{2/\alpha}\right)} d_l^{-\alpha}$.

As mentioned before, for a specific path-loss exponent α, the CDF of random variable I, $F_I(.)$, can be written as a function of the density parameter λ'. So the outage probability $q(\lambda_1)$ is:

$$q(\lambda_1) = 1 - F_I\left(\frac{r^{-\alpha}}{\beta_1}\right). \tag{3.23}$$

and, the optimized distance r is:

$$r = \left(\beta^* F_I^{-1}(1-\epsilon)\right)^{-\frac{1}{\alpha}}. \tag{3.24}$$

Here $F_I^{-1}(.)$ is the inverse of random variable I's CDF.

Next, substitute the distance r as a function of β^* into the transport capacity defined in (3.2), yielding the transport capacity of VN_1 with the optimized β and r as:

$$TC_1 = \lambda_1 (1-\epsilon) \log_2(1+\beta^*) F_I^{-1} (1-\epsilon)^{-\frac{1}{\alpha}} \beta^{*-\frac{1}{\alpha}}. \tag{3.25}$$

From (3.12), it is clear that the optimal β^* only depends on α; hence the transport capacity in (3.25) depends only on the density allocation of the system and each user's transmission power.

3.2.3.2 Three or more users sharing resource

Extending the result to three or more users, the transport capacity TC_i of the i^{th} virtual network is:

$$TC_i = \lambda_i F_{I_i}^{-1}(1-\epsilon)^{-\frac{1}{\alpha}} (1-\epsilon) \log_2(1+\beta^*) \beta^{*-\frac{1}{\alpha}}, \tag{3.26}$$

where $I_i \sim \sum_{X_k\in\Pi\left(\lambda_i+\sum_{j\neq i}\lambda_j\left(\frac{P_j}{P_i}\right)^{2/\alpha}\right)} d_k^{-\alpha}$, $i,j \in (1,N)$.

The detailed derivation is presented in Appendix A.

CHAPTER 4

RESOURCE ALLOCATION AND PRICING

In a virtual network environment, each user makes a network request of the substrate resource. In a wired network, the VN request might be the bandwidth of links and the CPU capacity of the nodes. In the model of this thesis, the VN request is set as the transport capacity, which is reasonable because it is an important performance metric of wireless networks. Since the exact total capacity of the system is dynamic, directly allocating the transport capacity to users is impossible. The transport capacity depends on the density parameter of the network and the transmission power. So setting the density parameter to allocate interference space to users is an effective method of resource allocation in interference-limited wireless networks. In this section, first an algorithm which can maximize the total satisfaction of all users sharing resources in the system is brought out, and then we present a method for setting a "price" for a new user.

The transport capacity depends both on the density of nodes and the transmitting power. From the application point of view, it is natural to think that adjusting each virtual network's transmitting power, which also can control each virtual network's performance, is a more flexible method to allocate resources of the wireless system. However, for the theoretical analysis in this thesis, we consider the situation that users can put down their own nodes according to the density assigned to them, and thus the node density is the determinant of our resource allocation algorithm. The reason that we do not choose to adjust the power is that, from (3.26), we know that it is the ratios of multiple virtual networks' transmitting power that matters instead of the exact powers. So in order to allocate power, it requires further analysis with a quite different approach or changing the performance metric from transport capacity to another parameter which directly depends on the transmitting power.

4.1 Utility function

Instead of providing the hard requested transport capacity of users, my approach is employing a utility function $U_i(.)$ to represent the satisfaction of user i to the network performance and $C_i(.)$ to represent the user cost. Then our goal is to maximize the summation of all users' utility minus cost. The method of utility maximization is often used as a efficient way to consider resource allocation problems in many fields. A tutorial [17] gives some basic concepts and a number of maximization algorithms.

In a wireless network, a natural utility $U_i(.)$ is a non-decreasing and concave function of the transport capacity. The utility function used here is:

$$U_i\left(TC_i\right) = Sigmoid\left(2^{TC_i}\right), \tag{4.1}$$

where $Sigmoid(x) = 1/1 + e^{-a(x-b)}$. The shape of the Sigmoid function captures the nature of users' satisfaction. The satisfaction grows fast when the performance is around the expectation of the user. When the performance is at a very low level, the satisfaction improves slowly since the quality of service is far from their expectation. And, when the performance is far beyond what the user needs, the satisfaction grows slowly since the user already is very satisfied with the service and thus further performance improvement provides little value. Because of these characteristics, this Sigmoid function is also used as network utility function in other research [23] which considers utility-based power control schemes.

A linear cost function is used in the algorithm:

$$C_i\left(\lambda_i\right) = \lambda_i P_i, \tag{4.2}$$

where P_i is the transmission power of user i. It is easy to see that $\lambda_i P_i$ is the power density, which is the total transmission power per m^2. Power is a valuable commodity in wireless networks, so this cost function well reflects a key resource. However, other cost functions will also work in the algorithm if they satisfy certain requirements shown below.

4.2 Allocation algorithm

The goal of the allocation algorithm is to maximize the total utility minus cost of the system users, or:

$$V(\lambda) = \sum\nolimits_{i \in users} \left(U_i \left(TC_i \right) - C_i \left(\lambda_i \right) \right). \tag{4.3}$$

Since $V(\lambda)$ is differentiable, the obvious method of maximization is to take a multi-dimensional gradient of $V(\lambda)$. But applying this method requires significant support from a central controller in the network. A decentralized algorithm should be more efficient in a wireless network and allow each user to make their own decision.

