AZTEC SURVEY OF THE CENTRAL MOLECULAR ZONE: MODELING DUST SEDS WITH HIERARCHICAL BAYESIAN ANALYSIS

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AZTEC SURVEY OF THE CENTRAL MOLECULAR ZONE: MODELING DUST SEDS WITH HIERARCHICAL BAYESIAN ANALYSIS

A Dissertation Presented
by
YUPING TANG

Submitted to the Graduate School of the University of Massachusetts Amherst in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY

May 2019

Department of Astronomy
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Approved as to style and content by:

______________________________
Grant W. Wilson, Chair

______________________________
Q. Daniel Wang, Member

______________________________
Mark Heyer, Member

______________________________
Erin Conlon, Member

Daniela Calzetti, Department Chair
Department of Astronomy
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My thanks also goes to my peers and the friends I have made during my graduate career and my supportive family who believed in me and helped me get to where I am today.
In this dissertation, we present a study based on the AzTEC/Large Millimeter Telescope (LMT) survey of dust continuum at 1.1mm on the central 200 parsecs (The Central Molecular Zone (CMZ)) of our Galaxy. Owing to its unusually high gas density and turbulence, strong magnetic field, and high cosmic ray flux, the CMZ represents an initial condition for star-formation typical of starburst galaxies in the distant universe. In order to understand dust properties in such an extreme environment. We perform a joint SED analysis of existing dust continuum surveys on the CMZ, from a wavelength of $\lambda = 160 \, \mu m$ to 1.1 mm. This analysis follows a Bayesian model incorporating the knowledge of Point Spread Functions (PSFs) in different maps, which enables full utilization of our high resolution (10.5") map at 1.1 mm and achievement of unprecedented detailed information on the spatial distribution of
dusty gas across the CMZ. There is a remarkable trend of increasing dust spectral index, from 2.0 – 2.5, toward dense peaks in the CMZ, indicating a deficiency of large grains or a fundamental change in dust optical properties. The latter scenario leads to an underestimate of dust temperature when using the conventional model. Depending on how optical properties of dust deviate from the conventional model, dust temperature could be underestimated by 10 – 50%, and potentially even higher. We further develop new methods to explore the temperature and density structures of the CMZ molecular clouds, based on Hierarchical Bayesian Analysis. We propose a phenomenological model for line-of-sight temperature decomposition and show that the temperature profile of dust evolves with orbital phases, in agreement with previous studies on gas temperature. Finally, we show that at the 0.5 parsec spatial resolution achieved by our study, the Probability Density Function of the Column Densities (N-PDF) provides a robust indicator of the density structure.
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CHAPTER 1
INTRODUCTION

Observations have revealed that the central \( \sim 200 \) parsec region (Central Molecular Zone, or CMZ) of our Galaxy has an extreme gaseous environment, which may be common to the nuclear regions of many galaxies [Morris & Serabyn(1996)]. The CMZ is characterized by dense \( (n_{H_2} \gtrsim 10^4 \text{ cm}^{-3}) \), warm \( (T \approx 60 - 100 \text{ K}) \) [Paglione et al.(1998), Oka et al.(2007), Ginsburg et al.(2016)] molecular gas with violent turbulent motions [Bally et al.(1987), Kauffmann et al.(2017A)]. The magnetic fields [Morris(2015), Federrath et al.(2016)] and the flux density of cosmic rays [Aharonian et al.(2006), Crocker & Aharonian(2011)] in the CMZ also achieve their maximum values throughout the Galactic disk. Such conditions are not unique. In fact, it is hard to ignore that the CMZ is a region sharing gas densities and kinematics in common with those observed in high redshift starburst galaxies [Mills(2017)], and therefore a natural local site to study the distribution and dynamics of stars and gas, as well as the star-formation mode and history under an extreme galactic environment.

Meanwhile, the CMZ is distinct from starburst galaxies by its current unusually low star-formation rate (SFR). It has been realized for years that the SFR in the CMZ is an order of magnitude lower than what is expected from empirical relationships established in both the local and the high redshift universe [Yusef-Zadeh et al.(2009), Longmore et al.(2013), Kruijssen et al.(2014), Kauffmann et al.(2017A)]. While individual star-formation regions such as Sgr B2 exists, most dense clouds and regions in the CMZ (e.g., G0.253+0.016, The Brick, with \( n_{\text{peak}} > 10^6 \text{ cm}^{-3} \)) show weak or no
sign of ongoing star-formation activity. [Kruijssen et al.(2014)] investigated various mechanisms that could potentially suppress star-formation in the CMZ in terms of the energy budget, and suggest that turbulent pressure plays a crucial role in preventing gas from collapsing. The importance of turbulence is confirmed by further detailed studies on individual CMZ clouds (e.g., G0.253+0.016, [Federrath et al.(2016)]). It is proposed that the star-formation in the CMZ is episodic, currently in a slump, but will be triggered toward a burst phase, possibly by tidal compression induced by the Galactic gravitational potential, as bar-driven inflows continuously accumulate gas and eventually the density threshold for star-formation will be achieved. In this picture, dense clouds in the CMZ could be geometrically placed in a star-forming sequence, where star-formation in each cloud is triggered during its pericentre passage, and most CMZ clouds are currently in a pre-burst phase.

Much progress has recently been made in the study of the GC. The CMZ in particular has been surveyed in all accessible bands: e.g., X-ray with Chandra and XMM-Newton [Wang et al.(2002), Ponti et al.(2015)] and near- to far-IR with HST, Spitzer, and Herschel [Molinari et al.(2010), Wang et al.(2010), Dong et al.(2011)], as well as in many wavelength ranges with ground-based telescopes (e.g., NANTEN2, [Riquelme et al.(2010)]; Mopra, [Jones et al.(2012)]). These surveys have provided an unprecedented panoramic view of stars and gas in the GC (e.g., Fig 1.1).

The rich combination of cloud characteristics and energetic objects also makes the CMZ an excellent and unique test ground for our understanding of cold gas and dust astrophysics in distant starburst galaxies. The tight connections between gas conditions in the CMZ and high-redshift starburst galaxies highlight the CMZ as a template for verification/calibration of dust models in such environments. In modeling of high-redshift starburst galaxies [Blain et al.(2002), Casey et al.(2014), Popping et al.(2017)], optical properties of dust grains are conventionally adopted from those inferred in the local environments. The dust absorption curve is normally
Figure 1.1. A multi-wavelength montage of the CMZ from [Kruijssen et al.(2015)]. The white dotted line shows the contemporary orbital model. The Red shows the HOPS NH$_3$(1,1) emission to indicate gas with a density $n >$ several $10^3$ cm$^{-3}$, green shows the MSX 21.3m from warm dust, blue shows the 8.28 $\mu$m emission from hot dust, young stellar object and evolved stars.

simplified as a single power-law from far infrared (FIR) to submillimeter wavelengths, characterized by a spectral index $\beta = 1.5 - 2$ (Figure 1.2). Our power to constrain dust properties in distant starburst galaxies is limited by the lack of spatial resolution [Casey(2012), Magnelli et al.(2012)]. As a matter of fact, even in the local universe, studies on the spectral energy distributions (SEDs) of dust emissions have not clearly established how the optical properties of dust vary in different environments. On small scales, $\beta \approx 1$ is observed in proto-planetary and proto-stellar disks [Draine(2006), Kwon et al.(2009)] and is commonly attributed to $\gtrsim 1$ mm size large grains. In dense molecular clouds and the diffuse ISM, a wide variety of $\beta$ is observed, from 0.8 to $> 2$ [Dupac et al.(2003), Paradis et al.(2011), Juvela et al.(2015)]. The origin of this diversity is debated. So far, observations suggest an anti-correlation between dust temperature and $\beta$ or a positive correlation between gas density $n_{H_2}$ and $\beta$ at long wavelengths $\lambda \gtrsim 500\mu m$, over the range from the diffuse ISM to cold
dense clumps. At short wavelengths $\lesssim 200 - 500 \mu m$, however, an inverse trend is observed [Ysard et al.(2012)], i.e., a flattening of the dust absorption curve toward dense regions. It is debated that radiative transfer effect [Shetty et al.(2009)] and parameter degeneracy [Juvela et al.(2013)] could be responsible. The wavelength dependent change of $\beta$ is intriguing, and cannot be reproduced by classic models of dust growth [Ossenkopf & Henning(1994), Köhler et al.(2012), Ysard et al.(2012), Ysard et al.(2013)], which predict a negative $n_{H2} - \beta$ correlation. Recently, two new models have been proposed to solve this problem, a) accretion of small hydrogenated amorphous carbon onto large grains with updated optical properties on hydrogenated amorphous carbons [Jones et al.(2013), Köhler et al.(2015)] and b) an intrinsic dependency of the dust absorption curve on the dust temperature [Meny et al.(2007), Paradis et al.(2014)]. These two models could be potentially distinguishable from observations, as the first scenario suggests a density dependency of $\beta$, and the second scenario suggests a temperature dependency. Nevertheless, observations with wide coverage in the $n_{H2} - T_{dust}$ plane is required. It is also possible that turbulence [Hirashita & Yan(2009)] could be another factor affecting dust properties, especially in systems like the CMZ and high-redshift starburst galaxies.

Submillimeter/millimeter observations sampling the Rayleigh-Jeans tail of the dust SED are crucial for constraining the dust absorption curve. During Early Science Cycle 2 (ES2) for the Large Millimeter Telescope (LMT), we carried out a 20 hours survey of the dust continuum at 1.1mm on the central $\approx 200$ pc of our Galaxy with the AzTEC bolometer array camera [Wilson et al.(2008)]. The AzTEC survey outperforms pre-existing FIR/submillimeter surveys (SPIRE/Herschel, Bolocam/CSO, HFI/Planck) with regard to spatial resolution (HPBW= 10.5″). Existing studies on the dust emission in the CMZ are mostly based on the Herschel Hi-GAL survey. Herschel/SPIRE instrument (160 $\mu m$-500 $\mu m$) has low spatial resolutions (HPBW$_{500\mu m}$ = 36″) and insufficient spectral coverage on the Rayleigh-
Figure 1.2. The dust extinction curve for three different models from [Galliano(2018)]. Also shown is a power-law approximation in dashed yellow, with $\beta = 1.79$.

Jeans tail of the dust SED. Adding a high-resolution survey at 1.1 mm to the current data set significantly enhances our capability to uncover small scale structures [Heyer et al.(2018)]. Under the conditions of heavy obscuration and turbulent gas motions in the CMZ, column mass density of dust grains can serve as a robust proxy of the total column density of the molecular gas. The conversion from an observed dust SEDs to a total gas column density is only dependent on the dust mass absorption coefficient, the gas metallicity, and the depletion factor of heavy elements. All three factors are expected to have small variations on a $\sim 100$ pc scale.

Achieving optimal spatial resolution with SED analyses requires proper treatment of multi-wavelength maps. Studies based on multi-wavelength maps suffer from resolution non-uniformity, a common approach is to dilute all maps to the lowest spatial resolution, which results in loss of information. The dilution of Point Spread Function (PSF) could be incorporated as a component of the model in a forward-fitting manner.
However, a direct convolution of a large map with a PSF is computationally expensive. Our first step in this study is, therefore, to build a computationally-economic forward-fitting approach to solve the problem of non-uniformity in spatial resolutions among different maps during the SED analysis.

We also aim to disentangle line-of-sight temperature structures and explore how temperature-variation affects the inference of parameters from SED models. A detailed 3D radiative-transfer analysis is generally a costly approach, especially under optically thick conditions ([Steinacker et al. (2013)] and references therein). Line-of-sight decomposition can be alternatively achieved by a 1-D integration (with self-shielding accounted being accounted for) over empirical density ([Plummer(1911)]) and temperature (polytropic, $T \propto n_{H_2}^{1-\gamma}$) profiles. Different components in our model could be eventually integrated into a single Bayesian framework with the CMZ components being separated from the fore/background in the Galactic disk.

The work presented in this dissertation is organized as follows. The observation strategy and data reduction for our AzTEC survey are discussed in Chapter 2. Also discussed in Chapter 2 is how Herschel-SPIRE/Planck-HFI/CSO-Bolocam observations are processed and included into our study. In Chapter 3 we present our SED analysis with a dust model that incorporates the knowledge of different PSFs at different wavelength bands. We focus on the optical property of dust in the CMZ and how parameters inferred from dust SEDs rely on different assumptions and priors. In Chapter 4 we develop new models and methods to explore the temperature and density structure of the CMZ clouds based on hierarchical Bayesian analyses. In Chapter 5 we draw our conclusions. Throughout this work, the distance to the Galactic center is assumed to be 8.5 kpc.
CHAPTER 2
THE AZTEC SURVEY OF THE CMZ

2.1 Observation

The AzTEC survey was conducted during ES2 for the 32 meter LMT in 2014. The survey covers the Galactic Center Region $l = [-0.7, 0.9], b = [-0.6, 0.5]$, which roughly extends from Sgr B2 to Sgr C. The target field was mosaiced by 6 square tiles in 2 different patterns, as shown in Figure 2.1. This observation strategy is adopted for two reasons. First, the CMZ has a maximum altitude of $\approx 30^\circ$ as seen from the LMT. During ES2, the telescope gain was a non-negligible function of the altitude and hence time at such low altitudes. By dividing our target field into small tiles, each tile can be calibrated with a roughly constant gain factor. Another reason is that observations with the LMT suffered from random shutdowns in scanning mode at a frequency of about 1-3 times per hour at the beginning of the season. This issue was solved later, but the survey was designed to avoid significant loss from such an accident.

Each tile is observed with a raster-scan mode, meaning that a square tile is scanned line by line, with a scanning speed of $200''/s$ and a step size of $30''$ perpendicular to the scanning direction. The parameters of all observed tiles are summarized in Table 2.1. Each pattern (with 6 tiles) takes $\approx 2$ hours to complete. The last tile is dropped when it is observed at a very low altitude ($< 28^\circ$) or missed when sunrise was approaching. In practice, we scanned the entire field once with either Pattern 1 or 2 in each night of observation, along with pointing observations. We use Sgr A* and a compact millimeter source BGPS G000.378 + 00.041 as pointing sources. Science observations
were interwoven with pointing observations between every two tiles for correcting pointing and focus offsets of the secondary mirror. The survey was conducted from Apr 17 to June 18, 2014, with a total integration time of \( \approx 20 \) hours. We obtain 9 individual maps covering the entire field, evenly distributed between Pattern 1 and Pattern 2.

As mentioned before, The altitude of the CMZ is low \((b = 28 - 32^\circ)\) as seen from the LMT. During ES2, this altitude is close to the lower-limit of altitude where the telescope could operate functionally. As a result, the effective beam size is enlarged. Since we have conducted pointing observation toward Sgr A*, which could be roughly viewed as a point source, these observations are used to determine the intrinsic beam size. We have found an intrinsic beam size of \( 9.5 \pm 0.5'' \). This value is used for unit conversion for comparison with other sets of data, i.e., conversion from Jy/beam to Jy/sr.