Let each user maximize their own utility minus cost instantaneously through a gradient ascent algorithm. The update rule is:

$$\lambda_i^{'} = \lambda_i + \dot{\lambda}_i, \tag{4.4}$$

where $\lambda_i^{'}$ is the updated density, and

$$\dot{\lambda}_i = k\left(\lambda_i\right) \frac{d\left(U_i(TC_i) - C_i(\lambda_i)\right)}{d\lambda_i}. \tag{4.5}$$

The differential gives the direction of ascent, and $k(\lambda_i)$ is a scaling function. Here $k(\lambda_i) = c\lambda_i$. The update step size can be controlled by the constant c.

The next problem to consider is whether the global optimum can be achieved when all users follow the gradient ascent algorithm update rule. From Theorem 3.2 in [17], it shows that the utility function, cost function and scaling function used here satisfy the requirements of globally asymptotical stability, since the derivative of $V(\lambda)$ is decreasing and has a unique zero point. This means that each user only needs to maximize their own utility, and the whole system will achieve the optimum point where the total utility of the system is maximized. Obviously, the choices of $U_i(.), C_i(.)$ and $k(.)$ are not unique, as many other potential choices also satisfy Theorem 3.2 in [17].

Although we seek a resource allocation algorithm for allocating the interference space, a similar idea might also be used for allocating the nodes in a deployed system to different users. Suppose a deployed wireless system is modelled by a Poisson point process model with density λ, and we want to find the optimized density allocation for N virtual networks sharing resources in the system, which means still finding $\lambda_1, \lambda_2, \cdots \lambda_N$, the densities of the corresponding virtual network, and satisfying $\sum_{i=1}^{N} \lambda_i = \lambda$. From a mathematical point of view, this is a constrained optimization problem, which could be complicated.

One method to solve the constrained problem is to first run our algorithm, which applies to the unconstrained problem, to find the densities of each virtual network with the requested transmitting power, $\lambda_1^{'}, \lambda_2^{'}, \cdots, \lambda_N^{'}$. Then calculate $P_i = \frac{\lambda_i^{'}}{\sum_{i=1}^{N} \lambda_i^{'}}$. The P_i is the probability that virtual network i gets a given node in the system, which means the node density of virtual network i is $\lambda_i = \lambda P_i$. This is a possible method for allocating nodes to multiple virtual networks coexisting in a constrained wireless system; however, whether it is the optimized solution requires more study and proof.

4.3 Pricing scheme

Next, a pricing scheme to charge users fairly when providing service to them is proposed. When a new user requests resources from the system, this user's interference will result in worse performance for the existing users of the system. So the basic idea is to measure the total detriment a user causes on others.

4.3.1 Exact results

From the allocation algorithm, the optimum point of a system which maximizes the total network utility can be calculated. When adding the new user with a hard request for a virtual network with node density λ_0 and power P_0, the interference variables of other users will change, and decrease the total utility of the existing users at the new optimal operating point. The amount of decrease is the price this user should pay for sharing resources of the system.

Let a system be operating at an optimum utility V_{op}. After the new user enters, the interference variable of user i already in the system changes to:

$$I_i^{'} \sim \sum_{X_k \in \Pi\left(\lambda_i + \lambda_0\left(\frac{P_0}{P_i}\right)^{2/\alpha} + \sum\limits_{j \neq i} \lambda_j \left(\frac{P_j}{P_i}\right)^{2/\alpha}\right)} d_k^{-\alpha}. \tag{4.6}$$

and the resource allocation algorithm of the previous section is run again to find the new optimal total utility $V_{op}^{'}$. So the price for the new user should be $M_0 = V_{op} - V_{op}^{'}$. However, in order to get $V_{op}^{'}$, all users need to run the allocation algorithm completely. This motivates finding a practical approximation of the price, which we discuss next.

4.3.2 Approximation

Finding an approximation of the exact price which requires less time and calculations is necessary since this price should be quickly provided to the new users with a certain level of accuracy. It is clear that in order to reach the new optimization point $V_{op}^{'}$, each user needs to update their network density. So if all users stay at the previous density without running the allocation algorithm when a new user enters the system, the total network utility of the original users V^* will be lower than $V_{op}^{'}$; that is, $V^* < V_{op}^{'}$.

Suppose λ_i^* is the density of user i at the system's stable point before the new user enters. After the new user is added with density λ_0 and power P_0, the interference variable of the original user changes to:

$$I_i^* \sim \sum\nolimits_{X_k \in \Pi\left(\lambda_i^* + \lambda_0\left(\frac{P_0}{P_i}\right)^{2/\alpha} + \sum\limits_{j \neq i} \lambda_j^*\left(\frac{P_j}{P_i}\right)^{2/\alpha}\right)} d_k^{-\alpha}. \tag{4.7}$$

Let $M_0^* = V_{op} - V^*$. Since $V^* < V_{op}^{'}$, $M_0^* > M_0$.

The approximation of the exact price used here is:

$$\widehat{M_0} = -\lambda_0 \sum_{i \neq 0} \frac{d\left(U_i(TC_i^*) - C_i(\lambda_i^*)\right)}{d\lambda_0}|_{\lambda_0=0}, \tag{4.8}$$

where TC_i^* is the transport capacity of user i after the new user enters, but at the previously optimized density λ_i^*. Each user needs only calculate and publish this derivative, which requires significantly less time and calculation than running the allocation algorithm. This approximated price is clearly the tangent of M_0^* at origin. Since M_0^* appears to be a concave function (shown in numerical analysis), $\widehat{M_0}$ is likely an upper bound of the exact price M_0.

4.4 Numerical Analysis

For numerical analysis, I set $\alpha = 4$, for which $Z_\alpha \equiv \sum_{X_i \in \Pi(\lambda)} d_i^{-\alpha}$ has the closed-form distribution expression, $F_{Z_4}(z) = 2Q\left(\frac{\pi^{3/2}\lambda}{\sqrt{2z}}\right)$. The outage probability constraint is $\epsilon = 0.55$, and the two parameters of the utility Sigmoid function are $a = 1, b = 2$. Figure 4.1 shows a surface for the total utility minus cost of a system with two users sharing resources. As expected, $V(\lambda)$ is a strictly concave function, and has a unique maximization point. Figure 4.2 shows the process of optimization via gradient ascent. As expected, the algorithm takes the system to the optimal point.

In Figure 4.3 and Figure 4.4, the exact prices M_0, M_0^*, and the approximation price $\widehat{M_0}$ are plotted versus the spatial interference density of the new user. For the examples considered, they tell that the function of M_0^* is concave, so the approximation price, which is a tangent to the curve, is always an upper bound. It is obvious that the approximation price is accurate for small λ_0 and the gap grows when λ_0 increases.

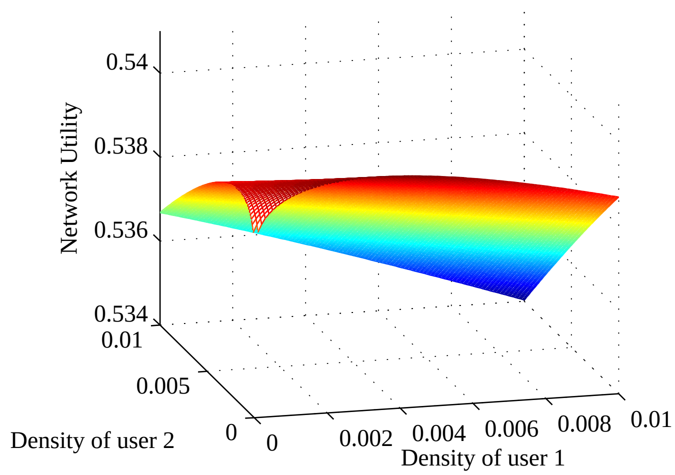

Figure 4.1. The total network utility of a system with two users transmitting with power $P_1 = 0.5, P_2 = 0.7$. The outage probability constraint is $\epsilon = 0.55$ and the pathloss exponent is $\alpha = 4$. The surface is concave as expected and has a unique maximization point.

Next, we run a simulation to implement our allocation algorithm and pricing schemes. Suppose there is a system with five users, whom enter and leave the system by a Poisson Process, transmitting with power $P_1 = 0.40, P_2 = 0.45, P_3 = 0.50, P_4 = 0.55$ and $P_5 = 0.60$. Each user picks a density parameter which is uniformly distributed on $[0, 0.0001]$ before entering. The density range seems unreasonable low, however, as mentioned before, it is not the density of the total nodes in each virtual network, but the density of the transmitting nodes. Then, we calculate the exact

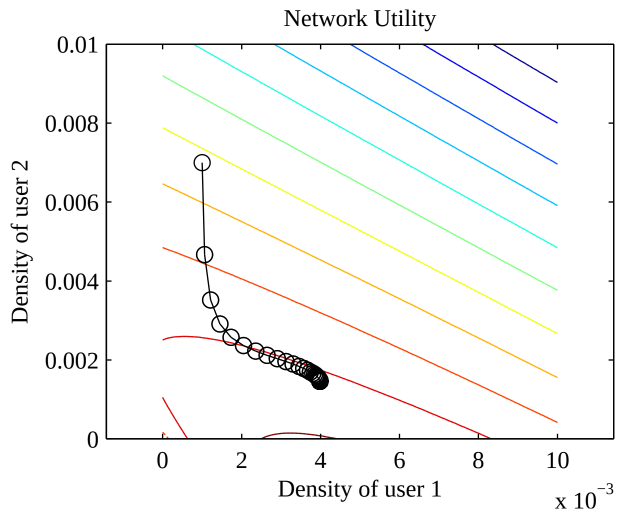

Figure 4.2. The update process of the resource allocation algorithm operating in the same system of Fig. 1. The beginning point is $\lambda_1 = 0.001, \lambda_2 = 0.007$. We first contour the surface in Fig. 1, and then calculate $V(\lambda)$ for each update step, which are shown by the spots in the graph. The path goes to the global optimal point.