A primary issue with the AzTEC observations during ES2 is that the scanning speed of the telescope could deviate from what was requested by the observer. The requested speed is \( 200''/s \), which should have been a constant. However, the actual

---

**Figure 2.1.** Two different patterns of tiles used to cover the target Galactic Center Region: \( l = [-0.7, 0.9], b = [-0.6, 0.5] \). In both patterns, all tiles have an identical size. The first pattern has a tile size of \( \approx 0.57^\circ \) and the second pattern has a tile size of \( \approx 0.62^\circ \).
Table 2.1. List of individual AzTEC raster maps

<table>
<thead>
<tr>
<th>Observation ID</th>
<th>RA</th>
<th>DEC</th>
<th>Pattern</th>
<th>Dimension</th>
<th>Date</th>
<th>Observation ID</th>
<th>RA</th>
<th>DEC</th>
<th>Pattern</th>
<th>Dimension</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>019329</td>
<td>17:45:49.85</td>
<td>-29:27:20.67</td>
<td>A</td>
<td>2080″</td>
<td>04/17/2014</td>
<td>021854</td>
<td>17:41:10.99</td>
<td>-29:00:42.31</td>
<td>A</td>
<td>2080″</td>
<td>06/02/2014</td>
</tr>
<tr>
<td>019330</td>
<td>17:47:04.09</td>
<td>-29:00:42.33</td>
<td>A</td>
<td>2080″</td>
<td>04/17/2014</td>
<td>021855</td>
<td>17:45:49.85</td>
<td>-29:27:20.67</td>
<td>A</td>
<td>2080″</td>
<td>06/02/2014</td>
</tr>
<tr>
<td>019331</td>
<td>17:48:17.71</td>
<td>-28:34:01.51</td>
<td>A</td>
<td>2080″</td>
<td>04/17/2014</td>
<td>021858</td>
<td>17:43:47.89</td>
<td>-29:11:03.30</td>
<td>A</td>
<td>2080″</td>
<td>06/02/2014</td>
</tr>
<tr>
<td>019336</td>
<td>17:43:47.89</td>
<td>-29:11:03.30</td>
<td>A</td>
<td>2080″</td>
<td>04/17/2014</td>
<td>022762</td>
<td>17:47:45.70</td>
<td>-28:35:17.77</td>
<td>B</td>
<td>2245″</td>
<td>06/15/2014</td>
</tr>
<tr>
<td>019404</td>
<td>17:43:47.89</td>
<td>-29:11:03.30</td>
<td>A</td>
<td>2080″</td>
<td>04/20/2014</td>
<td>022763</td>
<td>17:46:54.75</td>
<td>-28:59:28.02</td>
<td>B</td>
<td>2245″</td>
<td>06/15/2014</td>
</tr>
<tr>
<td>019413</td>
<td>17:47:45.70</td>
<td>-28:35:17.77</td>
<td>B</td>
<td>2245″</td>
<td>04/21/2014</td>
<td>022850</td>
<td>17:45:11.76</td>
<td>-28:45:43.94</td>
<td>B</td>
<td>2245″</td>
<td>06/16/2014</td>
</tr>
<tr>
<td>019415</td>
<td>17:47:45.70</td>
<td>-28:35:17.77</td>
<td>B</td>
<td>2245″</td>
<td>04/21/2014</td>
<td>022855</td>
<td>17:46:54.75</td>
<td>-28:59:28.02</td>
<td>B</td>
<td>2245″</td>
<td>06/16/2014</td>
</tr>
<tr>
<td>021017</td>
<td>17:45:11.76</td>
<td>-28:45:43.94</td>
<td>B</td>
<td>2245″</td>
<td>05/19/2014</td>
<td>022924</td>
<td>17:47:44.09</td>
<td>-29:00:42.33</td>
<td>A</td>
<td>2080″</td>
<td>06/17/2014</td>
</tr>
<tr>
<td>021021</td>
<td>17:46:16.41</td>
<td>-28:17:52.43</td>
<td>A</td>
<td>2080″</td>
<td>05/19/2014</td>
<td>022926</td>
<td>17:47:04.09</td>
<td>-29:00:42.33</td>
<td>A</td>
<td>2080″</td>
<td>06/18/2014</td>
</tr>
</tbody>
</table>

scanning speed oscillated between 50″/s and 400″/s for the first 10 hours integration time due to an un-noticed problem with the motors driving the telescope. At a scanning speed > 100″/s, the beam size is elongated along the scanning direction due to sparse sampling. The elongation is further amplified through down-sampling and low-pass filtering during data processing. An unfortunate consequence of this speed variation is a variable beam profile in the product maps. Our simulation shows that at scanning speeds of 200, 300 and 400″/s, an 8 beam is elongated to 8.7″, 9.6″ and 10.9″, respectively. In practice, we can compare the raster maps with the pointing observations toward Sgr A*. The latter are obtained with Lissajous scanning patterns with a low scanning speed, 50/s, as shown in Figure 2.2. The elongated beam size along the scanning direction is 10.5 on average. Since every pixel is scanned in eight different directions, we use 10.5″ as a rough estimate for the effective resolution for the product map. Notice that this larger 10.5″ beam size is due to under-sampling, which differs from the beam size used for flux calibration (unit conversion from Jy/beam to Jy/sr). The latter is determined from pointing observations, which is carried out with slow scanning speed, thus unaffected by under-sampling, and is found to be ≈ 9.5″, as mentioned before.
Figure 2.2. A comparison between a rapidly scanned raster map (pixel size=3") and a slowly scanned Lissajous map (pixel size=0.75"). The sampling rate is $v_{\text{scan}} \approx 200"/\text{s}$ for the raster map and $v_{\text{scan}} = 50"/\text{s}$ for the Lissajous map. The elongation of the beam along the scanning direction is clearly seen in the raster map. In this single case, the beam size along the scanning direction is elongated to 10.5", which is consistent with our simulation.

Another issue with our observations is associated with decreasing telescope gain at low altitudes. Flux calibrations for AzTEC images are performed on the pipeline level. A certain type of observation (named “Beammap”) for calibration purpose on planet/asteroid calibrators is carried out about 2-3 times per night of AzTEC observation. At the end of each season, these Beammaps are collected to build a calibration curve, which is implemented into the pipeline. However, during ES2, we do not have enough Beammaps observed at low latitudes ($\approx 30^\circ$), mainly due to a lack of availability toward the end of the night, when the CMZ was observed. As a result, the telescope gain is overestimated because it is mainly determined by extrapolation from calibrators observed at higher altitudes. This problem can be easily perceived from systematically decreasing flux in the pointing maps toward lower altitudes. In 2016, during Early Science Cycle 4 (ES4) for the LMT, we re-observed the CMZ along with two pointing sources, Sgr A* and BGPS G000.378+00.041. The telescope gain is significantly improved in ES4, and shows a flat gain curve around 30°. It is
found that the ES2 observations are under-calibrated by 30% comparing to the ES4 observations. Therefore, we multiply our ES2 maps by a factor of 1.30 to correct for the unaccounted latitude-dependent gain factor.

We eventually obtain a $1\sigma$ noise level of about 20 mJy/beam after 20 hours integration, this is significantly higher than what is proposed for this project, 5 mJy/beam, partially due to poor weather conditions and partially due to loss of sensitivities at low altitudes during ES2.

2.2 Data Reduction

2.2.1 Iterative Cleaning Based on Principle Component Analysis

The raw data were reduced using the standard AzTEC analysis pipeline, as described by [Scott et al.(2008)]. We use iterative Principle Component Analysis (PCA) to remove correlated signals among bolometers, which are primarily contributed by atmospheric emissions, emissions from the telescope itself and non-Gaussian noises associated with the secondary mirror and back-end instruments. Signals projected onto the highest $n_{\text{eig}}$ ranked eigenvectors in bolometer-bolometer space are viewed as non-astronomical signals and are removed, where $n_{\text{eig}}$ is a user-defined integer. Since astronomical signals of interests corresponding to extended structures are also correlated to a certain level, which is indistinguishable from non-astronomical correlated signals, a more aggressive (larger $n_{\text{eig}}$) PCA cleaning removes simultaneously more non-astronomical and astronomical signals. To compensate for this loss of information, our cleaning is performed iteratively, until the rms in the final noise map is consistent with no astronomical signal above $2.5\sigma$. The cleaning procedure is as follows:

1) PCA cleaning. The highest $n_{\text{eig}}$ ranked eigen-components of signals are removed. A cleaned map with a pixel scale of 3" is created and slightly smoothed
with a Gaussian kernel of FWHM= 6″.

2) Pixels above 2.5σ significance in the cleaned map are identified and are preserved. Here, the noise level is predicted from bolometer sensitivities, which are modeled based on a set of observations toward planet & asteroid calibrators throughout the season. In practice, we find that jackknifed noise realizations (constructed by multiplying each time-stream by ±1 to suppress astronomical signals, see Section 2.2.3.) indicate systematically higher noise, but within 120% of this empirical noise level, which suggests a actual S/N threshold of 2.2 – 2.5. The jackknifed noise realization provides more conservative estimates. However, this realization was not directly implemented on the pipeline level for the iterative PCA cleaning, hence we use noise predicted from bolometer sensitivities for this particular reduction procedure. In post-reduction analyses, we instead use noise estimates from jackknifed noise realization.

3) Every pixel is zero-valued except pixels above the 2.5 threshold and their neighboring pixels within an aperture of 9″. This map is named “total map”.

4) The total map is cast back to the time-stream and is subtracted from the raw data, the residuals are PCA cleaned and gridded to create a “residual map”, which is then added to the current total map. This latter product is named “current map”.

5) Any new pixels above 2.5σ in the residual map are preserved and are added to the total map.

6) If at Step 5), no more than 0.3% new pixels are found to be above 2.5σ threshold, the convergence is assumed to be reached and the current map is our final product.
We start iteration with an aggressive PCA cleaning, with \( n_{\text{eig}} \), \( j_{\text{of cut}} = 8 \), and reduce \( n_{\text{eig}} \), \( j_{\text{of cut}} \) by 1 at the beginning of each iteration down to a minimum of \( n_{\text{eig}} \), \( j_{\text{of cut}} = 2 \). Eventually, a total of 40 iterations are performed until convergence.

The product AzTEC map is shown in Figure 2.3, which has a pixel scale of 3′. For this study, the Sgr B2 area is excluded from analysis since its flux peak is saturated.

**2.2.2 Signal Decomposition with Linear Regression**

An alternative approach to separate astronomical signals from atmospheric and instrumental emissions is based on linear regression. The astronomical signals are
recurring signals at fixed locations in the sky. In principle, we can further use a set of basis function to model the atmospheric and instrumental emissions.

Suppose we have a total of $s$ time-stream scans, each scan is a 1D vector of raw signals. For the $k$th time-stream scan $d^k$, we have:

$$d^k = P^k m + B \alpha^k + n^k$$  \hspace{1cm} (2.1)$$

$d^k$ is a $r \times 1$ vector, $r$ is the total number of sampled signals (i.e. the length of the time-stream). $P^k$ is a $r \times w$ inverse-pointing matrix to project a sky map back into a time-stream. $w$ is the total number of pixels in the map. The $w \times 1$ vector $m$ is the vector of astronomical signals. The $t \times 1$ vector $\alpha^k$ is the weight vector of basis functions, $t$ is the total number of basis functions. $n^k$ is the $r \times 1$ noise vector. So we have:

$$
\begin{pmatrix}
    d^1 \\
    d^2 \\
    \vdots \\
    d^s
\end{pmatrix} =
\begin{pmatrix}
    (P^1, B) \\
    (P^2, B) \\
    \vdots \\
    (P^s, B)
\end{pmatrix}
\begin{pmatrix}
    m \\
    \alpha^1 \\
    m \\
    \alpha^2 \\
    \vdots \\
    m \\
    \alpha^s
\end{pmatrix}
+ 
\begin{pmatrix}
    n^1 \\
    n^2 \\
    \vdots \\
    n^s
\end{pmatrix}.
$$  \hspace{1cm} (2.2)$$

For convenience, we use $d$ to denote the vector on the left side of the above equation, use $A$ to denote the block matrix, use $v$ to denote the vector multiplied by the matrix and use $n$ to denote the last vector. So we have:

$$d = Av + n.$$
To get the maximum-likelihood estimate of \( \mathbf{v} \) we consider

\[
\chi^2 = (\mathbf{d} - \mathbf{Av})^t(N)^{-1}(\mathbf{d} - \mathbf{Av})
\]

where \( N = \mathbb{E}[\mathbf{mm}^t] \). We want to find the \( \alpha^1, \ldots, \alpha^s \) and \( \mathbf{m} \) which minimize \( \chi^2 \).

Since \( \mathbf{n}^1, \ldots, \mathbf{n}^s \) are independent, we have

\[
N^{-1} = \begin{pmatrix}
(N^1)^{-1} \\
(N^2)^{-1} \\
\vdots \\
(N^s)^{-1}
\end{pmatrix}
\]

(2.3)

where \( N^i = \mathbb{E}[\mathbf{n}^i(\mathbf{n}^i)^t] \). Therefore by straight computation we have:

\[
\chi^2 = \sum_{k=1}^s (\mathbf{d}^k - (P^k \mathbf{m} + B\alpha^k))^t(N^k)^{-1}(\mathbf{d}^k - (P^k \mathbf{m} + B\alpha^k))
\]

(2.4)

Suppose:

\[
\alpha^k = \begin{pmatrix}
\alpha^k_1 \\
\alpha^k_2 \\
\vdots \\
\alpha^k_t
\end{pmatrix}
\]

and:

\[
\mathbf{m} = \begin{pmatrix}
m_1 \\
m_2 \\
\vdots \\
m_w
\end{pmatrix}
\]

From \( \frac{\partial \chi^2}{\partial \alpha^k_t} = 0 \) we have

\[
B^t(N^k)^{-1}(\mathbf{d}^k - (P^k \mathbf{m} + B\alpha^k)) = 0
\]

(2.5)
for all \( k \). From \( \frac{\partial \chi^2}{\partial m_i} = 0 \) we have

\[
\sum_{k=1}^{s} (P^k)^t(N^k)^{-1}(d^k - (P^k m + B\alpha^k)) = 0.
\]  

(2.6)

By (Eq 2.5) we have

\[
\alpha^k = \left[ B^t(N^k)^{-1}B \right]^{-1}B^t(N^k)^{-1}(d^k - P^k m).
\]  

(2.7)

Plugging (Eq 2.7) into (Eq 2.6) we have

\[
m = \Phi^{-1} \left\{ \sum_{k=1}^{s} (P^k)^t(N^k)^{-1} \left[ I_{r_x r} - B[B^t(N^k)^{-1}B]^{-1}B^t(N^k)^{-1} \right] d^k \right\}
\]  

(2.8)

where

\[
\Phi = \sum_{k=1}^{s} (P^k)^t(N^k)^{-1} \left[ I_{r_x r} - B[B^t(N^k)^{-1}B]^{-1}B^t(N^k)^{-1} \right] P^k.
\]  

(2.9)

The performance of this technique relies on whether the chosen basis functions \( B \) correctly characterize the patterns of the atmospheric and instrumental emissions. Unfortunately, during ES2, we do not have a good choice for the basis functions. Furthermore, we identified recurring instrumental emissions with a frequency similar to that of the raster scanning pattern, which are misidentified as astronomical signals. It was not clear to us how to remove this component. Therefore, we decided to use the iterative PCA cleaning for signal decomposition throughout this study.

### 2.2.3 Noise Maps

After the last iteration of the PCA cleaning, a noise map is generated from residual signals. Typically, there is still a small, but a visible fraction of un-recovered extended emission in this residual map, these features can be further suppressed by multiplying each time-stream scan of residual signals (here, a scan is defined as all
samples between any two adjacent turning points in a raster pattern) by randomly +1 or −1. This procedure further removes middle-to-large correlated signals in the scan which survives PCA cleaning. The local standard deviation for each pixel is then calculated within a box of ±1′ centered at it. The standard deviation derived in such a way is systematically higher than, but within 120% of what is predicted from bolometer sensitivities, which is not unexpected, since we use a less aggressive PCA cleaning for our map compared to what we used to reduce calibration map, correlated noise corresponding to lower rank eigen-components is retained. The varying and high scanning speed throughout this set of observations should also contribute to the deviation from common sensitivity.

### 2.2.4 Correction for Pointing offsets and zero-flux offsets between Individual maps

Pointing offsets are corrected in two stages, a pipeline stage, and a post-pipeline stage. In the pipeline, the pointing offset of each map is estimated from the most recent pointing observation (for this project, it is either toward Sgr A* or toward BGPS G000.378 + 00.041). In the post-pipeline stage, after each individual raster map is PCA cleaned and reduced separately, the optimal pointing offsets and zero-flux offsets are obtained by minimizing connecting difference among individual maps. More specifically, we are minimizing:

$$\chi^2 = \sum_{i,j} \frac{\text{off}(i) - \text{off}(j) + \text{diff}_{(ij)}}{\sigma_{(ij)}}$$  \hspace{1cm} (2.10)

where \text{diff}_{(ij)} stands for relative offset in either position or flux between two adjacent maps, the $i$th and the $j$th map. \text{off}(i) and \text{off}(j) are the absolute offsets to be solved. This is achieved by solving:

$$\frac{\partial \chi^2}{\partial \text{off}(i)} = 0$$  \hspace{1cm} (2.11)
A more detailed description of this approach could be found in [Dong et al.(2011)].

For pointing offsets, each $\text{diff}_{(ij)}$ in eq 2.10 is derived by cross-correlating a pair of two adjacent raster maps $(i,j)$. Before cross-correlation, both maps are interpolated to a common grid, pixels below $3\sigma$ significance are zero-valued to exclude correlated noise. The cross-correlated image is convolved with a Gaussian kernel of FWHM $= 10.5''$ before peak-finding. Following [Kurtz & Mink(1998)], we set the uncertainty $\sigma_{(ij)}$ of the cross-correlated peak to $\frac{3w}{8(1+r)}$, where $w$ is the FWHM of the correlation peak and $r$ is the ratio of the correlation peak height to the amplitude of the noise.