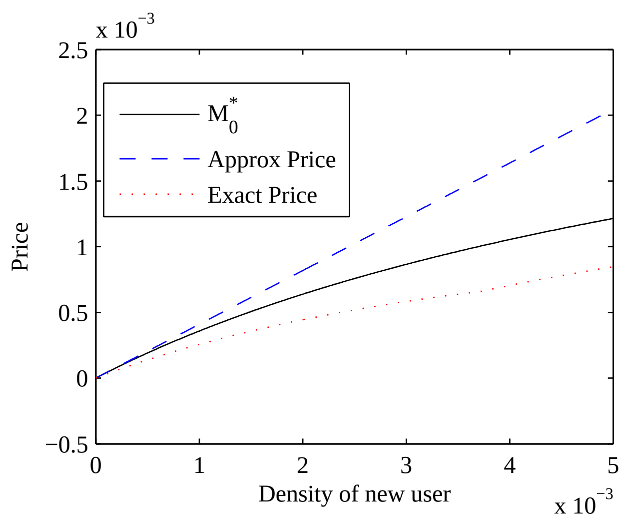

Figure 4.3. Price versus density λ_0 of a new user for a system with two original users. Here, the two original users have transmission powers $P_1 = 0.5, P_2 = 0.7$ and the new user transmits with power $P_0 = 0.6$. The curve of M_0^* is concave and the approximate price is an upper bound to the exact price.

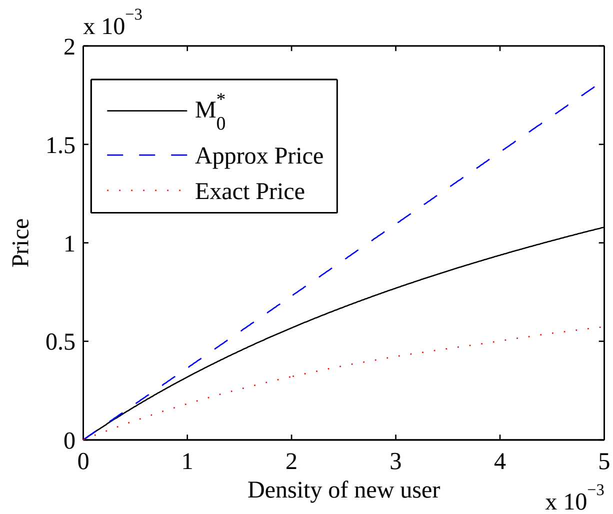

Figure 4.4. Price versus density λ_0 of a new user for a system with three original users. Here, the three original users have transmission powers $P_1 = 0.8, P_2 = 0.7, P_3 = 0.7$ and the new user transmits with power $P_0 = 0.5$. The curve of M_0^* is still concave.

price and the approximate price following the pricing scheme. In Table 4.1, we show the average prices of each user and how much the user is over charged by the approximate price. We can see that the approximation is accurate when the density request of new users is low.

Table 4.1. Simulation Results: The exact and approximate price and overprice percentage of five users with different transmission powers, who enter and leave the system by a Poisson Process.

User Number	**Transmit Power**	**Exact Price** ($\times 10^{-4}$)	**Approximate Price** ($\times 10^{-4}$)	**Overprice** (%)
1	0.40	0.1417	0.1811	27.83
2	0.45	0.1616	0.1937	19.84
3	0.50	0.1391	0.1776	27.66
4	0.55	0.2036	0.2389	17.35
5	0.60	0.1612	0.1886	17.02

Since the resource allocation algorithm requires each user to publish its transmitting power and densities, it is possible that a user might use higher power and higher densities than the amount it published. However, because of the interference and cost, using higher power and density actually decreases the user's utility. Numerical analysis establish this fact. The following two plots, Figure 4.5 and Figure 4.6, show the decrease of utility when the user uses a higher power and density then published. Also, since the other users only know and use the published power and density to calculate the density at the next iteration, this user's behavior will not affect other users' density allocation, but their performance would be worse.

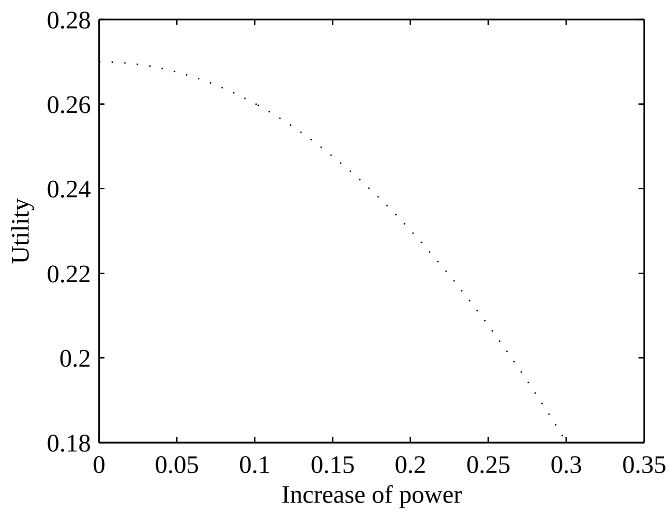

Figure 4.5. Utility versus the increase of power. In a system, two virtual network sharing resource and one of the users use higher transmitting power than published. This plot shows that this user does not gain any benefit from "cheating", but has a decreasing performance.