For relative flux offsets, similarly, two adjacent maps $(i,j)$ are first interpolated to a common grid. In this case, $\text{diff}_{(ij)}$ is the mean flux-offset in overlapped pixels. $\sigma_{(ij)}$ is proportional to Poisson noise, $\sqrt{n_{\text{overlapped}}}$, where $n_{\text{overlapped}}$ is the number of overlapped pixels.

The remaining median pointing offset is estimated to be $3.2''$ for each raster map after being corrected by pointing observations, and the median zero-flux offset is $1$ mJy/beam.

### 2.2.5 Processing of Herschel PACS/SPIRE maps

To construct dust SEDs, we take advantage of existing Herschel PACS/SPIRE 160 $\mu$m, 250 $\mu$m, 350 $\mu$m and 500 $\mu$m maps from the Hi-GAL survey [Molinari et al.(2010)]. The PACS 70 $\mu$m map is excluded. The CMZ clouds are dense, and even FIR dust emission can be self-shielded. In Herschel maps it is visually apparent that typical CMZ clouds are dark at 70 $\mu$m, in contrast with their bright surrounding medium, indicating that 70 $\mu$m emission originates from warm diffuse components that are spatially distinct from massive clouds.

By default, the Herschel and Planck data calibrate the flux densities assuming an artificial power-law spectrum ($S \propto \nu^{-1}$). Color corrections must be performed to obtain correct monochromatic flux densities in each band. Furthermore, SPIRE
maps are internally calibrated to units of Jy/beam. The beam area is not constant across a broad wavelength band. For Herschel maps, we downloaded the Photometer Calibration Products from the ESA Herschel Science Archive, which contains filter responses and aperture efficiencies for each band. The 160 $\mu$m - 1.1 mm maps are first fitted with a modified black-body model, pixel by pixel, without color correction. The color correction factor for each pixel is then calculated from the best-fitted SED by convolving it with the product of the filter response and the aperture efficiency. Further iteration is not needed since the color corrections are insensitive to temperature and $\beta$ variation across the map. The standard deviation of the color corrections across the CMZ region is at most 2.5%, in the PACS 160 $\mu$m band. It is verified that for given spectral shapes (parameterized by temperature T and spectral index $\beta$), our inferred color corrections are consistent with tabulated values in the PACS and SPIRE handbooks.

The errors in the Herschel/Planck maps are dominated by calibration uncertainties, which could be divided into relative calibration uncertainties and absolute calibration uncertainties. In some existing works, the absolute calibration uncertainties are directly added to the relative calibration uncertainties, pixel-by-pixel, despite that the absolute uncertainties should be correlated across each map and all bands of each instrument. In Section 3.8, it will be demonstrated by simulation that, while absolute calibration uncertainties could be appropriately accounted for by modeling the calibration offsets as additional parameters for each map [Kelly et al.(2012)], such statistically proper treatment, however, could lead to questionable results, if a wrong assumption about dust absorption curve is made. A widely adopted, but a potentially problematic assumption is that a single power-law could approximate the wavelength dependence of the dust absorption curve. Besides, the Hi-GAL data has been cross-calibrated with Planck and IRAS [Molinari et al.(2016)]. We choose to ignore any remaining absolute calibration uncertainty. The main reason is that it is difficult
to differentiate between the calibration uncertainties and the model uncertainty of the dust absorption curve $\kappa_\nu$. We adopt a relative calibration uncertainty of 2\% for all SPIRE bands, and a relative uncertainty of 5\% for the PACS 160 $\mu$m band [Bendo et al.(2013), Balog et al.(2014)].

The errors in the 1.1 $mm$ compound map are even more complicated. In addition to the statistical and calibration uncertainties in each component map, the combining procedure of the maps, the removal of the CO J=3-2 contribution, all introduce further uncertainties to the 1.1 $mm$ map. In this work, we arbitrarily adopt a 10\% relative calibration uncertainty for the 1.1 $mm$ compound map, roughly estimated from the uncertainty of the beam area in the AzTEC map, plus an additional 20 mJy/beam random noise, estimated from the jackknifed noise realization of the AzTEC map.

### 2.2.6 Processing of Planck HFI maps

We create a compound 1.1 $mm$ map from the AzTEC 1.1 $mm$ map, the Planck/HFI 353 GHz map (Planck 2013 data release (PR1)) and the CSO/Bolocam 1.1 $mm$ map [Aguirre et al.(2011), Ginsburg et al.(2013)], to compensate for the large scale emission filtered out by the PCA cleaning in the AzTEC map. The Planck/HFI (353GHz or 850$\mu$m) map is scaled to 1.1mm to match the wavelength of the AzTEC & Bolocam maps before merging. We apply uniform color correction and scaling factor from 850 $\mu$m to 1.1 $mm$ for the entire map, assuming $T = 25 K$ and $\beta = 1.8$, which are inferred from a fitting to high latitudes extended emissions. For density peaks in the CMZ, $T$ and $\beta$ could be potentially different, however, the combined flux is predominantly contributed by the AzTEC map and the Bolocam map on density peaks, the variation of Planck/HFI color correction and scaling factor propagate to a smaller uncertainty (< 1\%).
Figure 2.4. Planck/HFI map of the CMZ after contamination from CO J=3-2 is removed, stripe-like artifacts are produced during this procedure.

Besides, the Planck 353 GHz map is contaminated by CO J=3-2 emission. In the CMZ, this contamination is particularly significant since kinematic temperatures are high. A set of CO J=3-2 correction maps is available from the Planck archive. Here we use their Type 1 correction map, which is based on the precise knowledge of the difference in filter response among individual bolometers, without any further assumption. This estimated CO J=3-2 map was downloaded from the Planck Legacy Archive and subtracted from the original map. The CO J=3-2 line emission contributes up to 30% of the integrated flux over the entire band. The cleaned Planck map after the removal of the CO contribution is shown in Figure 2.4, where large-scale, artificial stripes introduced by this process are visible.

To create a compound map at 1.1mm, we first merge the AzTEC map and the Bolocam map:
\[ M(AB) = M(A) + [M(B) - (M(A) \otimes \text{beam}(B - A))] \quad (2.12) \]

We degrade the resolution of the AzTEC map \( M(A) \) (10.5\arcsec) to that of the Bolocam map \( M(B) \) (33\arcsec), where \( \otimes \) stands for convolution, and \( \text{beam}(B - A) \) stands for a Gaussian kernel with \( \sigma_{B-A} = \sqrt{(33^2 - 10.5^2)^\arcsec} \). The degraded AzTEC map is subtracted from the Bolocam map. This differential map with an “excess” component at the scale of the Bolocam beam (33\arcsec) is added back to the original AzTEC map to produce a combined map \( M(AB) \).

Similarly, to further incorporate the Planck map:

\[ M(ABP) = M(AB) + [M(P) - (M(AB) \otimes \text{beam}(P - B))] \quad (2.13) \]

Again, the “excess” component at the scale of the Planck beam (29\arcsec) is added back to the combined AzTEC+Bolocam map. As mentioned before, we arbitrarily adopt a 10\% relative calibration uncertainty for the compound map, roughly estimated from the uncertainty of the beam area in the AzTEC map. The relative calibration uncertainties of the Bolocam and the Planck/HFI maps are smaller. The 1 – \( \sigma \) statistical noise is 20 mJy/beam in the AzTEC map, the statistical noises in the Bolocam and the Planck/HFI map are relatively negligible.
CHAPTER 3
MODELING DUST PROPERTIES WITH BAYESIAN ANALYSIS

In this chapter, we carry out a Bayesian analysis of the dust SEDs from the CMZ to explore the optical properties of dust grains. To start with, we first present a forward modeling strategy to fit a dust model to multi-band maps, each diluted by a different instrumental PSF. The goal is to achieve higher spatial resolutions. We start with a simple model. The dust SEDs follow a standard modified black-body radiation with a single temperature approximation, the wavelength-dependent dust absorption curve $\kappa_\lambda$ is assumed to be a single power-law, characterized by a spectral index $\beta$. This model is referred to as STMB hereafter.

3.1 Dust Model

An STMB model for the entire CMZ relies on three cellular-based parameter grids: a temperature grid $T$, a column density grid $N_{H_2}$ and a grid of the dust spectral index $\beta$. The cellular size of all parameter grids is set uniformly to 14”, the choice of this number is discussed in Section 3.3. The surface brightness $F_i(\nu_j)$ at pixel(i) and frequency $\nu_j$ is given by:

$$F_i(\nu_j) = [1 - \exp(-\tau_{i,\nu_j})]B_{\nu_j}(T_i)\Omega_j$$  \hspace{1cm} (3.1)

where $\Omega_j$ is the beam area in the jth band. $B_{\nu_j}(T_i)$ is the Planck function. $\tau_{i,\nu_j}$ is the optical depth at frequency $\nu_j$, which is given by:
\[ \tau(i, \nu_j) = \kappa_0 \left( \frac{\nu_j}{\nu_0} \right)^{\beta} \mu m_H \times N_{H_2} \times 1\% \quad (3.2) \]

where \( \kappa_0 \) is the absorption cross section per unit mass at frequency \( \nu_0 \). We adopt \( \kappa_0 = 1.37 \text{ cm}^2/\text{g} \) and \( \nu_0 = c/1000 \text{ \mu m} \) from [Ossenkopf & Henning(1994)] for coagulated dust grains with thin ice mantles (their Table 1). We also adopt a mean molecular weight \( \mu = 2.8 \) and a dust-to-gas mass ratio of 1% to convert from \( N_{H_2} \) to column dust mass density. This model is not restricted to an optically thin approximation \( (\kappa \propto \nu^\beta) \).

The raw flux map \( F(\nu_j) \) calculated above is diluted to the instrumental resolution of each wavelength band to match the data:

\[ \text{Model}(\nu_j) = F(\nu_j) \otimes \text{beam}_j \quad (3.3) \]

where \( \otimes \) refers to convolution. All beams profiles are approximated as Gaussian profiles. The Full Width Half Maximum (FWHM) of the beams are 12" at 160\( \mu \text{m} \), 18" at 250 \( \mu \text{m} \), 25" at 350 \( \mu \text{m} \), 36" at 500 \( \mu \text{m} \) and 10.5" at 1.1 \( \text{mm} \), respectively.

### 3.2 MCMC Analysis: Sampling Strategy

We perform Markov chain Monte Carlo (MCMC) analysis to derive optimal parameters for our model, using a Slice-within-Gibbs sampling strategy. In statistics, Gibbs sampling is a sampling strategy to obtain samples from the joint distribution of a posterior \( P(x_1, x_2, x_3...) \) by sampling from the full conditional posterior for each parameter \( x_j \) in turn. Meaning that, if we start from the \( i \)th sample \( (x_1^i, x_2^i, x_3^i...) \) of a posterior distribution \( P(x_1, x_2, x_3...) \), the \( i+1 \)th sample \( (x_1^{i+1}, x_2^{i+2}, x_3^{i+3}...) \) is obtained by sampling from each conditional posterior \( P(x_i | x_1^i, x_2^i, ...x_{i-1}^i, x_{i+1}^i, x_{i+2}^i...) \) in turn.
In this way, sampling from a multi-dimensional posterior distribution is reduced to sampling from multiple 1-D conditional posterior distributions.

Following [Neal R. M. (2003)], we further use slice sampling with a stepping out and shrinkage strategy to sample from the full conditional posterior of each parameter $(T_{ix,iy}/N_{H2,ix,iy}/\beta_{ix,iy})$. This procedure is composed of two steps, stepping out, which is illustrated by Figure 3.1, and shrinkage, which is illustrated by Figure 3.2.

The log-posterior $lnP(x)$ for $x \in (T_{ix,iy}, N_{H2,ix,iy}, \beta_{ix,iy})$ is given by:

$$lnP(x) \propto \sum_{j,ix,iy} \frac{(Mod(x,ix,iy,\nu_j) - Map(ix,iy,\nu_j))^2}{2 \times \sigma_{ix,iy,\nu_j}^2} \quad (3.4)$$

where $j$ refers to the $j$th band, $Mod(x,\nu_j) = F(x,\nu_j) \otimes beam_j$. In practice, one can avoid a full convolution of $F(\nu_j)$ with $beam_j$ when updating each parameter. Instead, if the current model map $Mod(x_i,\nu_j)$ is recorded, $Mod(x_{i+1},\nu_j)$ can be
Figure 3.2. The shrinkage procedure in slice sampling, adapted from [Neal R. M. (2003)].

Input: \[ \log P = \text{The natural logarithm of the full conditional posterior of parameter } i \]
\[ x_0 = \text{the current value of parameter } i \]
\[ y = \text{a vertical line defining the slice} \]
\[ (L, R) = \text{the interval to sample from} \]

Output: \[ x_1 = \text{the new point} \]

Here exponential(-1) is exponential distribution \( p(x)dx = \exp(-x)dx \)

\[ L' = L, \quad R' = R \]

Repeat:
\[ U \sim \text{Uniform}(0, 1) \]
\[ x_1 = L' + U(R' - L') \]
\[ e \sim \text{Exponential}(-1) \]
\[ y = \log P(x_0) - e \]
\[ \text{if } y < \log P(x_1): \text{Accept } x_1 \text{ and exit loop} \]
\[ \text{if } x_1 < x_0 \text{ then } L' = x_1 \text{ else } R' = x_1 \]

The advantage of the Gibbs sampling strategy is that, since one single parameter in a single cell is updated per step, we can avoid frequent convolutions of the entire image with its PSF, which is computationally expensive. Instead, the global likelihood is modified within an area of \( 4\sigma \) around the cell to be sampled at each step, in each map. Furthermore, a new updating is always accepted by a Gibbs sampler, while a multivariate random-walk Metropolis updating with high dimensionality could have meager acceptance rate and low sampling efficiency. We have developed a C++ package to perform this MCMC analysis. The sampler is run for 6000 steps for each parameter. The convergence is examined by both trace plots and auto-correlations, which is rapidly achieved after 1000 steps. The optimal value of each parameter is derived by the median value of the last 3000 steps.
Our procedure shares the same advantage of the PPMAP procedure applied to previous Hi-GAL surveys [Marsh et al.(2015)], meaning that spatial resolutions of the best-fitted parameter grids are improved by incorporating the PSF knowledge into the Model. Although we cannot computationally afford to have a large number of independent temperature components along each line of sight, which can be achieved in the PPMAP, we are not restricted to optically thin conditions assumed by the PPMAP in order to retain the linearity and solvability of the model. By using a Gibbs sampler, we can avoid convolving the entire map frequently, and an optically thin condition is not necessarily required.