There is another possibility that a user uses lower density and power than published in order to make other users have lower density. Figure 4.7 shows that using lower density than published can not get higher utility. But Figure 4.8 shows that using lower power than published actually can makes this user's utility increased.

4.5 Simulation with randomness

As mentioned before, the random nature of wireless networks is the main focus when considering resource allocation problems in this thesis. So far, all of the analysis

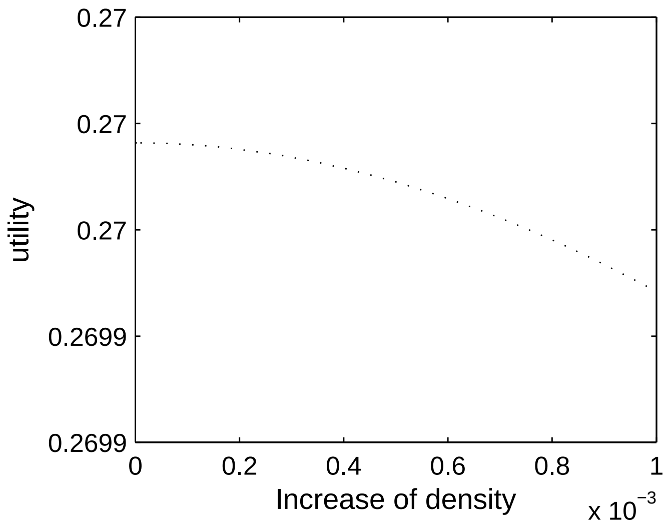

Figure 4.6. Utility versus the increase of density. In a system, two virtual network sharing resource and one of the users use higher density than published. This plot shows that this user does not gain any benefit from “cheating”, but has a decreasing performance.

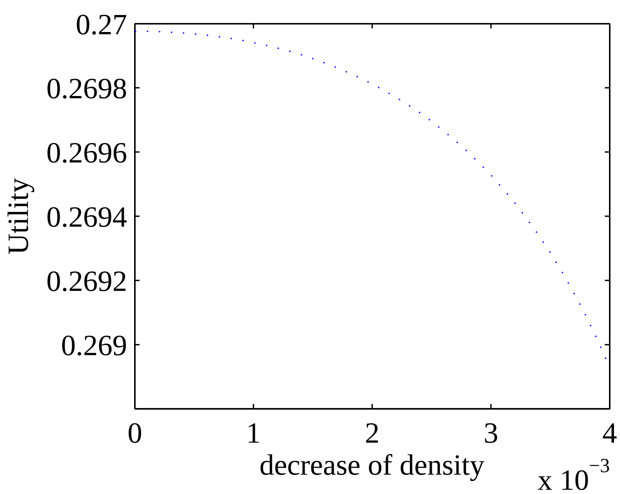

Figure 4.7. Utility versus the decrease of density. In a system, two virtual network sharing resource and one of the users use lower density than published. This plot shows that this user does not gain benefit from “cheating”, but has a decreasing performance.

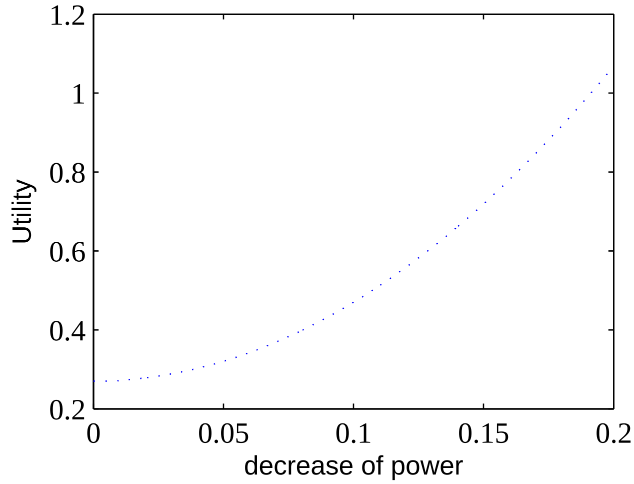

Figure 4.8. Utility versus the decrease of power. In a system, two virtual network sharing resource and one of the users use lower power than published. This plot shows that this user has a better performance because of the “cheating”.

is based on the mathematical foundation of stochastic geometry. Here, a simulation is built, in order to test how the allocation algorithm works in a system with randomness.

First, I calculate the optimized densities and transport capacities of a virtual networks with given transmitting powers. Then I set up a system supporting these virtual networks where nodes in each virtual network form a Poisson point process with the densities calculated by the algorithm. All the nodes in the system employ ALOHA as the medium access control protocol. After letting the system run for a certain time, I measure the average transport capacity of each virtual network. Comparing the exact measured performance and the optimized performance calculated by the algorithm could show how the algorithm captured the randomness of wireless system.

Table 4.2 and Table 4.3 show the simulation results for the system of size 100×100. Table 4.4 and Table 4.5 show the results for the system of size 120×120.

From the tables, it is clear that the exact measured transport capacities are very close to the optimized transport capacities calculated by the algorithm, which means

Table 4.2. Results of the simulations in which two virtual networks with nodes employing ALOHA share resources in the system of size 100×100. P is the transmit power assigned to the corresponding virtual network. λ and TC_o are the optimized node densities and transport capacities, respectively, derived from the resource allocation algorithm. TC_e is the exact transport capacity of the simulated system.