3.3 Smoothness Prior

Depending on the cellular size of the parameter grid relative to the sizes of the PSFs, overfitting could occur, which manifests as high-frequency fluctuations among neighboring cells in the best-fitted parameter maps. For $T$ and $\beta$, overfitting is more likely, since both parameters are constrained by the entire SED. On the other hand, $N_{H_2}$ is predominantly determined by the 1.1mm map, which has the highest spatial resolution. This overfitting issue due to the incorporation of the PSF knowledge into the model could be reduced by regularized Bayesian inference [Warren & Dye(2003)]. We choose the simplest gradient form of regularization, which can be expressed as:

$$\ln P(x_{ix,iy}) = \ln P(x_{ix,iy}) + \lambda G$$

(3.5)

$$G = \sum_{+, -} \frac{(x_{ix,iy} - x_{ix\pm1,iy})^2 + (x_{ix,iy} - x_{ix,iy\pm1})^2}{2}$$

(3.6)

where $\ln P(x_{ix,iy})$ is the logarithmic of the full conditional posterior for parameter $x_{ix,iy}$, where $x$ can be $N_{H_2}/T/\beta$. The regularization term $G$ can be regarded as a prior specifying the smoothness of parameter $x$. $\lambda$ is a user-defined parameter controlling the weight of the smoothness prior. One can rewrite $\lambda$ as $\lambda = \frac{1}{2\sigma^2}$, where $\sigma$ could be
Table 3.1. Parameters and Smoothness Prior for Different Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Free Parameters</th>
<th>Prior</th>
</tr>
</thead>
<tbody>
<tr>
<td>STMB</td>
<td>$lg(N_{H_2})$, $ln(T)$, $\beta$</td>
<td>$\sigma_\beta = 0.2$</td>
</tr>
<tr>
<td>STMB with fore/background subtracted</td>
<td>$lg(N_{H_2})$, $ln(T_{cmz})$, $\beta_{cmz}$</td>
<td>$\sigma_{lg(N_{H_2}[cm^{-2}])} = 0.2$</td>
</tr>
<tr>
<td>STMB with a multivariate prior</td>
<td>$lg(N_{H_2})$, $ln(T)$, $\beta$, $\mu$, $\Sigma$</td>
<td>$(ln(T), \beta) \sim \text{Student}(\mu, \Sigma)$, $\sigma_{lg(N_{H_2}[cm^{-2}])} = 0.1$</td>
</tr>
<tr>
<td>MTMB with fore/background subtracted</td>
<td>$n_0$, $T_0$, $\gamma$, $\mu$, $\Sigma$</td>
<td>$\sigma_{n_0} = 0.2$, $\sigma_{T_0} = 0.2$, $\sigma_{\mu_{cm^{-2}}} = 100$</td>
</tr>
<tr>
<td>MTMB with a multivariate prior</td>
<td>$lg(N_{H_2})$, $ln(T)$, $\beta_1$, $\beta_2$, $\lambda_{bk}$</td>
<td>$\sigma_{n_0} = 0.2$, $\sigma_{T_0} = 0.2$, $\sigma_{\mu_{cm^{-2}}} = 100$</td>
</tr>
<tr>
<td>TLS model</td>
<td>$lg(N_{H_2})$, $ln(T)$</td>
<td>$\sigma_{ln(T[K])} = 0.1$</td>
</tr>
</tbody>
</table>

(a) Smoothness prior characterized by a standard deviation between two adjacent cells

viewed as an \textit{a priori} mean standard deviation for all adjacent pairs in the parameter grid cell. In order to obtain better performance, $\sigma$ or $\lambda$ could be trained using numerical simulations or interferometer observations. In this study, we focus on large scale structures and use arbitrary, but reasonably weak priors.

Figure 3.3 shows how the spatial resolution is improved with PSF being modeled and regularized by a smoothness prior. The two objects in the images are the 20 km/s cloud (right) and the 50 km/s cloud (left). In this analysis, the 160 $\mu m$-1 $mm$ maps are interpolated to with a common grid with a pixel scale of 8$''$. The smoothness priors are set to: $\sigma_{lg(N_{H_2}[cm^{-2}])} = 0.1$, $\sigma_{ln(T[K])} = 0.1$ and $\sigma_\beta = 0.1$. Improvement of resolution in $N_{H_2}$ is clearly seen, compared to the results derived with all maps degraded to the lowest Herschel 500 $\mu m$ resolution (36$''$).

To analyze the entire CMZ region, we adopt a larger cellar/pixel size, 14$''$, and a prior on $\beta$ only, with $\sigma_\beta = 0.2$, without any prior to $T$ and $N_{H_2}$. These priors are moderately weak, only to avoid strong fluctuations among neighboring cells. Later in this study, we will constantly use this technique to relieve the problem of over-parameterization for different models. The smoothness priors for different models are summarized in Table 3.1.

### 3.4 Hierarchical Bayesian Model

A common problem encountered in physical modeling is parameter degeneracy. The global distribution of the estimated parameters could be viewed as a convolu-
Figure 3.3. Comparison between best fitted $N_{H_2}$, $T$ and $\beta$ derived with PSFs being modeled (left), and those derived by degrading 160 $\mu m$-1.1 $mm$ maps to the lowest spatial resolution (HPBW(500$\mu$m) = 36$''$).
tion of their natural distribution with the probability distributions of their estimated values propagated from measurement uncertainties. Given that measurement uncertainty is always present, parameter degeneracy leads to correlated probability distributions, which dilute the apparent distribution of the best-fitted parameters toward a false correlation. In SED analysis with an STMB model, $T$ and $\beta$ are known to have a strong degeneracy, which manifests as a banana-shaped posterior distribution. Recovering the intrinsic $T - \beta$ distribution can be difficult if it is much more peaked than the posterior distribution. [Juvela et al.(2013)] examine several existing techniques aiming to explore the intrinsic $(T, \beta)$ relation and conclude that all techniques suffer from some bias.

Hierarchical Bayesian Analysis has been proposed to recover the intrinsic correlation between $T$ and $\beta$, by implementing the natural distribution of parameters as a prior to the model [Kelly et al.(2012), Galliano(2018)]. Following [Kelly et al.(2012)], we adopt a multivariate Student-t distribution as a prior for the $\ln(T) - \beta$ distribution. The posterior for the $i$th grid cell can be written as:

$$P(ln(T_i), \beta_i|D) = P(D|ln(T_i), \beta_i)P(ln(T_i), \beta_i|\mu, \Sigma)$$

$$P(ln(T_i), \beta_i|\mu, \Sigma) \propto \frac{1}{|\Sigma|^{1/2}} \times [1 + \frac{1}{d}(x_i - \mu)^T \Sigma^{-1}(x_i - \mu)]^{-(d+2)/2}$$

$$x_i = (\ln(T_i), \beta_i)$$

where $D$ is the data, $\mu$ is the global mean of $(\ln(T_i), \beta_i)$. $\Sigma$ is the covariance matrix of $(\ln(T_i), \beta_i)$. Following [Kelly et al.(2012)], the number of the degrees of freedom $d$ is set to 8. The covariance matrix can be decomposed as:

$$\Sigma = SRS$$
Figure 3.4. Distributions of $T$ and $\beta$ recovered from a simulated set of SEDs, using hierarchical (coral pink) and non-hierarchical (black) Bayesian analysis. The SEDs are simulated from a 3x3 multivariate normal distribution of $(lg(N), ln(T), \beta)$, with $(\mu_{lg(N)}^{(cm^{-2})} = 22.5, \mu_{ln(T[K])} = ln(20), \mu_\beta = 2), (\sigma_{lg(N)}^{(cm^{-2})} = 0.2, \sigma_{ln(T[K])} = 0.2, \sigma_\beta = 0.1)$ and $(\rho_{lg(N),ln(T)} = -0.5, \rho_{lg(N),\beta} = 0.5, \rho_{ln(T),\beta} = 0.3)$. The black dots mark values estimated from a non-hierarchical Bayesian model, which show a weak anti-correlation, the coral pink dots mark values estimated from a hierarchical Bayesian analysis, with $(ln(T), \beta)$ following a prior of a 2x2 multivariate Student-t distribution. The green contours correspond to the simulated marginal distribution of $(ln(T), \beta)$.
where $S$ is the diagonal matrix of the standard deviations and $R$ is the correlation matrix.

With $P(lg(N_i), ln(T_i), \beta_i|\mu, \Sigma)$ as an additional prior, we have 5 hyperparameters: $(\mu_{ln(T)}, \mu_\beta, \sigma_{ln(T)}, \sigma_\beta$ and $\rho_{ln(T),\beta})$, which are sampled along with $(lg(N_i), ln(T_i), \beta_i)$ using a slice-within-Gibbs strategy. Since the covariance matrix $\Sigma$ is a 2x2 matrix, it is always positive-definite as long as the correlation coefficient $-1 < \rho_{ln(T),\beta} < 1$.

We can simply place a uniform prior on $\rho_{ln(T),\beta}$ between $-1$ and $1$, we further place uniform priors on the rest of parameters: $\mu_{ln(T)} \sim U(ln(5), ln(60)), \mu_\beta \sim U(0.5, 3.0), \sigma_{ln(T)} \sim U(0.02, 0.4), \sigma_\beta \sim U(0.02, 0.4)$.

An illustration of hierarchical Bayesian analysis is shown by Figure 3.4. Here, we simulate a sample of 22400 dust SEDs from a 3x3 multivariate normal distribution, with $(\mu_{lg(N_{cm^{-2}})} = 22.5, \mu_{ln(T[K])} = ln(20), \mu_\beta = 2), (\sigma_{lg(N_{cm^{-2}})} = 0.2, \sigma_{ln(T[K])} = 0.2, \sigma_\beta = 0.1)$ and $(\rho_{lg(N),ln(T)} = -0.5, \rho_{lg(N),\beta} = 0.5, \rho_{ln(T),\beta} = 0.3$. The signal-to-noise ratios are identical to our observed data. While $\ln(T)$ and $\beta$ are simulated to have a positive correlation coefficient $\rho_{ln(T),\beta} = 0.3$. The best-fitted distribution of $ln(T)$ and $\beta$ derived from a regular Bayesian analysis shows an apparent anti-correlation. For this simulation, we adopt an prior (2x2 multivariate normal distribution) that correctly characterize the natural distribution (3x3 multivariate normal distribution) and is able to accurately recover the intrinsic correlation between $T$ and $\beta$. We caution that the natural $T - \beta$ distribution in molecular clouds is unlikely to follow a multivariate bell-shaped distribution as we simulated. However, based on a magnetohydrodynamical (MHD) simulation of molecular clouds, [Juvela et al.(2013)] has shown that a multivariate prior retains the information of the correlation coefficient given reasonable noise levels.
3.5 Fore/Background Subtraction

Now we add fore/background subtraction as an extra component to the above model. The purpose is to separate the column densities of the CMZ clouds, $N_{cmz}$, from the fore/background column densities $N_{fb}$. We assume that along each line of sight, the dust emission from the fore/background is only a function of Galactic latitude. We further assume that the fore/background column density and temperature exponentially decrease away from the Galactic plane:

$$N_{fb} = N_{fb,0} \times exp\left(-\frac{|b - b_{0N}|}{\sigma_N}\right)$$ (3.11)

$$T_{fb} = T_{fb,0} \times exp\left(-\frac{|b - b_{0T}|}{\sigma_T}\right)$$ (3.12)

where $N_{fb,0}$ and $T_{fb,0}$ are peak column density and peak temperature, $b_{0\{N,T\}}$ and $\sigma_{\{N,T\}}$ are offsets and scale heights, respectively.

The spectral index $\beta$ is fixed to 2.0 for fore/background dust emission, this value is derived from SED-fitting to the Herschel 160 – 500 $\mu m$ and Planck 353GHz maps degraded to the lowest resolution of the Planck map, which shows a uniform $\beta$, $\approx 2.0 \pm 0.04$. Notice that for this particular analysis we do not scale the Planck 353Ghz map to 1.1mm. In order to constrain $N_{fb,0}$, $T_{fb,0}$, $b_{0\{N,T\}}$ and $\sigma_{\{N,T\}}$, we defines three regions at high Galactic latitudes as “pure fore/background”, as shown in Figure 3.5: $\{0.15^\circ < l < 0.3^\circ; 0.24^\circ < b < 0.98^\circ\}$, $\{359.58^\circ < l < 359.81^\circ; 0.24^\circ < b < 0.73^\circ\}$ and $\{359.45^\circ < l < 359.84^\circ; -0.83^\circ < b < 0.42^\circ\}$. These low-flux regions are visually selected based on the 500 $\mu m$ and 1.1 mm maps. $N_{bk,0}$, $T_{bk,0}$, $b_{0\{N,T\}}$ and $\sigma_{\{N,T\}}$ are only constrained by dust emissions in these three regions, assuming there is no CMZ component. The best-fitted fore/background column density and temperature distributions are listed in Table 3.2. The quality of this fitting is shown in Figure 3.6. The scale height of the fore/background column density is $\approx 100$ pc. In comparison, the CO disk has a scale height of 60-110 pc in the inner Galaxy. Note
Figure 3.5. Herschel 500 $\mu$m map. The white rectangles show high-latitudes regions used to constrain fore/backgrounds.

Figure 3.6. The best-fitted exponential background (red-crosses) compared with observed $\pm 1\sigma$ flux densities around the median values at different latitudes (blue-shaded area), data points are gathered from the three selected “pure fore/background” regions used for fitting.
Table 3.2. Fore/Background dust column density and temperature distributions

<table>
<thead>
<tr>
<th>lg(N)</th>
<th>peak</th>
<th>offset</th>
<th>scale height</th>
</tr>
</thead>
<tbody>
<tr>
<td>cm$^{-2}$</td>
<td>22.15</td>
<td>-0.0547°</td>
<td>-0.452°</td>
</tr>
<tr>
<td>T</td>
<td>23.57[K]</td>
<td>-0.1911°</td>
<td>1.965°</td>
</tr>
</tbody>
</table>

that although the peak temperature shows a significant offset from the zero latitude and this offset is suspicious, the scale height of the temperature is very large, which results in flat temperature distribution.

The above approach of background subtraction is performed in the \( \{N, T\} \) space rather than on each flux map, such as that used by [Battersby et al. (2011)]. In this way, we take advantage of the knowledge that flux densities in different bands are correlated to follow an approximated modified black-body SED. It could be dangerous to apply background subtraction on individual flux maps here since we are essentially extrapolating high galactic latitudes emission to low galactic latitudes, small temperature offsets could lead to significant flux offset at 160 \( \mu m \).

If a CMZ component is present, the total flux along any line of sight is the sum of the CMZ component and the fore/background component. With self-shielding being accounted for, the total flux is:

\[
F_{tot} = F_{bg} \times \exp(-\tau_{cmz} - \tau_{fg}) + F_{cmz} \times \exp(-\tau_{fg}) + F_{fg}
\]

(3.13)

where \((F_{bg}, F_{cmz}, F_{fg})\) are unextincted flux, corresponding to the background, the CMZ and the foreground component, respectively. We can further assume that the fore/background emissions along each line of sight are identical: \(F_{bg} = F_{fg} = (1 - \exp(-\frac{1}{2}\tau_{fb}))B_{\nu}(T_{fb}), \tau_{bg} = \tau_{fg} = \frac{1}{2}\tau_{fb}\).

For the purpose of MCMC sampling, it is more convenient to use the integrated column density, \(N_{tot}\) along each line of sight as a free parameter, instead of \(N_{cmz}\). In this way, a smoothness prior could be applied to \(N_{tot}\), and we set \(\sigma_{lg(N_{tot}[cm^{-2}])}=0.2\).
\[ F_{\text{tot}} = \begin{cases} F(N_{\text{tot}}, \{T, \beta\}_{fb}), & \text{if } N_{\text{tot}} \leq N_{fb} \text{ (a)}, \\ F(\{T, N, \beta\}_{bg}) \times \exp(-\tau_{\text{cmz}} - \tau_{fg}) \\ + F(\{T, N, \beta\}_{cmz}) \times \exp(-\tau_{fg}) \\ + F(\{T, N, \beta\}_{fg}), & \text{if } N_{\text{tot}} > N_{fb} \text{ (b)}, \end{cases} \]

where \( F(\{T, N, \beta\}) = (1 - \tau(N, \beta))B_{\nu}(T), N_{cmz} = N_{\text{tot}} - N_{fb}, \tau_{bg} = \tau_{fg} = \frac{1}{2} \tau_{fb}, N_{bg} = N_{fg} = \frac{1}{2} N_{fb}, T_{bg} = T_{fg} = T_{fb} \) and \( \beta_{bg} = \beta_{fg} = 2.0 \). Note that fore/background flux are not completely fixed to extrapolated fluxes from high latitudes region. In (a), while \( T \) is always fixed to extrapolated values \( T_{fb} \) from high latitudes region and \( \beta \) is fixed to \( \beta_{fb} = 2.0 \), \( N_{\text{tot}} \) could take values smaller than \( N_{fb} \). In other words, low column density pixels are not elevated to the average extrapolated fore/background level.

### 3.6 Results: Single Temperature Modified Blackbody Model (STMB)

The product maps of \( T, N_{H_2} \) and \( \beta \) before and after fore/background subtraction are shown in Figure 3.7. Overall, the ranges of \( T \) and \( N_{H_2} \) are similar to those derived by [Molinari et al.(2011)] with DUSTEM. The temperature of dense clouds are typically \( \lesssim 20K \), and the peak column density \( N_{H_2} \) is \( \approx 10^{23.5} \text{ cm}^{-2} \). Below \( N_{H_2} \approx 10^{22} \text{ cm}^{-2} \), the fluxes are dominated by fore/background emission.