Transmitting power $P(P_1, P_2)$	**Density** λ $(\lambda_1, \lambda_2) \times 10^{-2}$	**Optimized TC** TC_o $(TC_{o1}, TC_{o2}) \times 10^{-2}$	**Exact TC** TC_e $(TC_{e1}, TC_{e2}) \times 10^{-2})$
$(0.3, 0.3)$	$(1.12, 1.12)$	$(3.19, 3.19)$	$(2.87, 3.26)$
$(0.3, 0.4)$	$(1.14, 0.47)$	$(3.84, 1.70)$	$(3.95, 1.87)$
$(0.3, 0.5)$	$(1.17, 0.22)$	$(4.14, 0.87)$	$(4.69, 0.70)$
$(0.4, 0.6)$	$(0.66, 0.19)$	$(2.97, 0.94)$	$(2.97, 1.19)$
$(0.4, 0.7)$	$(0.64, 0.10)$	$(3.10, 0.56)$	$(2.96, 0.69)$

Table 4.3. Results of the simulations in which three virtual networks with nodes employing ALOHA share resources in the system of size 100×100. P is the transmit power assigned to corresponding virtual network. λ and TC_o are the optimized node densities and transport capacities, respectively, derived from the resource allocation algorithm. TC_e is the exact transport capacity of the simulated system.

Transmitting power P (P_1, P_2, P_3)	**Density** $\lambda \times 10^{-2}$ $(\lambda_1, \lambda_2, \lambda_3)$	**Optimized TC** $TC_o \times 10^{-2}$ $(TC_{o1}, TC_{o2}, TC_{o3})$	**Exact TC** $TC_e \times 10^{-2}$ $(TC_{e1}, TC_{e2}, TC_{e3})$
$(0.5, 0.6, 0.7)$	$(0.40, 0.22, 0.07)$	$(1.98, 1.14, 0.40)$	$(1.66, 0.82, 0.26)$
$(0.5, 0.5, 0.5)$	$(0.32, 0.32, 0.32)$	$(1.38, 1.38, 1.38)$	$(1.40, 1.40, 1.08)$
$(0.6, 0.7, 0.8)$	$(0.27, 0.16, 0.07)$	$(1.59, 0.98, 0.43)$	$(1.26, 1.04, 0.51)$
$(0.7, 0.8, 0.9)$	$(0.19, 0.12, 0.06)$	$(1.32, 0.86, 0.44)$	$(1.30, 1.49, 0.51)$
$(0.8, 0.7, 0.7)$	$(0.09, 0.18, 0.18)$	$(0.60, 1.12, 1.12)$	$(0.81, 0.93, 1.25)$

Table 4.4. Results of the simulations in which two virtual networks with nodes employing ALOHA share resources in the system of size 120×120. P is the transmit power assigned to the corresponding virtual network. λ and TC_o are the optimized node densities and transport capacities, respectively, derived from the resource allocation algorithm. TC_e is the exact transport capacity of the simulated system.

Transmitting power $P(P_1, P_2)$	**Density** λ $(\lambda_1, \lambda_2) \times 10^{-2}$	**Optimized TC** TC_o $(TC_{o1}, TC_{o2}) \times 10^{-2}$	**Exact TC** TC_e $(TC_{e1}, TC_{e2}) \times 10^{-2})$
$(0.3, 0.3)$	$(1.12, 1.12)$	$(3.19, 3.19)$	$(3.39, 3.12)$
$(0.3, 0.4)$	$(1.14, 0.47)$	$(3.84, 1.70)$	$(3.92, 1.85)$
$(0.3, 0.5)$	$(1.17, 0.22)$	$(4.14, 0.87)$	$(4.30, 0.87)$
$(0.4, 0.6)$	$(0.66, 0.19)$	$(2.97, 0.94)$	$(2.99, 1.08)$
$(0.4, 0.7)$	$(0.64, 0.10)$	$(3.10, 0.56)$	$(2.79, 0.69)$

Table 4.5. Results of the simulations in which three virtual networks with nodes employing ALOHA share resources in the system of size 120×120. P is the transmit power assigned to corresponding virtual network. λ and TC_o are the optimized node densities and transport capacities, respectively, derived from the resource allocation algorithm. TC_e is the exact transport capacity of the simulated system.

Transmitting power P (P_1, P_2, P_3)	**Density** $\lambda \times 10^{-2}$ $(\lambda_1, \lambda_2, \lambda_3)$	**Optimized TC** $TC_o \times 10^{-2}$ $(TC_{o1}, TC_{o2}, TC_{o3})$	**Exact TC** $TC_e \times 10^{-2}$ $(TC_{e1}, TC_{e2}, TC_{e3})$
$(0.5, 0.6, 0.7)$	$(0.40, 0.22, 0.07)$	$(1.98, 1.14, 0.40)$	$(1.83, 1.15, 0.24)$
$(0.5, 0.5, 0.5)$	$(0.32, 0.32, 0.32)$	$(1.38, 1.38, 1.38)$	$(1.26, 1.31, 1.08)$
$(0.6, 0.7, 0.8)$	$(0.27, 0.16, 0.07)$	$(1.59, 0.98, 0.43)$	$(1.53, 1.23, 0.14)$
$(0.7, 0.8, 0.9)$	$(0.19, 0.12, 0.06)$	$(1.32, 0.86, 0.44)$	$(1.53, 0.96, 0.40)$
$(0.8, 0.7, 0.7)$	$(0.09, 0.18, 0.18)$	$(0.60, 1.12, 1.12)$	$(0.55, 1.00, 0.92)$

this resource allocation algorithm can work well for the wireless system with probabilistic characteristics.