On large scales, there are two pronounced correlations: A negative correlation between \( N_{H_2} \) and \( T \) and a positive correlation between \( N_{H_2} \) and \( \beta \). It is notable that \( T \) and \( \beta \) maps are over-resolved (with apparent cell-to-cell fluctuations). While \( N_{H_2} \) strongly depends on the compound 1.1\( mm \) map, which has the highest resolution, \( T \) and \( \beta \) are more dependent on the entire set of multi-wavelength data. There is no straightforward way to define an “optimal” pixel scale since different parameters are
Figure 3.7. Upper: Best-fitted temperatures (upper left), column densities (upper right), $\beta$ (lower left) and optical depths at 160 $\mu$m (lower right), obtained with a single temperature, modified black-body approximation. The last map shows the integrated optical depths along the line of sight at 160 $\mu$m, which are inferred from $N_{H_2}$, $\kappa_0$ and $\beta$. The white and red circles show two objects identified as foreground objects by their $V_{lsr}$. Besides, the blue circle shows an object possibly associated with the HII region SH-20. Lower: Similar maps after fore/background subtraction. The last map is instead a smoothed map of $\beta$ with a Gaussian kernel of $\sigma = 3$ pixel.
resolved to different degrees. The cellular size of 14” is determined practically, since we find that with pixel scale smaller than 14”, the best-fitted T and $\beta$ maps are over-resolved, showing strong fluctuations between neighboring pixels, unless a stronger smoothness prior is adopted. $N_{H_2}$, on the other hand, is primarily determined by the high resolution 1.1 mm map, therefore suffers least from over-fitting.

The marginal distributions of the best-fitted $T$, $N_{H_2}$ and $\beta$, before and after fore/background subtraction, are plotted in Figure 3.8, in each panel we also plot three typical projected sampled posteriors at different locations in the parameter space. Cells with high galactic latitudes ($b < -0.19^\circ$ or $b > 0.09^\circ$) are excluded for high-lighting the CMZ region. Measurement uncertainties partially induce the apparent correlation between estimated $T$ and $\beta$, which propagate into a banana-shaped posterior distribution. However, the sampled posterior distributions suggest that a genuine anti-correlation between $T$ and $\beta$ is likely to present. The hierarchical Bayesian analysis also supports a natural $T$-$\beta$ anti-correlation. Figure 3.10 shows the $T-\beta$ distribution derived by modeling the natural $T-\beta$ distribution as a multivariate Student-t prior distribution (Section 3.4). The $T-\beta$ distribution does not significantly change other than a reduction of high-temperature cells. The estimated correlation coefficient $\rho_{\ln(T),\beta} = -0.81$ indicates a strong anti-correlation.

In Figure 3.9 we plot the histograms of the ratios between the best-fitted flux and the observed flux, $F_{fitted}/F_{observed}$. There is a systematical offset of $\approx 20\% - 25\%$ at 1.1mm. This large systematic offset in contrast with Herschel bands is partially due to more substantial uncertainties at 1.1mm. It is not clear whether this systematic offset is model-driven or due to calibration error. We will come back to this issue in Section 3.8. We do not see any systematical difference between high-density pixels and low-density pixels, which might be anticipated if the variation of $\beta$ is related to the filtering effect in the Bolocam and the AzTEC map at 1.1mm. Indeed, dense clouds should be less affected by the filtering effect. As demonstrated by Figure 3.11,
Figure 3.8. Upper: Correlations between the best-fitted values of different parameters, derived from an STMB model. Only the low Galactic latitude ($-0.19^\circ < b < 0.09^\circ$) region is shown. Dark dots represent low column density pixels $N_{H_2} < 10^{22.8}$ cm$^{-2}$ and blue filled circles represent high column density pixels $N_{H_2} > 10^{22.8}$ cm$^{-2}$. Also plotted are three typical sampled posteriors at different locations in the parameter space, with confidence intervals of 1 and 3$\sigma$. The red-dash line in $\beta$ v.s. $N_{H_2}$ plot is a best-fitted model for the relationship between $\beta$ and $N_{H_2}$, assuming a four parameter smoothly broken power-law function. The relatively isolated stripe-like feature corresponds to regions where 1.1mm emission is completely filtered out in the AzTEC map and the Bolocam map. Lower: Similar plots after fore/background subtraction.
Figure 3.9. Histograms of the ratios $F_{fitted}/F_{observed}$. Blue bars correspond to all pixels satisfying $(\log(N_{H_2}[cm^{-2}]) > 22.2)$ and red bars correspond to high column densities pixels only $(\log(N_{H_2}[cm^{-2}]) > 22.8)$. Pixels with high galactic latitudes ($b < -0.19^\circ$ or $b > 0.09^\circ$) are excluded for high-lighting the CMZ region.

Figure 3.10. $T - \beta$ distribution derived from a hierarchical Bayesian analysis (coral pink), compared with that derived from a non-hierarchical Bayesian analysis (same as Figure 3.8, black), from an STMB model with no fore/background subtraction. The contours correspond to a $3\sigma$ confidence level. By modeling the natural $\ln(T) - \beta$ distribution hierarchically as a multivariate Student-t distribution, we derive an anti-correlation of $\rho_{\ln(T),\beta} = -0.81$. 
with the 1.1mm map being removed, we performed the same analysis on the Herschel 160 – 500 $\mu$m maps. We derive a distribution of $\beta$ qualitatively identical to that in Figure 3.7. We also notice that some identified foreground objects in the CMZ shows no sign of elevated $\beta$. [Deguchi et al.(2012)] suggest that the dark cloud G359.94 is composed of two clouds in the foreground, with $V_{lsr} = 0$ km/s and 15 km/s. The comet-like feature near Sgr C complex ($l = 359.64, b = 0.24$) is associated with a foreground HII region RCW 137 [Russeil et al.(2003), Tanaka et al.(2014)] 1.8kpc away. These two regions are marked in Figure 3.7.

The CMZ is moderately optically thick at 160$\mu$m, partially due to high column densities in the CMZ and partially due to the steep slope of the dust absorption curve. The highest optical depth is $\tau_{160} \approx 1$. $\tau_{160}$ is irrelevant to our choice of the amplitude of $\kappa_0$, since $\kappa_0$ and $N_{H_2}$ are completely degenerate (exchangeable).
Figure 3.12. The dust absorption curves at different temperatures, derived from a TLS model [Paradis et al.(2014)], for the Galactic diffuse environment (dashed line) and the Galactic cold dense environment (solid lines). The power-indices $\beta$ from 500 – 1100 $\mu m$ are in the range of 1.3-1.7 for the diffuse environment and 1.0-2.1 for the cold dense environment.
3.7 Discussion: Increased $\beta$ in Dense Clumps

We identified an anti-correlation between $N_{H_2}$ and $\beta$. While this trend is qualitatively in agreement with recent observations [Dupac et al.(2003), Paradis et al.(2011), Juvela et al.(2015)]. Increased $\beta$ up to 2.5 towards density peaks could not be easily explained by existing dust models. We noticed that [Lis & Menten(1998)] reported this steep absorption curve in the CMZ based on ISO observations. The origin of this trend deserves some discussion.

We recognize that some studies on the molecular cloud “Brick” [Marsh et al.(2016), Rathborne et al.(2015)] adopted a different $\beta = 1.2$ for modeling dust emission, which is significantly lower than our results. This low value of $\beta$ was proposed based on a comparison between the Herschel 500 $\mu$m map and the Atacama Large Millimeter/submillimeter Array (ALMA) 3mm dust continuum by [Rathborne et al.(2014)], who find that by adopting $\beta = 1.2$, the scaled Herschel 500 $\mu$m map best recovers the missed large scale emission at 3 mm in the spatially filtered ALMA map. This comparison was not quantitatively detailed, and the uncertainty is not clear. Such low values would suggest grain growth and formation of millimeter size grains. [Schnee et al.(2014)] find similarly low $\beta$ ($\approx 1$) toward OMC 2/3 from a comparison between 1.2mm and 3.3mm observation, but later study by [Sadavoy et al.(2016)] suggest a higher $\beta$ ($1.7 - 1.8$) based on an independent dust SED analysis between 160 $\mu$m-2 mm, and argue that previous low $\beta$ might be due to elevated 3mm flux either by contamination or deviation from a flattened power law in the absorption curve beyond 2 mm. A flattening of dust spectral index in the millimeter portion of the SED has been reported by Herschel and Planck studies in some environments [Goldsmith et al.(1997), Planck Collaboration et al.(2011)]. The origin of such flattening is not clearly understood, potential candidates are discussed in [Planck Collaboration et al.(2011)], including 1. an extra cold dust component; 2. dust Growth in very dense regions 3. magnetic dipole emission and 4. low energy
transitions in amorphous solids. Therefore, even if the spectral index from 500 $\mu m$ to 3 $mm$ indicates a lower $\beta$ value, it does not necessarily contradict our results.

The total column densities we measured are similar to those derived by previous studies [Longmore et al.(2012), Rathborne et al.(2015)]. Taking into account that the metallicity in the Galactic Center is probably 2 times higher than the solar metallicity we assumed here [Shields & Ferland(1994), Najarro et al.(2009)], and that the column densities are likely underestimated by a factor of $<2$ using a single temperature approximation, there is a factor of $<3$ uncertainty other than the uncertainty due to measurement uncertainties.

It is not a trivial task to recover the intrinsic $T - \beta$ relation. [Shetty et al.(2009)] have discussed spurious correlation due to temperature mixing along the line of sight, however, this effect is more likely to suppress $\beta$ with additional cold components on the Rayleigh-Jeans tail, which cannot explain the increase of $\beta$ in the dense clouds. Since the Rayleigh-Jeans tail is sampled down to 1.1 $mm$, the weak dependence of the spectral slope on the temperature at long wavelength also helps us to break this degeneracy. Finally, we have shown with hierarchical Bayesian analysis that the observed anti-correlation is not likely solely due to the degeneracy.

The intrinsic spectral index of dust absorption could be changed via environment dependent dust evolution, such as dust growth (e.g., accretion & coagulation, [Kruegel & Siebenmorgen(1994), Ossenkopf & Henning(1994)], or dust destruction (e.g., shattering & sputtering, [Draine & Salpeter(1979)]). In dense molecular clouds, dust growth is usually expected due to high-frequency collision & sticking with low relative velocities. Class models of dust growth suggest that this process leads to a lowering of $\beta$ in submillimeter/millimeter wavelengths [Ossenkopf & Henning(1994), Köhler et al.(2012), Ysard et al.(2012), Ysard et al.(2013)]. A recent model developed by [Jones et al.(2013), Köhler et al.(2015)] with updated optical properties of hydrogenated carbon grains could, however, reproduce the increase of $\beta$ from FIR
to submillimeter ($\gtrsim 500\,\mu m$) by accretion of small hydrogenated carbon grains onto larger grains and further coagulation of core-mantle large grains. Still, this model does not suggest $\beta$ as high as $\gtrsim 2$. It is also questionable that coagulation could occur in the dense region in the CMZ, where the velocity dispersion is enhanced by a factor of a few [Shetty et al.(2012), Kauffmann et al.(2017A)]. Recently, [Hankins et al.(2017)] use DUSTEM to study the 3.6 $-$ 70 $\mu m$ dust SEDs of the Arched Filaments in the CMZ, and suggest a depletion of large dust grains, which is in line with our finding that there is a millimeter deficit instead of an excess.

Laboratory experiments on “astrophysically relevant dust analogs” suggest complex relationships between the FIR-mm spectral index and the chemical composition or the physical structure (e.g., amorphous v.s. crystalline) of dust grains [Boudet et al.(2005), Coupeaud et al.(2011), Demyk et al.(2017A)]. In these studies, an anti-correlation between $T$ and $\beta$ for amorphous dust is continuously reported. This behavior could be reproduced by the TLS (two-level system) model proposed by [Meny et al.(2007)], who adopt a disorder charge distribution (DCD) on the nanometer scale and two-level systems on the atomic scale to describe the optical properties of dust. The absorption due to the DCD process is temperature independent and the combined absorption due to the TLS process, including resonant absorption, tunneling, and hopping, increases with temperature. This model is later applied by [Paradis et al.(2011), Paradis et al.(2014)] to successfully reproduce the SEDs of ultracompact HII regions and cold clouds observed with Herschel/PACS & SPIRE and CSO/Bolocam. Both [Paradis et al.(2014)] and [Juvela et al.(2015)] reported an anti-correlation between $T$ and $\beta$ from large samples of cold clouds, which at least suggests that dust growth is not always a dominant factor in determining the spectral index. Our results confirm that this anti-correlation still exists in the more extreme CMZ environment.
The original TLS model is over-parameterized for it is physically motivated. [Paradis et al.(2014)] provide simplified models, describing the dust absorption curve as a function of the temperature: $Q_{abs} = Q_{abs}(\lambda, T)$, with additional three parameters related to grain physics being constrained from Far IR to submillimeter SEDs, separately, for the Galactic diffuse environment (FIRAS/WMAP) and the Galactic cold dense environment (Archeops). Their models for both environments are shown in Figure 3.12. The power-indices $\beta$ from $500 - 1100 \, \mu m$ are in the range of 1.7-1.3 for the diffuse environment and 2.1-1.0 for the cold dense environment, decreasing with increasing temperature, from $T = 10 - 40 \, K$.

We can test to what extent the TLS model could be used to describe the dust emission in the CMZ. Here we focus on dense regions, between $-0.19^\circ < b < 0.09^\circ$, and we adopt the TLS model with grain properties being constrained from the Galactic dense environments (solid lines in Figure 3.12). The best-fitted maps and the goodness of the fitting are shown in Figure 3.13. The deviation from the best-fitted model to the observations is most significant in the 1.1mm band, with a factor of close to 50% deficit. This is not surprising, while temperature mixing along the line of sight could also partially explain why the TLS model could not achieve higher $\beta$ and lower temperature, with grain properties characterizing the Galactic cold dense regions, $\beta_{500\mu m-1.1mm}$ is at most 2.1 at a temperature of 10K and 1.8 at a temperature of 20K. At shorter wavelengths, between $100 - 300 \, \mu m$, the TLS model suggest little difference for different temperatures. This result might suggest a further difference between the grain properties in the CMZ and those in typical Galactic dense environments.

Dust mass absorption coefficient at millimeter wavelengths could also be lowered if dust grains have a crystalline structure (Agladze 1996, Mennella 1998). However, the formation of crystalline dust grains usually requires condensation or annealing process with $T \gtrsim 1000 \, K$, and crystalline silicate, with identifiable spectral features, are primarily identified in the circumstellar environment, occasionally in diffuse ISM.
Figure 3.13. Upper:The best-fitted column densities and temperatures derived from a TLS model with grain properties resembling those in the Galactic cold dense environment. Lower:The flux ratios $F_{\text{fitted}}/F_{\text{observed}}$ for best-fitted TLS models. Blue bars correspond to $\log(N_{H_2}[\text{cm}^{-2}]) > 22.2$ pixels and red bars correspond to pixels with $\log(N_{H_2}[\text{cm}^{-2}]) > 22.8$ only. High galactic latitude regions ($b < -0.19^\circ$ or $b > 0.09^\circ$) are excluded for high-lighting the CMZ region.
where shocks are present [Wright et al.(2016)]. Furthermore, it is expected that crystalline dust undergoes amorphization in the ISM environment [Kemper et al.(2004)]. It is then questionable how in dense clouds crystalline structure could be dominant. We conclude that an intrinsic $T - \beta$ anti-correlation, as well as a possible impediment to dust growth or shattering of large dust grains in the turbulent environment in the CMZ, remains the most plausible candidate scenario.

### 3.8 Discussion: Dust Absorption Curve


$$
\kappa_\lambda = A\left(\frac{\lambda}{\lambda_t}\right)^{-\beta_1}\left\{\frac{1}{2}\left[1 + \left(\frac{\lambda}{\lambda_t}\right)^{\frac{1}{2}}\right]\right\}^{(\beta_1 - \beta_2)\delta}
$$

(3.15)

For simplicity, we fix $\delta$ to 0.1, this leads to a sharp transition from $\beta_1$ to $\beta_2$ at wavelength $\lambda_t$. Limited by the five-bands SEDs, $\beta_1$ is almost complete degenerate with the dust temperature. We fix $\beta_1$ to a value of either 1.5 or 2.0 to investigate its impact on the measurement of the temperature and the column density. To fit this new $\kappa_\lambda$ to the CMZ maps, again we apply smoothness priors to the parameter grids to avoid over-fitting. We set $\sigma_{\beta_2} = 0.2$ and $\sigma_{\lambda_t} = 100\ \mu m$. The best-fitted parameter distributions for a $\beta_1 = 2.0$ are shown in Figure 3.14. We find that $\beta_2 = 2 - 3$ and $\lambda_t \approx 500\ \mu m$ across the map, which is within the range of those suggested by recent experimental studies on astrophysically relevant dust analogs [Boudet et al.(2005), Coupeaud et al.(2011), Demyk et al.(2017A), Demyk et al.(2017B)], a summary is
Figure 3.14. Best-fitted parameters derived from a model with a broken power-law dust absorption curve. Upper left: column densities. Upper right: dust temperature. Middle left: $\beta_1$ (fixed to 2). Middle right: $\beta_2$. Lower: transition wavelengths $\lambda_t$. The white circles again show two objects identified as foreground objects by their $V_{lsr}$. Besides, the blue circle shows an object possibly associated with the HII region SH-20.
Figure 3.15. Upper: The differences between column densities derived from single and broken power-law dust absorption curves. Lower: The ratios between dust temperature derived from single and broken power-law dust absorption curves. The comparison is limited to low Galaxy Latitudes $-0.19 < b < 0.09$. The parameter maps are smoothed with a Gaussian kernel of $\sigma = 1$ pixel before comparison.
given by Table 1 in [Demyk et al. (2013)]. Figure 3.15 provides a comparison between the temperature and column densities derived with single/broken power-law absorption curves. This comparison illustrates that temperature determination is very sensitive to the assumption of $\kappa_{\lambda}$. The temperatures derived with a $\beta_1 = 1.5$ are systematically higher by $20 - 30\%$ compared to those derived with a $\beta_2 = 2.0$, as a result of the reduced absorption coefficient at short wavelengths, which also leads to lower optical depths at $160 \, \mu m$. Interestingly, this could provide a potential explanation to the discrepancy between the dust temperature and the gas temperature in the CMZ.

### 3.9 Absolute Calibration Uncertainties

One implication of the above discussion is that the deviation of the dust model from reality could be misinterpreted as other factors. Another commonly encountered situation is when absolute calibrations are accounted for during the fitting.

To illustrate this problem, we carry out simulations on dust SEDs, with a temperature variation along the line of sight. The temperature profile is exponential, $T = T_{out} e^{-\tau_{160\mu m}/\tau_0}$. $T_{out}$ is fixed to $30 \, K$. $\tau_{160\mu m}$ is the optical depth at $160 \, \mu m$. The inner temperature follows a circularly symmetric profile, linearly increasing from $15 \, K$ to $30 \, K$ from the center of the map to the corners of the map. The column density decreases from $lg(N_{H_2}[cm^{-2}]) = 23.2$ to 22.2 from the center to the corners. The scale length for the exponential temperature profile, $\tau_0$, is fixed to 0.2. The parameters are chosen to represent a typical dense cloud in the CMZ. The distribution of $N_{H_2}$, $T_{in}$, $T_{out}$ and the mean temperature integrated over each line of sight, $T_{avg}$, are shown in the upper panels of Figure 3.16.

The simulated dust SEDs are fitted with an STMB model with a single power-law absorption cure, but with absolute calibration offset introduced as additional
free parameters for each band [Kelly et al.(2012)]. This is achieved by rewriting the posterior:

$$\log P(x) \propto \sum_{j,ix,iy} \frac{(\text{Mod}(x,ix,iy,\nu_j) - \text{Map}(ix,iy,\nu_j))^2}{2 \times \sigma_{ix,iy,\nu_j}^2}$$ \hspace{1cm} (3.16)

as:

$$\log P(x) \propto \sum_{j,ix,iy} \frac{(\text{Mod}(x,ix,iy,\nu_j) - C_j\text{Map}(ix,iy,\nu_j))^2}{2 \times \sigma_{ix,iy,\nu_j}^2}$$ \hspace{1cm} (3.17)

where j refers to the jth band, \textbf{Mod}(x,\nu_j) is the model map, x refers to \((N_{H_2}, T, \beta)\), and \textbf{Map}(ix,iy,\nu_j) is the observed map. \(C_j\) is the calibration offset in the jth wavelength band and \(\log(C_j) \sim N(0, \sigma_{\text{cal},j})\). \(N(0, \sigma_{\text{cal},j})\) is a normal distribution centered at 0. Since the calibration offsets for the three SPIRE bands are correlated, and we further eliminate the degree-of-freedom corresponding to the normalization of the SED, we have only two calibration offsets left as free parameters, for which we choose \(C_0(160\mu m)\) and \(C_4(1.1mm)\). The calibration uncertainty \(\sigma_{\text{cal},j}\) is 10% for the 160 \(\mu m\) band [Gordon et al.(2014)] and 10% for the 1.1 \(mm\) band. The latter is estimated from a compact AzTEC source G0.378+0.04, observed as a calibrator for a monitoring program by AzTEC on the Sgr A*.

As a first test, we fix simulated \(\beta\) to 2. The only discrepancy between the model and the simulated SEDs is the temperature mixing. The simulated 160 \(\mu m\) - 1.1 \(mm\) SEDs have no intrinsic calibration offsets.

We fit an STMB with a single power-low dust absorption curve to the simulated SEDs. The absolute calibration offsets are modeled as free parameters. The best fits indicate small absolute calibration offsets, 2% at 160 \(\mu m\) and 3% at 1.1 \(mm\). The offsets of the best-fitted parameters from their true values are shown in the lower panels of Figure 3.16. The simulated parameter distributions are well recovered, which is not surprising since this toy model has only a moderate temperature variation.
As a second test, we change the intrinsic dust absorption curve to a broken power-law, with $\beta_1 = 1.4$ for $\lambda < \lambda_t = 400 \mu m$ and $\beta_2 = 2.0$ for $\lambda > \lambda_t$. Again, we fit an STMB to this new simulation. The results are shown in Figure 3.17. The simulated parameter distributions are now poorly recovered, especially when absolute calibration offsets are modeled as free parameters (Lower panels). In this case, we derive an absolute calibration offset of 30% at 160 $\mu m$ and 3% at 1.1mm. The temperature is underestimated by $7 - 8K$, and $\beta$ is overestimated by 0.3.

We conclude that as long as the dust absorption curve is not clearly understood, it is dangerous to account for absolute calibration uncertainties simply as additional free parameters during SED analysis.

### 3.9.1 Summary

To explore dust properties in the CMZ, we combined the AzTEC 1.1 mm map with existing Herschel and Plank surveys from 160 $\mu m$ to 1.1 mm and carried out...
Figure 3.17. Continue from Figure 3.16, offsets of the best fitted STMB parameters from their true values, when SEDs are simulated with a broken power-law dust absorption curve. Best fitted values are derived with upper: absolute calibration uncertainties not accounted for, and lower: absolute calibration uncertainties accounted for. The distributions of the parameters in this simulation are identical to those presented in the first row of Figure 3.16, except that \( \kappa(\lambda) \) follows a broken power-law, with \( \beta_1 = 1.4 \) and \( \beta_2 = 2.0 \), and a transition wavelength at 400 \( \mu m \).
a joint SED analysis. We develop an MCMC analysis tool which incorporates the knowledge of the PSFs to improve the spatial resolution. Equipped with this technique, we can explore the spatial variation of the dust properties, such as the column density, the dust temperature, and the dust spectral index $\beta$ in the CMZ. We find that:

1) The spectral index $\beta$ of the dust absorption curve increases from 1.8 to 2.5 from intermediate column densities ($N_{H_2} \approx 22.5 \text{ cm}^{-2}$) to high densities ($N_{H_2} \approx 23.5 \text{ cm}^{-2}$). We confirm with hierarchical Bayesian analysis that this correlation is not due to model degeneracy. We also derive a similar distribution of $\beta$ by only using Herschel/Planck maps. Furthermore, we notice an absence of increased $\beta$ toward foreground dense clouds in the same field. Therefore, the increase of $\beta$ is likely genuine.

2) The positive correlation between $N_{H_2}$ and $\beta$ can be qualitatively, but not quantitatively, explained by contemporary dust models. This correlation could also be partially owing to a lack of dust growth, or even shattering due to the grain-grain collision in a turbulent environment. The high $\beta$ values cannot be reproduced by either the dust growth model [Köhler et al.(2012), Köhler et al.(2015)] or the TLS model [Meny et al.(2007), Paradis et al.(2014)].

3) The inferred dust temperature is strongly dependent on the assumed dust absorption curve. We show that, with different (but both reasonable) assumptions for the dust absorption curve, the column density could differ by $\approx 0.2$ dex and the temperature could differ by $\approx 50\%$. This model uncertainty might be partially responsible for the decoupling between the gas temperature and the dust temperature previously observed in the CMZ (e.g., [Krieger et al.(2017)]). We also demonstrate by simulations that when a broken power-law absorption curve is present, incorporating
absolute calibration offsets into the SED analysis could lead to misinterpretation of the physical parameters.
So far, we have only considered a conventional, simple dust model and focused on the optical properties of dust grains in the CMZ. We developed a Bayesian framework that allows us to extend our study to more complex models. In this chapter, we propose two new models, in order to resolve the temperature and density structures of the CMZ molecular clouds.

4.1 Line-of-Sight Temperature Decomposition

The STMB model assumes that a luminosity-weighted average temperature could characterize the dust SED. It has been shown in Section 3.6 that CMZ clouds are moderately optically thick at 160 µm, with $\tau_{160\mu m} \lesssim 1$. This is a critical range of optical depth since a hot core inside a moderately optically thick shell could be potentially differentiated from a cold core. In this section, we explore this possibility by proposing a modified blackbody model with multi-temperature line-of-sight components (hereafter MTMB), which takes into account temperature mixing and self-shielding.

The density and temperature profile from the surface to the center of a cloud are simplified to empirical functional forms. We adopt a plummer-like density profile, which is expressed as:
\[ n_{H_2}(r) = \left[ \frac{n_0}{1 + (\frac{r}{r_f})^2} \right]^{p/2} \] (4.1)

This density profile has a central flat radius \( r_f \) and an exponent \( p \) at large radii. This profile is originally used for globular clusters, but has been extended to studies of protostellar cores and dense filaments [Plummer(1911), Kacharov et al.(2014), Myers(2017)]. A hydrostatic isothermal cylinder has \( p = 4 \) [Ostriker(1964)], observations of dense filaments show typical values of \( p = 1.5 - 2.5 \).

We further assume a polytropic equation of state for the dust temperature:

\[ T = T_0 \left[ \frac{n_0}{n_{H_2}(r)} \right]^{\gamma - 1} \] (4.2)

Notice that \( T \) is \( T_{dust} \), and there is a known discrepancy between the dust temperature and the gas temperature in the CMZ. It is still debated whether gas and dust are completely decoupled [Clark et al.(2013), Krieger et al.(2017)]. For example, [Krieger et al.(2017)] find a weak positive correlation between \( T_{dust} \) and \( T_{gas} \). Nevertheless, this discrepancy does not prevent us from using this functional form for the temperature profile of dust.

The integrated flux density along the line of sight at a given frequency \( \nu \) is:

\[ F_\nu = \int_{-r_{100}}^{r_{100}} \mu m_H n_{H_2}(r) \times 0.01 \times \kappa_0\left( \frac{\nu}{\nu_0} \right)^\beta B_\nu(T(r))S_\nu e^{-\tau_\nu(r)} dr \] (4.3)

where \( B_\nu \) is the Planck function, \( S_\nu \) is the area of the beam at frequency \( \nu \).

If \( T \) is a constant, this equation converges back to an STMB. The integration is performed over the range \((-r_{100}, r_{100})\), where \( r_{100} \) is the radius where \( n_{H_2} \) decreases down to \( n_{H_2}(r_{100}) = 100 \text{ cm}^{-3} \). The density and temperature profiles are symmetric: \( n(r) = n(-r), T(r) = T(-r) \). \( \tau_\nu(r) \) is the optical depth at frequency \( \nu \) and radius \( r \).
\[
\tau_\nu(r) = \int_r^{r_{100}} \mu m_H n_{H_2}(r) \times 0.01 \times \kappa_0(\frac{\nu}{\nu_0})^\beta dr \tag{4.4}
\]

This model has 5 free parameters \((n_0, r_f, T_0, p \text{ and } \gamma)\), which subjects to complex
degeneracies. First, \(p\) and \(\gamma\) are almost completely degenerate, \(F_\nu\) in Eq 4.3 can be
alternatively expressed as a function of \(T(\tau_{\nu,r})\), where \(\tau_{\nu,r} = \tau(\nu, n(r))\). However, we
choose not to express our model as a function of \(T(\tau_{\nu,r})\), since this profile should be
flattened when \(r \to 0\) and truncated when \(r\) is large. However, it is not clear where
(at which \(\tau_{\nu,r}\)) \(T(\tau_{\nu,r})\) should be flattened or truncated. We choose to fix \(p\) to values
inferred from [Kauffmann et al.(2017B)], who derive a constant exponent of \(\approx 1.3\) for
the density profile of most of the clouds in our map, except Sgr C, which shows an
exponent of 2.0. Their analysis is based on the projected column density distribution
inferred from the Submillimeter Array (SMA) and the Herschel observations, on scales
of \(0.1 – 10\) pc. The line-of-sight density exponent might differ from the above values
due to tidal-compression [Kruijssen et al.(2019)]. By fixing \(p\), any variation in density
profiles is transformed into a variation of \(\gamma\). We further fix \(r_f\) to 0.01 pc, as \(r_f\) is also
degenerate with \(p\) and \(\gamma\), which simplifies our density profile to a power-law profile
down to the size of a protostellar core. Therefore, our model is left with three free
parameters: \(n_0, T_0\) and \(\gamma\).

In practice, integration over the line of sight is computationally slow since nested
numerical integration is performed. SEDs are pre-calculated and tabulated in a fits
file. During MCMC analysis, a 3-dimensional interpolation is applied to recover SEDs
from this table for any given combination of parameters. The range and the number
of nodes for each parameter are summarized in Table 4.1.

The above model is added to the global fore/background determined previously
in Section 3.5. Meaning that \(F_{cmz}\) and \(\tau_{cmz}\) in Eq 3.13 is replaced by \(F_\nu\) and \(\tau_\nu\)
given by Eq 4.3 and Eq 4.4. Since it is only possible to disentangle the line-of-
sight temperature structure under optically thick conditions, we apply this model to
Table 4.1. Pre-calculated SED table for line-of-sight density/temperature decomposition

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scale</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
<th>Number of Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_0$</td>
<td>$\log_{10}$</td>
<td>$lg(n_0[cm^{-3}]) = 3.5$</td>
<td>$lg(n_0[cm^{-3}]) = 7.0$</td>
<td>30</td>
</tr>
<tr>
<td>$T_0$</td>
<td>$\ln$</td>
<td>$T = 5K$</td>
<td>$T = 80K$</td>
<td>50</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Linear</td>
<td>$\gamma = 0.7$</td>
<td>$\gamma = 1.3$</td>
<td>30</td>
</tr>
</tbody>
</table>

cells with column densities derived from a fore/background subtracted STMB model $N_{H_2,cmz} > 10^{22.6} \text{ cm}^{-2}$, which correspond to $\tau_{160} \approx 0.1$ (Section 3.6). Still, due to the PSF dilution, for cells just above this threshold, we need to account for the contribution to the flux from its neighboring cells below the threshold. The SEDs for the cells below the threshold are adopted from the best-fits of the STMB model. We further apply a smoothness priors on $\gamma$ to avoid overfitting: $\sigma_{\gamma} = 0.02$. For quiescent molecular clouds and filaments, $0.9 \lesssim \gamma \lesssim 1$ [Scalo et al.(1998), Spaans & Silk(2000), Palmeirim et al.(2013)].

To account for the variation of $\beta$, we approximate $\beta$ as a function of $N_{H_2}$, based on the results from the STMB model(Section 3.6). This function is assumed to be a four-parameters smoothly broken power-law:

$$\beta = A\left(\frac{N_{H_2}}{N_b}\right)^{-\alpha_1}\left\{\frac{1}{2}[1 + \left(\frac{N_{H_2}}{N_b}\right)^{\delta}]\right\}^{(\alpha_1 - \alpha_2)\delta}$$ (4.5)

where $A$ is a normalization factor, $\alpha_1$ and $\alpha_2$ are two power law indices, $N_b$ is the transition column density and $\delta$ is a smoothness factor controlling the transition from $\alpha_1$ to $\alpha_2$, $\delta$ is fixed to 2. We ignored correlated uncertainties between $\beta$ and $N_{H_2}$, it is shown that (Figure 3.8) the correlation is weak over the entire range of parameter space.