CHAPTER 5

CONCLUSION AND FUTURE WORK

5.1 Conclusion

In this section, the main work of this thesis is summarized.

- *A system model capturing randomness.* First, a PPP system model is brought out in this thesis which take the randomness of nodes' locations into account. Since node mobility is the most important characteristic of the wireless environment, this model make it possible to gain insight into the virtualization problem in wireless scenarios. Another big advantage of this model is that the mathematical tools from stochastic geometry can be applied and provide a framework for network performance analysis.

- *Decentralized dynamic algorithm for resource allocation.* This decentralized algorithm drives the system to the optimal point where the total utility, which is defined as the satisfaction degree of the users, of the virtual networks is maximized. The "resource" allocation here does not means allocating physical hardware, such as substrate nodes and links, but as the ability to cause a certain amount of interference in the network.

- *Pricing scheme.* The pricing scheme employs a new idea for charging a new user for network resources fairly. Due to the interference property of wireless channels, a new user sharing the resources of the system will result in a performance decrease for the existing users. Hence, this scheme measures the total detriment caused by this user on others. In addition, a practical method for approximating this price is also presented, which is effective for low user densities.

5.2 Future work

In this thesis, the resource allocation problem and pricing are considered separately. However, the price, which is also a cost to the user, can affect multiple virtual networks' behavior and request. So taking the price into account when solving the resource allocation problem could yield a better way for the practical virtualization environment.

APPENDIX A

TRANSPORT CAPACITY OF A SYSTEM WITH THREE OR MORE USERS

Applying the same method as in the system with two users, it is easy to extend the previous results to three or more users sharing network resources.

First, to establish the pattern, suppose there are three users sharing resources in a system. Each of them has corresponding virtual network VN_1, VN_2, VN_3 with nodes transmitting with power P_1, P_2, P_3, respectively. Transmissions of nodes in VN_2 and VN_3 are treated as interference for nodes in VN_1. So the outage probability of VN_1 is:

$$q(\lambda_1) = \mathbb{P}\left(\frac{r^{-\alpha}}{I_1 + I_2 + I_3} < \beta_1\right), \tag{A.1}$$

where $I_1 \equiv \sum_{X_m \in \Pi(\lambda_1)} d_m^{-\alpha}$, $I_2 \equiv \sum_{X_n \in \Pi(\lambda_2)} \left(\frac{P_2}{P_1}\right) d_n^{-\alpha}$, $I_3 \equiv \sum_{X_p \in \Pi(\lambda_3)} \left(\frac{P_3}{P_1}\right) d_p^{-\alpha}$ are three independent random variables.

Applying the characteristic function method from the two user scenario yields:

$$\lambda^{'} = \lambda_1 + \lambda_2 \left(\frac{P_2}{P_1}\right)^{2/\alpha} + \lambda_3 \left(\frac{P_3}{P_1}\right)^{2/\alpha}. \tag{A.2}$$

So, for the interference variable $I \equiv I_1 + I_2 + I_3$, it is easy to get:

$$I \sim \sum_{X_l \in \Pi\left(\lambda_1 + \lambda_2 \left(\frac{P_2}{P_1}\right)^{2/\alpha} + \lambda_3 \left(\frac{P_3}{P_1}\right)^{2/\alpha}\right)} d_l^{-\alpha}. \tag{A.3}$$

It is obvious that adding a network with density λ and transmission power P to the system adds to a term $\lambda \left(\frac{P}{P_1}\right)^{2/\alpha}$ to $\lambda^{'}$.

Thus in general, for N virtual networks, VN_1, VN_2, VN_3,..., VN_N, each with corresponding density λ_i and node transmission power P_i, the transport capacity of VN_i is:

$$TC_i = \lambda_i F_{I_i}^{-1}(1-\epsilon)^{-\frac{1}{\alpha}}(1-\epsilon)\log_2(1+\beta^*)\beta^{*-\frac{1}{\alpha}}, \tag{A.4}$$

where $I_i \sim \sum_{X_k \in \Pi\left(\lambda_i + \sum\limits_{j \neq i} \lambda_j \left(\frac{P_j}{P_i}\right)^{2/\alpha}\right)} d_k^{-\alpha}$, $i, j \in (1, N)$.

APPENDIX B

THE TABLE OF NOTATIONS USED IN THE THESIS

Table B.1. Summary of Notations

Notation	**Description**
$a \equiv b$	a is defined to equal b
λ	spatial density of attempted transmitters
$\Pi = \{X_i\}$	Poisson Point Process of density λ of transmitter locations
α	pathloss exponent ($\alpha > 2$)
β	SIR/SINR requirement for successful reception
r	distance separating each transmitter and receiver pair
$q(\lambda)$	outage probability
ϵ	outage probability constraint
TC	Transport capacity
P	transmission power
I	interference variable
$U(.)$	utility function
$C(.)$	cost function
$V(\lambda)$	network utility (total system utility minus cost)
M	price

BIBLIOGRAPHY

[1] Baccelli, F., B.Blaszczyszyn, and Muhlethaler, P. An aloha protocol for multihop mobile wireless networks. *IEEE transaction on Information Theory 52*, 2 (Feb 2006), 421–436.