4.2 Results: Line-of-Sight Temperature Decomposition

Figure 4.1 shows the best-fitted parameters derived from the MTMB model, assuming a plummer-like density profile and polytropic temperature. Overall, $n_0$ has a
similar large scale distribution to that of $N_{H_2}$ derived from the STMB model. However, this new model reveals more detailed information about temperature structures in the maps of $T_0$ and $\gamma$.

To interpreted these results, it should be first noted that all parameters are degenerate. The marginal distribution and posteriors of $n_0$, $T_0$ and $\gamma$ are plotted in Figure 4.2. Under the optically thin condition, the uncertainty of $\gamma$ approaches $\infty$. Toward higher optical depth, a degeneracy emerges between the interior temperature $T_0$ and the temperature exponent $\gamma$. The apparent spatial correlation between $T_0$ and $\gamma$ could be partially due to this degeneracy. Again, we address this issue with a hierarchical Bayesian analysis, by adopting a multivariate Student-t distribution as a prior for the natural distribution of $T_0$ and $\gamma$. Similar to Eq 4.6-Eq 4.8, we have:

\begin{equation}
\begin{align*}
P(ln(T_{0,i}), \gamma_i|D) &= P(D|ln(T_{0,i}), \gamma_i)P(ln(T_{0,i}), \gamma_i|\mu, \Sigma) \\
P(ln(T_{0,i}), \gamma_i|\mu, \Sigma) &\propto \frac{1}{|\Sigma|^{1/2}} \times [1 + \frac{1}{d}(x_i - \mu)^T\Sigma^{-1}(x_i - \mu)]^{-(d+2)/2} \\
x_i &= (ln(T_{0,i}), \gamma_i)
\end{align*}
\end{equation}

where $D$ is the data, $\mu$ is the global mean of $(ln(T_{0,i}), \gamma_i)$. $\Sigma$ is the covariance matrix of $(ln(T_{0,i}), \gamma_i)$. The degrees of freedom $d = 8$. Along with the prior in Eq 4.6, a smoothness prior is applied to $N_{H_2}$, $\sigma_{lg(N_{H_2}|cm^{-2})} = 0.1$.

Figure 4.3 shows the $T_0 - \gamma$ distribution derived from the hierarchical Bayesian model. It seems that the positive correlation between $T_0$ and $\gamma$ is real, with a correlation coefficient of 0.88. The range of $\gamma$ is $0.85 - 1.0$ for dense clouds, which agree with values for quiescent, non-star forming clouds [Scalo et al.(1998), Spaans & Silk(2000), Palmeirim et al.(2013)]. A positive correlation between $T_0$ and $\gamma$ is physically ex-
Figure 4.1. Best-fitted maps of interior number density $n_0$, interior dust temperature $T_0$, and temperature exponent $\gamma$. The density exponent $p$ is fixed to values inferred by [Kauffmann et al.(2017B)]. The pixel scale is $14''$, corresponding to 0.5 $pc$. The dense CMZ clouds generally show shallow, negative temperature gradients on such a scale, with $0.85 < \gamma < 1$. The red circles mark CMZ clouds with the strongest evidence of ongoing star-formation: Sgr B1-off, Dust Ridge C, 20 km/s cloud and Sgr C, from left to right [Lu et al.(2015), Walker et al.(2018), Lu et al.(2019)]. No strong evidence for internal heating is present in the maps of $T_0$ and $\gamma$, however, a trend of increasing $T_0$ and $\gamma$ can be perceived along the “dust-ridge” (from the Brick to Sgr B1-off).
Figure 4.2. Correlations between the best-fitted values of different parameters, derived from a model with a plummer-like density and temperature profiles (MTMB). Only the low Galactic latitude (−0.19° < b < 0.09°) region is shown. Also plotted are three typical sampled posteriors at different locations in the parameter space, with confidence intervals of 1 and 3σ.

Expected because higher interior temperature $T_0$ could indicate either an internal heating source or a smaller optical depth if the cloud is heated externally, both result in a shallower temperature gradient and thus $\gamma$ closer to or larger than 1. We also show in Figure 4.4 that the global distributions of $T_0$ and $\gamma$ derived from a hierarchical Bayesian model are similar to that in Figure 4.1.

There is a trend of increasing $\gamma$ along the “dust-ridge”, from the Brick to Sgr B1-off. This trend is in agreement with the previous study by [Krieger et al.(2017)] on NH$_3$(3,3) temperature distribution. Higher $T_0$ and $\gamma$ values are also observed in the 20 km/s cloud and toward the very dense peak of Sgr C, these two clouds, along with Sgr B1-off, are the three clouds with highest ongoing star-formation rates [Lu et al.(2019)] in our field. It should be noted that the best-fitted parameters for Sgr C have large uncertainties. Sgr C has a steep density profile, the model flux in the density peaks are contaminated by the neighboring cells below the $N_{H_2} = 10^{22.6}$ cm$^{-2}$ threshold due to the PSF dilution. Since the density exponent $p$ is fixed, increasing $\gamma$ up to one indicates either a shallower temperature gradient or a shallower density
Figure 4.3. $\ln(T_0) - \gamma$ distribution derived from a hierarchical Bayesian analysis (coral pink), compared with that derived from a non-hierarchical Bayesian analysis (same as Figure 4.2, black), for a MTMB model with background subtraction. The dash contours correspond to a $3\sigma$ confidence level. By modeling the $\ln(T_0) - \gamma$ distribution hierarchically with a multivariate Student-t distribution, we derive a positive-correlation of $\rho_{\ln(T_0),\gamma} = 0.88$. 
Figure 4.4. Same as Figure 4.1, but derived from a hierarchical Bayesian model with $ln(T_0)$ and $\gamma$ following a multivariate Student-t prior distribution. A smoothness prior is applied to $N_{H_2}$, $\sigma_{lg(N_{H_2}[cm^{-2}])} = 0.1$. 
gradient than what is assumed ($p = 2.0$ for Sgr C and $p = 1.3$ for everywhere else). Higher interior temperature could explain the apparent high $\gamma$ values in star-forming clouds, but geometry effects can also be important. For example, it is possible that the high column density of the Brick is partially a geometry effect. [Mills et al.(2018)] find that the dense gas fraction of the Brick is slightly smaller than that of the 50 km/s and the 20 km/s clouds. [Kruijssen et al.(2019)] suggest that the Brick could have a “pancake” shape due to tidal compression triggered during its last pericenter passage. Nevertheless, it is encouraging to see that the spatial variations of $T_0$ and $\gamma$ are consistent with existing observations.

In Figure 4.5, we carry out a cell-by-cell comparison between parameters derived from the STMB and the MTMB model. Before comparison, all parameter maps are median-filtered with a $3 \times 3$ kernel. $N_{H_2, \text{plummer}}$ are systematically higher than $N_{H_2, \text{sing-T}}$ by $0.1 - 0.2$ dex at both low and high column densities. This offset is a natural result of the temperature decomposition since the temperature derived with the STMB is luminosity-weighted, higher temperature components always gain more weights as $L \propto T^4$. The luminosity-weighted temperature is therefore always higher than the mass-weighted temperature. The lower panel of Figure 4.5 shows that $T_{\text{sing-T}}$ is bracketed by the interior temperature $T_0$ and the outer temperature at a density of $100 \text{ cm}^{-3}$. $T_0/T_{\text{sing-T}}$ is relatively constant from low to high column densities, in agreement with Figure 4.2, showing that $\gamma$ does not evolve with $N_{H_2}$.

4.3 A Hierarchical Analysis on the the Probability Distribution Function of the Column Density (N-PDF)

In this section, we study the N-PDF of the CMZ clouds. We assume a power-law functional form for the N-PDF at high column densities, as expected from typical gravitational collapse processes, and a log-normal functional form for the N-PDF at
Figure 4.5. Upper: A comparison between column densities derived from the STMB model and the MTMB model. The temperature exponent $\gamma$ satisfy $\gamma < 1$ for almost all cells. Therefore, $\gamma$ closer to 1 indicate a smoother temperature profile as a function of the radius. Lower: The ratios between dust temperatures derived from the STMB and the MTMB model. For the MTMB model, two temperatures are considered. The interior temperature $T_0$ and the outer temperature $T_{100}$ at a density of 100 cm$^{-3}$. All parameter maps are smoothed with a $3 \times 3$ median filter before comparison.
low column densities, as expected from a turbulent environment. This functional form can be expressed as [Myers(2015), Burkhart et al.(2017)]:

\[
P(\ln(N_{cmz})|P_{\text{high}}) = \begin{cases} 
A_1 \times \exp(-u^2), & \text{if } \ln(N_{low}) < \ln(N_{cmz}) < \ln(N_t), \\
A_2 \times \exp\{-\Gamma[\ln(N_{cmz}) - \ln(N_t)]\}, & \text{if } \ln(N_{cmz}) > \ln(N_t), 
\end{cases}
\]

(4.9)

where \( u = \frac{\ln(N_{cmz}) - \ln(N_m)}{\sqrt{2}\sigma_{\ln}} \). \( P_{\text{high}} = (N_m, \sigma_{\ln}, N_t, \Gamma) \) are hyperparameters for the N-PDF. \( \ln(N_{cmz}) \) is the column density of the CMZ component in logarithmic scale. \( \ln(N_{cmz}) = \lg(N_{cmz}) \times \ln(10) \). \( \ln(N_m) \) and \( \sigma_{\ln} \) are the mean and the standard deviation for the log-normal portion of the N-PDF, \( N_t \) is the transition column density from the log-normal portion to the power law portion. \( N_{low} \) is the lower limit of the column density at which the N-PDF is truncated. We arbitrarily set \( N_{low} \) to \( 10^{21.2} \) cm\(^{-2} \), which is \( \lesssim 1\sigma \) uncertainty of the column density before background subtraction.

Since \( N \) is in logarithmic scale, the power-law PDF:

\[
P(N_{cmz}) \propto (N_{cmz}/N_t)^{-\alpha}
\]

becomes:

\[
P(\ln(N_{cmz})) \propto \exp[-\Gamma(\ln(N_{cmz}) - \ln(N_t))]
\]

(4.11)

where \( \Gamma = \alpha - 1 \).

The integration \( \int_{-\infty}^{+\infty} P(\ln(N_{cmz}))d(\ln(N_{cmz})) = 1 \) and condition of continuity at \( \ln(N_t) \) require that:
\[ A_1 = \left\{ \frac{\pi}{2} \right\}^{\frac{1}{2}} \sigma_{lgn} \left[ erf(u_t) - erf(u_{low}) \right] + \frac{1}{\Gamma} \text{exp}(-u_t^2) \right\}^{-1} \]

\[ A_2 = A_1 \text{exp}(-u_t^2) \]

where \( u_t = \frac{\ln(N_t) - \ln(N_m)}{\sqrt{2}\sigma_{lgn}} \) and \( u_{low} = \frac{\ln(N_{low}) - \ln(N_m)}{\sqrt{2}\sigma_{lgn}} \). Normally, the assumed functional form of the N-PDF is fitted to the histogram of the column densities [Ellsworth-Bowers et al. (2015), Kainulainen et al. (2009)], the later can be inferred from either emission lines from various molecular species, Near IR extinction or dust SEDs. This approach has two shortcomings. First, it is not clear how measurement uncertainties propagate into the best-fitted parameters of the N-PDF, and second, the best-fitted N-PDF could suffer from the Eddington bias, meaning that there are more low column density pixels scattered to higher column density pixels than the reverse. Furthermore, the CMZ is characterized by high foreground and background dust emission. Therefore a procedure of fore/background subtraction has been introduced, which adds even more uncertainties into the inferred column densities of the CMZ clouds. With histogram fitting, one should either rely on pixels which are significantly (4-5 \( \sigma \)) above the fore/background level or try to obtain a complicated response matrix as a function of \( T, N, \beta \) through simulations.

We apply a statistically more elegant approach, hierarchical Bayesian Analysis, to infer the optimal N-PDF function. This means that Eq 4.9 is applied as a prior to the column densities during the SED fitting. By doing so, we have two levels of parameters: \((N_{i,j}, T_{i,j}, \beta_{i,j})\) as local parameters and \((N_m, \sigma_{lgn}, N_t, \Gamma)\) as hyperparameters. The full posterior is:

\[
P(p_{low}, p_{high}|Data) = P(Data|p_{low})P(p_{low}|p_{high})P(p_{high}) \tag{4.12}
\]

where \( p_{low} = (N_{i,j}, T_{i,j}, \beta_{i,j}) \), and \( p_{high} = (N_m, \sigma_{lgn}, N_t, \Gamma) \). \( P(Data|p_{low}) \) is the product of likelihoods for all cells. \( P(p_{low}|p_{high}) \) is eq(4.9), and \( P(p_{high}) \) is the prior.
Figure 4.6. Our Hierarchical Bayesian model for the N-PDF. The raw (un-convolved with PSF) flux densities \( F(160\mu m - 1.1mm, i, j) \) of each pixel are determined by the column density \( N(i,j) \), the temperature \( T(i,j) \), the dust spectral index \( \beta(i,j) \) in the corresponding grid cell. \( N(i,j) \) further follows a log-normal + power-Law prior. The raw flux densities of pixels surrounding each pixel \( (i,j) \) (within 4\( \sigma \) of the PSF) are convolved with the PSF to produce the model map flux densities \( M(160\mu m - 1.1mm) \) at \( (i,j) \).

For the hyperparameters \( p_{high} \). Here we simply adopt a trivial prior that \( P(p_{high}) \) is a multi-dimensional uniform distribution, with loose lower and upper bounds. The structure of our hierarchical model is illustrated by Figure 4.6.

We further truncate the N-PDF arbitrary at \( N_{cmz} = 10^{21.2} cm^{-2} \), this truncation column density is \( \lesssim 1\sigma \) for the faintest pixels in our map derived with an STMB model. With the fore/background components taken into account, the posterior probability Eq 4.12 becomes:
where \( \text{err}_{\nu,i} \) is the uncertainty at frequency \( \nu \) in the \( i \)th pixel. In case (b), no CMZ component is present, the posterior probability simplifies to the product of the likelihoods for all cells. In case (c), a CMZ component is present, the posterior probability is penalized by the N-PDF prior. Where \( P(\text{ln}(N_{cmz})|\{N_m, \sigma_{lgn}, N_t, \Gamma\}) \) is again eq(4.9).

### 4.4 Results: The N-PDF of the CMZ

The histogram of \( N_{cmz} \) is shown in Figure 4.7, along with the best-fitted N-PDF. The low end of the N-PDF is best-fitted with a broad log-normal function, with \( \sigma_{lgn} = 0.8 \). This log-normal portion extends to a column density \( \text{lg}(N_{cmz}[cm^{-2}]) \approx 22.4 \). We find that the mean of the log-normal portion, \( \text{lg}(N_m[cm^{-2}]) = 22.35 \), is almost equal to the transition column density \( \text{lg}(N_1[cm^{-2}]) = 22.37 \). Meaning that the entire N-PDF could be viewed as a power-law with a log-normal decay at low column densities. One should be cautious when interpreting the log-normal portion of the N-PDF. Although a log-normal N-PDF is suggested for a turbulent non-star-forming cloud [Ostriker et al.(2001)], the left side of the log-normal function can be
Figure 4.7. The histograms of the column densities for the low Galactic latitude region $-0.19 < b < 0.9$. The best-fitted N-PDF has a power-law index of 1.7. The vertical dash lines mark the transition column density from the log-normal portion to the power-law portion ($lg(N_t) = 22.4$). The blue band shows the best-fitted N-PDFs within $\pm 1\sigma$ uncertainty. Notice that the N-PDF in each bin is normalized by the total number and y-axis values are numbers in each bin.
affected by fluctuations from different sources (e.g., statistical noise, intrinsic turbulence, fore/background fluctuation). [Lombardi et al. (2015)] argued that the low end of the N-PDF could not be described by log-normal functions, but instead power-law functions with truncation at low densities. [Alves et al. (2017)] also find that the log-normal part vanishes if the PDF is defined within the last contour of the molecular clouds. Nevertheless, it is evident that above \( \ln(N_{\text{cmz}}) \approx 22.4 \), the N-PDF follows a power-law. There is also an apparent drop-off beyond \( \ln(N_{\text{cmz}}) \approx 23.5 \), suggesting that a single power-law could not describe the high column density portion of the N-PDF.

It is therefore better to model the N-PDF as a broken power-law distribution in the high end. The N-PDF, eq(4.9), then becomes:

\[
P(N_{\text{cmz}}|p_{\text{high}}) = \begin{cases} 
A_1 \cdot \exp(-u_1^2), & \text{if } \ln(N_{\text{low}}) < \ln(N_{\text{cmz}}) < \ln(N_1), \\
A_2 \cdot \exp[-\Gamma_1(\ln(N_{\text{cmz}}) - \ln(N_1))], & \text{if } \ln(N_{\text{cmz}}) > \ln(N_1) \text{ and } \ln N_{\text{cmz}} < \ln(N_2), \\
A_3 \cdot \exp[-\Gamma_2(\ln(N_{\text{cmz}}) - \ln(N_2))], & \text{if } \ln(N_{\text{cmz}}) > \ln(N_2). 
\end{cases} \tag{4.14}
\]

where \( u_1 = \frac{\ln(N_m) - \ln(N_1)}{\sqrt{2}\sigma_{\ln}} \) and \( u_2 = -\Gamma_1(\ln(N_2) - \ln(N_1)) \). Hyperparameters are \( p_{\text{high}} = \{N_m, \sigma_{\ln}, N_1, \Gamma_1, N_2, \Gamma_2\} \).

Similarly, the integration \( \int_{-\infty}^{+\infty} P(\ln(N_{\text{cmz}})) d(\ln(N_{\text{cmz}})) = 1 \) and condition of continuity at \( \ln(N_1) \) and \( \ln(N_2) \) require that:

\[
A_1 = \left\{ \frac{\pi}{2} \right\}^{\frac{1}{2}} \sigma_{\ln} \left[ \text{erf}(u_1) - \text{erf}(u_{\text{low}}) \right] \\
+ \frac{1}{\Gamma_1} \exp(-u_1^2)[1 - \exp(u_2)] + \frac{1}{\Gamma_2} \exp(-u_1^2) \exp(u_2)^{-1} \\
A_2 = A_1 \exp(-u_1^2) \\
A_3 = A_2 \exp(u_2) 
\]
Figure 4.8. Same as Figure 4.7, but the N-PDF follows a broken power-law in the high end. The best-fitted power law indices are 1.51 and 3.83 at low and high column densities. The vertical dash lines mark the two transition column densities \( \log(N_1[cm^{-2}]) = 22.3 \) and \( \log(N_2[cm^{-2}]) = 23.2 \).

where \( u_{low} = \frac{\ln(N_{low}) - \ln(N_m)}{\sqrt{2} \sigma_{\ln}} \).

The histogram and the best-fitted N-PDF are shown in Figure 4.8. We derive transition column densities \( \log(N_1[cm^{-2}]) = 22.30, \log(N_2[cm^{-2}]) = 23.20 \) and power-law indices \( \Gamma_1 = 1.51, \Gamma = 3.83 \). Again we find that \( \log(N_m[cm^{-2}]) = \log(N_1[cm^{-2}]) = 22.3 \). From our MTMB model, the transition column density \( N_2 = 10^{23} \) cm\(^{-2} \) can be converted to an interior density \( n_0 \approx 10^6 \) cm\(^{-3} \). A lack of high-density components is consistent with the recent finding by [Kauffmann et al.(2017B)] that CMZ clouds only show shallow density gradients, despite their high average densities. However, it is questionable whether the drop-off toward high densities is owing to our limited spatial resolution. While this issue should be addressed with observations at higher
resolutions, in the next section we discuss whether this transition is intrinsic by investigating the N-PDF of individual clouds.

4.5 Discussion: The N-PDFs of Individual CMZ Clouds

The N-PDF of the entire CMZ shows a power-law tail with a slope of 1.5-1.7 at intermediate column densities and a slope of $\approx 4$ above a column density of $N = 10^{23.2} \text{ cm}^{-2}$. An index of 2 is typical for star-forming clumps [Schneider et al.(2015), Myers(2015), Pokhrel et al.(2016)], which can be developed from gravitational collapse [Federrath et al.(2016)]. Although the drop-off toward high column densities could be due to poor spatial resolution [Alves et al.(2017)], we can further examine whether this behavior is universal for individual clouds, and more importantly, whether the N-PDF is related to other properties of a cloud, such as the SFR or the orbital location. An evolving sequence is predicted by the orbital model proposed by [Kruijssen et al.(2015), Kruijssen et al.(2019)], who suggest that collapse is triggered by tidal compression when clouds pass through the pericenter of the CMZ potential. In this scenario, the CMZ clouds should present an evolving sequence according to their orbital phase. This sequence starts from the Brick, which just passed the pericenter of the Galactic gravitational potential about 0.2 Myr ago. From the Brick to Sgr B2, The clouds are showing higher and higher SFR, SgrB2 marks the maximum of on-going star-formation. However, it is not clear whether the clouds at negative longitudes, especially the 20 km/s and the 50 km/s clouds, can be placed into a similar evolving sequence. For example, the 20 km/s cloud is suggested to be right before its next pericenter passage. However, it is already actively forming stars [Lu et al.(2019)]. The locations of the 20 km/s and the 50 km/s clouds are also debated. [Henshaw et al.(2016)] point out that the 20 km/s and the 50 km/s clouds are probably interacting with the circum-nuclear disk around Sgr A*.
We select a total of six CMZ clouds/complex to investigate their N-PDFs, which are: Sgr B1-off, the Brick, three-little-pigs, the 50 km/s cloud, the 20 km/s cloud and Sgr C. For the N-PDF of the entire CMZ, we find that \( N_m = N_1 \approx 10^{22.35} \text{ cm}^{-2} \). In order to simplify the discussion, we assume that the N-PDF of each individual cloud also satisfies \( N_m = N_1 \). In practice, we found that for individual clouds, the distribution of the intermediate column densities with \( N_{H_2} \approx 10^{22.35} - 10^{23} \text{ cm}^{-2} \) could usually be best-fitted as either a log-normal distribution or a power-law distribution, due to small number statistics. By fixing \( N_m = N_1 \), we only use power-law indices to quantify the slope change in the N-PDF so that cloud-cloud comparisons can be easily made.

A broken power-law N-PDF with a log-normal decay is applied as a prior for the column densities in each of the six regions, as shown in the upper panel of Figure 4.9. The lower panel shows the histograms of the column densities and the best-fitted N-PDF for each cloud/complex. In the discussion below we comment on each cloud/complex concerning the relationship between their best-fitted N-PDF and their properties, such as their density structure and SFR.

### 4.5.1 Sgr B1-off

The N-PDF above \( N = 10^{22.9} \text{ cm}^{-2} \) could be fitted as a single power-law distribution, with a flat slope \( \Gamma_2 = 2.4 \). Six \( \text{H}_2\text{O} \) and class II \( \text{CH}_3\text{OH} \) masers have been identified in this cloud [Lu et al.(2019)], four of which, along with a compact H II region identified from the VLA 1.3cm continuum (tracing free-free emission) are overlapped with the peaks of dust continuum as revealed by the SMA observation. The flat N-PDF is consistent with this picture of on-going star-formation. [Kauffmann et al.(2017B)] has estimated the density exponents from the mass-size relations for the CMZ clouds. They find that the density exponent is \( \approx 1.3 \) in SgrB1-off and most other CMZ clouds.
Figure 4.9. The histograms of the column densities and the best-fitted N-PDFs for six individual CMZ clouds. Upper: The 1100\(\mu\)m maps with white rectangles indicating the six clouds under investigation. Lower: The N-PDFs are assumed to be log-normal at low column densities and a broken power law at high column densities, with \(N_m = N_1\). The vertical dash lines mark the two transition column densities. The blue band shows the best-fitted N-PDFs within \(\pm1\sigma\) uncertainty. The N-PDF in each bin is normalized by the total number and y-axis values are numbers in each bin.
4.5.2 The Brick

The N-PDF of the Brick has been studied previously by [Rathborne et al.(2014)] using ALMA/Herschel observations and also by [Johnston et al.(2014)] using SMA and SCUBA observations. Both suggest a log-normal drop-off toward high column densities. Which is consistent with the steep slope we derived ($\Gamma_2 = 7.6$). The lack of high-density substructure is visually evident in the ALMA map. [Lu et al.(2019)] identified three H$_2$O masers, only one of which has a counterpart in the dust continuum and is on the periphery of the cloud. The N-PDF we derived is similar to that derived by [Johnston et al.(2014)], showing a flat plateau at intermediate and low column densities. The N-PDF derived with the ALMA observations by [Rathborne et al.(2014)] was not fore/background subtracted and shows a deficit of low column densities.

4.5.3 Three-Little-Pigs

The recent SMA survey of the CMZ [Battersby et al.(2017)] reveals an evolutionary sequence of density structures in this complex, a trend of increasing high-density substructures are found, from east to west, suggesting that this complex is at an initial stage of collapsing. The entire complex shows a rapid drop-off above a column density of $N = 10^{23.1}$ cm$^{-2}$, with $\Gamma_2 = 4.8$.

4.5.4 The 50 km/s Cloud

There is a complex of four compact H II regions at the south-east edge of the cloud [Mills et al.(2011)]. [Lu et al.(2019)] identified two H$_2$O masers that are likely associated with protostellar cores, only one of which overlaps with a peak in the dust continuum. The ongoing SFR traced by H$_2$O masers seems to be lower than that traced by the compact H II regions by a factor of $> 10$, suggesting that the cloud has evolved significantly within a time scale of $\lesssim 0.3$ Myr. The N-PDF shows a rapid drop-off above $N = 10^{23.0}$ cm$^{-2}$, with $\Gamma_2 = 5.5$. 

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4.5.5 The 20 km/s Cloud

The N-PDF shows a plateau up to a column density of \( N = 10^{23.2} \text{ cm}^{-2} \), and drops off with a slope of \( \Gamma_2 = 3.6 \). It is not clear whether such a plateau feature, seen also in the N-PDF of the Brick, is simply a result of beam averaging. Another possibility is that the plateau is a mixing of multiple fragments/clumps characterized by distinct mean column densities. The 20 km/s cloud has the highest SFR among the six clouds [Lu et al.(2019)]. However, in terms of SFR v.s. cloud mass, the 20 km/s cloud is similar to that of the 50 km/s cloud, Sgr B1-off and the Brick (Figure 6 in [Lu et al.(2019)]). [Kauffmann et al.(2017B)] also find that the 20 km/s cloud shows a mass-size slope similar to that in Sgr B1-off, the Brick, and the 50 km/s cloud.

4.5.6 Sgr C

Sgr C is distinct from other clouds studied here by its high SFR (by a factor of 10) relative to its gas mass [Lu et al.(2019)] and its steep density profile [Kauffmann et al.(2017B)]. Sgr C is considered as a successor of Sgr B2 in the current orbital model [Kruijssen et al.(2015)] and shows SFR per unit gas mass and mass-size slope, along with Sgr B2, similar to that in nearby molecular clouds. Its N-PDF also shows a power-law exponent typical of star-forming clouds, \( \Gamma_2 = 2.0 \) [Schneider et al.(2015), Myers(2015), Pokhrel et al.(2016)]. The slope of the N-PDF flattens above a density a column density of \( N = 10^{23.0} \text{ cm}^{-2} \), this is likely a beam averaging effect since this cloud is very compact.

Overall, we find that the slope of the N-PDF steepens at a column density of \( N = 10^{22.9-23.3} \text{ cm}^{-2} \). While limited spatial resolution could be partially responsible, a physical transition must occur. Future studies should focus on the physical origin of this transition. We find that the power-law index \( \Gamma_2 \) above the transition column den-
sity, derived from a Hierarchical Bayesian fitting to the N-PDF of the CMZ clouds at a spatial resolution of \( \approx 0.5 \) pc, serves as a robust indicator of their density structures and are consistent with their current SFR as traced by \( \text{H}_2\text{O} \) masers [Lu et al.(2019)]. The relative values of the power-law indices we derived are in good agreement with the exponents of the density profiles found by [Kauffmann et al.(2017B)]. The cloud showing the most flattened N-PDF at high densities, Sgr C, is also their cloud with the steepest density gradient. On the other hand, the two clouds with no clear sign of a power-law tail at high densities, the Brick and the 50 km/s, are also showing the lowest density gradients in [Kauffmann et al.(2017B)].

4.6 Summary

In this section, we develop two new methods based on hierarchical Bayesian Analysis to study the temperature and density structures of the CMZ clouds. We find that:

1. With a plummer-like line-of-sight density profile and an assumption of polytropic temperature, we can reasonably disentangle line-of-sight temperature profiles. Assuming density profiles inferred from the projected column density distribution by [Kauffmann et al.(2017B)], the derived temperature exponents of the CMZ clouds agree with values for quiescent molecular clouds.

2. The relative distribution of the temperature exponent \( \gamma \) is consistent with that derived from gas temperature based on \( \text{NH}_3(3,3) \) transitions. Along the dust ridge molecular clouds, we find increasing \( \gamma \) from the Brick to Sgr B1-off, suggesting either an intrinsic shallower temperature profile due to internal heating or a shallower line-of-sight density profile toward Sgr B1-off, possibly due to tidal compression. Both scenarios are in agreement with the contemporary orbital model for the CMZ clouds.
3. The column densities derived from an MTMB model is 0.1-0.2 dex higher than that derived from an STMB model. The offset is relatively constant within a range of $N = 10^{22.6-23.6}$ cm$^{-2}$. The temperature variation along the line-of-sight is roughly a factor of 2.

4. We show that the power-law index of the N-PDFs at a spatial resolution of $\approx 0.5$ pc serves as a good indicator of the density structure. The power-law indices in individual clouds are consistent with the mass-size distributions studied by [Kauffmann et al. (2017B)] and is correlated with the most recent SFR traced by H$_2$O masers. The N-PDF of the entire CMZ shows a transition column density around $N_{H_2} = 10^{23.2}$ cm$^{-2}$ to a drop-off toward higher column densities, which could be translated to an interior density $n_0 = 10^6$ cm$^{-3}$ based on the MTMB model. While beam averaging effect transition column density should not be ignored, this transition density, if intrinsic, provides a key clue about the mechanism that suppresses further collapse of the CMZ cloud.
The optical properties of dust grains and their environmental dependence remains a critical, yet poorly understood ingredient necessary for modeling the FIR-submillimeter portion of the SEDs in both the local and the distant universe. The environmental dependence is remarkable, as we see in this study. There is growing evidence that the environment dependent change of the dust absorption curve has a second order wavelength dependency. We have shown in Section 3.8 that parameter inference is strongly affected by this environmental uncertainty. The ultimate goal, therefore, is to understand how the dust absorption curve varies, as a function of both wavelength and environmental parameters, including density and turbulence. To achieve this goal, we need to expand the sample of spatially resolved dusty clouds with high density and strong turbulence, in terms of both wavelength coverage and sample size. The successor of the AzTEC, the TolTEC camera, with its three-band coverage from 1.1 mm to 2.0 mm, significantly extend the wavelength coverage of dust SEDs on the Rayleigh-Jeans tail, where $T - \beta$ degeneracy is minimal. On the other hand, a candidate sample of resolved dusty clouds under similarly extreme conditions to the CMZ is the core of nearby spiral galaxies. A pilot study has already successfully been carried out using ALMA observations [Ando et al.(2017)] toward the central 200 pc region of NGC 253. Observations of such distant targets is challenging due to the resolution limitation. The technique of SED fitting developed for this study applies to general models requiring proper incorporation of instrumental effects.
We use a simple phenomenological model with empirical density and temperature profiles for line-of-sight decomposition. A radiative transfer analysis, accounting simultaneously for the stellar radiation and the dust reprocessing, will be eventually required to obtain a contemporary understanding of both the optical properties of dust and the temperature/density distribution. Such an analysis can also provide an independent measurement of the 3-dimensional distribution of the CMZ clouds. Given that the stellar distribution in the CMZ has been studied in detail [Launhardt et al.(2002)]. The existing data sets for the CMZ from near-IR to millimeter already allow for a radiative transfer analysis of dust extinction and emission. We are further investigating hydrogen recombination lines in the submillimeter wavelengths from ALMA observations, which, in combination with recombination lines in the near-IR wavelengths mapped by the HST, also provide a constraint of dust extinction. Independent constraints from different sources should be eventually integrated into a Hierarchical Bayesian model for both dust extinction and emission for a more restrictive constraint on dust properties.
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