[2] Belbekkouche, Abdeltouab, Hasan, Md. Mahmud, and Karmouch, Ahmed. Resouece discovery and allocation in network virtualization. *IEEE Communications Surveys and Tutorials 14*, 4 (2012), 1114–1128.

[3] Chowdhury, N.M. Mosharaf Kabir, and Boutaba, Raouf. Network virtualization: State of the art and research challenges. *IEEE Communication Magazine 47*, 7 (July 2009), 20–26.

[4] Chowdhury, N.M. Mosharaf Kabir, and Boutaba, Raouf. A survey of network virtualization. *Computer Networks: The International Journal of Computer and Telecommunications Networking 54*, 5 (April 2010), 862–876.

[5] Chowdhury, N.M. Mosharaf Kabir, Rahman, Muntasir Raihan, and Boutaba, Raouf. Virtual network embedding with coordinated node and link mapping. In *IEEE, INFOCOM* (April 2009), pp. 783–791.

[6] Fischer, Andreas, Botero, Juan Felipe, Beck, Michael Till, de Meer, Hermann, and Hesselbach, Xavier. Virtual network embedding: a survey. *IEEE Communications Surveys and Tutorials pp*, 99 (Feb 2013), 1–19.

[7] F.Xue, and P.kumar. Scaling laws for ad hoc wireless networks: An information theoretic approach. *Ser. Foundations and Trends in Networking, Now Publishers* (2006).

[8] Gupta, P., and P, Kumar. The capacity of wireless network. *Information Theory, IEEE transaction on 46*, 2 (Mar 2000), 388–404.

[9] Haenggi, M., Andrews, J. G., Baccelli, F., and Dousse, O. Stochastic geometry and random graphs for the analysis and design of wireless networks. *IEEE Journal on Selected Areas in Communications 27*, 7 (August 2009), 1029–1046.

[10] Jovicic, A., P.Viswanath, and S.Kulkarni. Upper bounds to transport capacity of wireless networks. *IEEE Trans. Inf. Theory 50*, 11 (Nov 2004), 2555–2565.

[11] Leivadeas, A., Papagianni, C., Paraskevas, E., and Androulidakis, G. An architecture for virtual network embedding in wireless systems. In *Network Cloud Computing and Applications* (Nov 2011), pp. 62–68.

[12] Leveque, O., and Teletar, I.E. Information-theoretic upper bounds on the capacity of large extended ad hoc wireless network. *IEEE Trans. Inf. Theory 51*, 3 (Mar 2005), 858–865.

[13] Lv, Pin, Cai, Zhiping, and Xu, Jia. Multicast service-oriented virtual network embedding in wireless mesh networks. *IEEE Communication letters 16*, 3 (March 2012), 375–377.

[14] m. Park, K., and k. Kim, C. A framework for virtual network embedding in wireless network. In *4th International Conference on Future Internet Technologies* (2009), Seoul,Kerea.

[15] Nolan, J. P. *Stable Distributions-Models for Heavy Tailed Data*. Birkhauser, Boston, 2013.

[16] Peterson, L., Shenker, S., and Turner, J. Overcoming the internet impasse through virtualization. *Computer 38*, 4 (April 2005), 34–41.

[17] Shakkottai, S., and Srikant, R. Network optimization and control. *Foundations and Trends in Networking, Now Publishers 2*, 3 (month 2007), 271–379.

[18] Sousa, E. Performance of a spread spectrum packet radio network link on a poisson field of interferers. *IEEE Tran. Inf. Theory 38*, 6 (Nov 1992), 1743–1754.

[19] Stezenbach, D., Hartmann, M., and Tutschku, K. Parameters and challenges for virtual network embedding in the future internet. In *Network Operations and Management Symposium (NOMS), 2012 IEEE* (April 2012), pp. 1272–1278.

[20] Turner, J., and D.Taylor. Diversifying the internet. In *IEEE Global Telecommunications Conference* (2005), vol. 2.

[21] Weber, Steven, G.Andrews, Jeffrey, and Jindal, Nihar. An overview of the transmission capacity of wireless networks. *Communications, IEEE Transactions on 58*, 12 (December 2010), 3593–3604.

[22] Win, M. Z. A mathematical theory of network interference and its applications. *Proceedings of the IEEE 97*, 2 (Feb 2009), 205–230.

[23] Xiao, Mingbo, Shroff, Ness B., and Chong, Edwin K. P. A utility-based power-control scheme in wireless cellular systems. *IEEE/ACM Transactions on networking 11*, 2 (April 2003), 210–221.

[24] Yun, Donggyu, Ok, Jungseul, Shin, Bongjhin, Park, Soobum, and Yi, Yung. Embedding of virtual network requests over static wireless multihop networks. *Computer Networks 57*, 5 (April 2013), 1139–1152.